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A traffic responsive control framework for signalized junctions based on hybrid traffic flow representation

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***Abstract*—The paper proposes a traffic responsive control framework based on a Model Predictive Control (MPC) approach. The framework focuses on a centralised method, which can simultaneously compute the network decision variables (i.e., the green timings at each junction and the offset of the traffic light plans of the network). Furthermore, the framework is based on a hybrid traffic flow model operating as a prediction model and plant model in the control procedure. The hybrid traffic flow model combines two sub-models: an aggregate model (i.e., the Cell Transmission Model; CTM) and a disaggregate model (i.e., the Cellular Automata model; CA), using a transition cell to connect them. The whole framework is tested on a signalised arterial, performing several analyses to calibrate the MPC strategy and evaluate the traffic control approach using fixed and adaptive control strategies. All analyses are made in terms of total time spent, network total delay, queue lengths and degree of saturation.**

Keywords: traffic lights; model predictive control; network; interacting junctions; traffic flow model.

I. Background and motivation

Sustainable urban mobility is one of the main challenges facing cities in the EU and a matter of concern for many citizens. Road transport is one of the main causes of air pollution and greenhouse gas emissions in urban areas. With this in mind, diverse strategies can be adopted to mitigate traffic congestion.

The most effective approaches recognised in the literature consist in Advanced Traffic Control Strategies (Dresner & Stone, 2008; Ma et al., 2018; Yuan et al., 2019). Relevant methods are based on dynamic control through the application of traffic-responsive strategies activated by real-time flows/arrival detection; they include SCOOT, SCATS, PRODYN, RHODES, OPAC, TUC and GLADI (Dinopoulou et al., 2000; Farges et al., 1994; Gartner et al., 2001; Mirchandani & Wang, 2005; Robertson & Bretherton, 1991; Sims & Dobinson, 1980). Such adaptive strategies can follow three different approaches:

- centralised (or synchronisation): which simultaneously achieves the optimal values of all decision variables (e.g., by optimising the green timings and the offset of all junctions in the case of network optimisation);
- distributed (or coordination): which has a sequential procedure to obtain the optimal values of the decision variables (e.g., by first optimising the green timings at each junction within a network, and then the offsets for the whole network);
- decentralised: which requires only local information to optimise the decision variables of each junction independently of others on the same network.

Many centralised approaches have been proposed in the literature, most of them based on control theory. The traffic control problem considering interacting junctions (arterial and sub-networks/networks) is an integrated problem that couples the traffic signal setting with traffic flow models (Cai et al., 2009). The main applications refer to mixed-integer linear programming (Gartner et al., 1975; Jeong & Kim, 2014; Memoli et al., 2016; Xu et al., 2019), game theory (Villalobos et al.,2008), multicriteria optimisation (Li & Sun, 2019; Zhang et al., 2019; Di Pace, 2020), the reinforcement learning model (Genders & Razavi, 2019; Xu et al., 2020) and Model Predictive Control (MPC) (Aboudolas et al., 2009; Geroliminis et al., 2013; Chow et al., 2019).

Some studies on traffic control refer to decentralised methods for the low computational effort, especially back-pressure strategy (Varaiya 2013; Wongpiromsarn et al. 2012; Kulcsár et al., 2015; Lammer et al., 2006; Tassiulas & Bhattacharya, 2000) in which the algorithm time complexity is very low. The literature also proposes contributions in decentralised control (Zaidi et al., 2016; Le et al., 2015). However, to the best of the authors' knowledge, few studies have dealt with distributed control, making it worthy of investigation (Chow and Sha, 2016). Finally, some studies focus on the comparison between centralised and decentralised control approaches (Chow et al., 2019) to i) quantify the gap between them and ii) identify the specific conditions (e.g., network layout, path choice behaviour, etc.) to reduce this gap. Moreover, considering the gap between performances (in terms of total delay, capacity etc.) and computational effort, the centralised strategy is still more effective than the decentralised method. This paper focuses on an application of a centralised Model Predictive Control strategy.

With reference to traffic light optimisation, one of the main issues to be investigated in the literature concerns the traffic flow model usually adopted in traffic control. Indeed, whatever the traffic control strategy, a traffic flow model is necessary when considering interacting junctions in a network (Cantarella et al., 2015; Di Gangi et al., 2016; Memoli et

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al., 2017). The model reproduces the impacts of the control strategies from upstream to downstream and computes the values of the objective functions (e.g., total delay, total time spent, queues, etc.) to consider during the control procedure. Traffic flow representation models have historically been classified according to their level of aggregation of the variables of demand and supply. Thus, it is possible to identify three main groups: macroscopic, mesoscopic and microscopic models (Treiber and Kesting, 2013). These models may be combined, producing hybrid traffic flow models that have also been extensively studied. In this case, the traffic flows are represented by coupling different models, even with a variable aggregation level, switching between a macroscopic, mesoscopic or microscopic approach. Such models have been generally explored for multi-scale applications (Joueiai et al., 2014; 2013) to provide traffic analysis at local and network levels.

Furthermore, recent advances in vehicle-to-X communication make it possible to develop more effective and reliable traffic management strategies using the information provided by connected vehicles and ever-faster communication protocols (e.g., Fajardo et al., 2011). To date, several applications have been developed in urban contexts, mainly focusing on signalised junctions and, in general, on the control of vehicles approaching junctions (Yu et al., 2018; Guler et al., 2014; Yang et al., 2016). The literature contains a plethora of studies that specifically concern the optimisation of traffic lights (Stevanovic et al., 2009; Priemer and Friedrich, 2009; He et al., 2012; He et al., 2014; Goodall et al., 2013, 2014; Lee et al., 2013; Feng et al., 2015, 2016; Islam and Hajbabaie, 2017; Beak et al., 2017; Ban et al., 2018; Wang et al., 2021) whilst other studies focus on combining a fixed-time control and an adaptive speed/acceleration (Green Light Optimized Speed Advisory - GLOSA) or trajectory optimisation of vehicles (see Tajalli and Hajbabaie, 2021; Niroumand et al., 2020). Recently, increasing attention has been paid to combining different strategies that may improve traffic operations (Yu et al., 2018; Yang et al., 2016; Guo et al., 2019; Li and Zhou, 2017; Tajalli et al., 2020) and traffic performance in terms of total delay and number of stops (see Ma et al., 2017) while directly reducing fuel consumption (see Katsaros et al., 2011) and emissions.

Nevertheless, proper representation of vehicle (driver) behaviour using the information provided by connected vehicles requires specific traffic flow models to support them. Thus, it is necessary to collect and process information and, at the same time, provide vehicles or drivers with consistent information[1]. This problem is usually addressed with microscopic models, as they can deal with each vehicle separately. However, they are highly detailed, require several parameters to be set, and are computationally demanding, a drawback that grows exponentially as the network size increases. The hybrid model can be considered an alternative approach to overcome such issues.

Summing up, the use of the hybrid traffic flow model enables: i) multiscale applications in the presence of human-driven vehicles and connected vehicles (at different penetration rates) and ii) applications of enhanced traffic management strategies in the presence of connected vehicles by collecting and processing real-time information provided by vehicles approaching junctions.

In this paper, the adopted hybrid traffic flow model (i.e., H-CTM&CA, Storani et al., 2021; 2022) combines a microscopic model for node representation and a macroscopic model for link representation to support the optimisation of junctions. In detail, the Cellular Automata model (CA; Nagel and Schreckenberg, 1992) was adopted as a disaggregated microscopic model, and the Cell Transmission Model (CTM; Daganzo, 1994) was adopted as an aggregated macroscopic model. In more detail, the proposed H-CA&CTM aims to deal with three main issues:

- First of all, following the literature, the meso-micro model is preferred. However, in terms of network scalability and within-day dynamic network applications, the macro approach is more suitable than the mesoscopic one;
- Secondly, to overcome the difficulties in the combination of the continuous macroscopic model with the discrete microscopic model, discrete macroscopic modelling must be preferred;
- Finally, to facilitate the transition from macroscopic to microscopic model and reduce the whole computational effort, the Cellular Automata model (a simplified microscopic model) is considered.

The contribution of this paper is to develop a centralised MPC based traffic control framework integrated with a hybrid traffic flow model (H-CA&CTM), considered as a prediction and plant model. The whole framework makes it possible: i) the multiscale applications in presence of human-driven vehicles and connected vehicles (at different penetration rates) and ii) the application of the strategy in presence of connected vehicles by collecting and processing real-time information provided by vehicles approaching the junctions. The integration of the hybrid traffic flow model with a centralised traffic control strategy provides a framework that is suitable for the V2I applications regarding the urban network context.

The paper is organised as follows: Section II describes the adopted modelling framework based on Model Predictive Control (MPC; Hegyi, 2004; Hegyi et al., 2005; Papageorgiou et al., 2008; Carlson et al., 2010) with some details about the centralised approach; Section III presents numerical results and the discussion; finally, Section IV comments on future work to be undertaken.

II. Modelling framework

This section focuses on the characteristics and parameters of the MPC and provides the main details of applying the hybrid traffic flow model within the MPC. The hybrid traffic flow model in question was recently proposed elsewhere and further details about model specification and calibration, as well as comparison with respect to some benchmark models, can be found in Storani et al. (2021; 2022).

[1]That is, the traffic state that the driver will experience (Cascetta, 2006; Bifulco et al., 2009; de Luca and Di Pace, 2015).

A. Model predictive control

MPC can optimise traffic signal settings, adapting them to the system prediction, starting from its current state. To this end, the controller receives information (measurements) from the system, and via a control loop applies an optimisation procedure on a prediction model over a prediction horizon. During this prediction horizon, several control inputs may be obtained (solutions of the optimisation procedure) for a control horizon. Only the first sample is implemented, and the horizon is thus shifted by one sample, restarting the optimisation with new measurements.

The main MPC settings are:

- the prediction horizon N_p: is the time over which the controller would predict future states of the system on a prediction model if the set of control inputs of the optimisation procedure were applied to the system. This value should be long enough to consider all the dynamics of the system, but at the same time not so long as to increase computational complexity. It is expressed in controller time steps;
- the control horizon N_c: it is the number of controller time steps for which the control inputs are optimised. The setting of this parameter is a trade-off between low computational complexity (a shorter control horizon) and better controller performance (a longer control horizon). After the control horizon has passed, the control signal is assumed constant over the rest of the prediction horizon;
- parameters of the objective function: as stated above, the single objective function to be minimised is the total time that all vehicles spend on the network, as a unique value. However, it is possible to consider a more complex objective function – using a weighted sum approach – or several functions that could be in conflict. In this case, it is necessary to appropriately choose a set of weights to consider the trade-off between different objectives.

Other important parameters are:

- T simulation time interval for the prediction model: this is the time interval in which the prediction model (i.e., the hybrid model) yields an estimate of the system state. In this case, it is equal to 1 s. Its step counter is denoted as k;
- T_c control time interval: this is the time interval in which junctions within the subnetwork can communicate with each other and be synchronous. The first sample of the optimisation procedure is implemented during this interval. Its step counter is denoted as k_c.

The relation between T and T_c is given by:

$$T_\mathrm{c} = M \cdot T \tag{1}$$

where M is a constant integer. Since there is a difference between the simulation time interval T and the control time interval T_c, the traffic states are estimated M times more than the variation in the control inputs.

The optimisation strategy uses a metaheuristic procedure as a solution algorithm that must be selected based on the trade-off between the effectiveness of the algorithm related to the space solution exploration, the parameters to be set, and the computational effort depending on the algorithm complexity (see Cantarella et al., 2015). Some detailed considerations are provided below about Simulated Annealing (SA), Genetic Algorithms (GAs), and Differential Evolution (DE). In SA the number of the parameters to be set is lower than in GAs, and in terms of computational effort, the performance is generally better. Furthermore, SA can easily handle changes in the objective function, while, with GAs, utilising a fitness function to select the solution may complicate the search for the optimum. On the other hand, for a space of solution of higher dimension GAs are generally more effective than SA, resulting from their flexibility due to the two genetic operators, crossover and mutation at the cost of a higher number of parameters to be set. Because of this, GAs ensure exploration of a wider range of the solution space, allowing computation of an approximation of the entire Pareto front. This provides higher resilience in multi-criteria optimisation problems. Finally, DE has the advantage of being a stochastic and population-based optimisation algorithm like GAs, comparing each individual with another obtained by a mutation and recombination of a successful solution from the previous generation, and selecting the one with the best function value to keep to the next generation. The selected individual may thus be conserved across several generations, therefore improving or remaining with the same best function value until a stopping criterion is reached. The mutation and recombination procedures of other randomly selected individuals from the population avoid stopping at a maximum/minimum local while the algorithm keeps exploring the search space.

In this paper, the Differential Evolution (DE) method is applied as the metaheuristic optimisation strategy (DE; Price, 2013; Brabazon et al., 2006). In a comparison by Storn and Price (1997) DE outperforms simulated annealing and genetic algorithms in terms of the number of function evaluations required to locate a global minimum of the test functions. Ali and Torn (2004) found that, given the feature that all points in the set are possibly updated for each generation, the DE-type algorithms were more robust than the Controlled Random Search (CRS; Price, 1997; 1978) and GA-type algorithms. Lampinen and Storn (2004) demonstrated that the DE is easy to implement and use, effective, efficient and robust, which makes it an attractive and widely applicable approach for solving practical engineering design problems.

In this research, DE was applied with the following parameter settings:

- Population size: variable depending on each scenario
- Combination probability: 0.90
- Scale factor F: $0.50 \cdot (1 + rand)$
- Maximum iterations: 1000

The population size is set at 5 times the number of variables, multiplied by the control horizon (Storn and Price, 1997). The control variables are the duration of the green light at each approach and the absolute offset of the traffic light plans of each junction (minus one used as reference). $rand$ is a random value generated with a uniform distribution between 0 and 1, resulting in a DE with a random scale factor to reduce the risk of stagnating at a local optimum (Das et al., 2005).

III. Numerical results and discussion

This section uses a layout comprising a two-way arterial with three successive signalised junctions to test different applications. First, the strategy based on Model Predictive Control (MPC; Lin, 2011; Zegeye, 2011) was specified and calibrated (see Section III-A).

To properly analyse the MPC approach, the latter was also compared with a fixed time strategy, and an adaptive control approach called SCOOT (Hunt et al., 1981), while using a CTM as a traffic flow model (see Section III-B). Additional analyses were carried out in terms of computational effort and throughput (vehicle in-out) by comparing the results obtained using a CTM and the H-CTM&CA, considering the MPC strategy (see Section III-C).

Furthermore, the centralised traffic control framework based on the hybrid traffic flow model in question may be further developed, for instance, for multi-objective traffic light optimisation (including minimisation of the energy consumption of EVs) and combined signal settings design with a speed optimisation procedure (Green Light Optimized Speed Advisory, GLOSA; Katsaros, 2011). To highlight the traffic flow model suitability in the presence of connected and automated vehicles, an additional application to integrate traffic light and vehicle speed optimisation (advisory), still referring to the same layout, was introduced (see Section III-D).

The results of the various tests were analysed with respect to different indicators:

- *Maximum and mean queue (MMQ):* obtaining the queue length at each time step for each link, counting the number of vehicles stopped in the CA and the stopped-flow in the CTM, to then calculate the maximum and mean value on the whole simulation interval for each road.
- *Saturation degree (SD)***:** considering the ratio between the number of incoming vehicles on each road, and the maximum number of vehicles that could exit each link during the green time of the traffic signal.
- *Total time spent (TTS):* the sum of the number of vehicles on each link for each time step, for the whole simulation interval. It is equivalent to adding the total time spent by each vehicle on each link. By adding the TTS of each link, a single value for the whole network may be obtained.
- *Total delay (TD):* the extra time spent by each vehicle on a link, due to congestion or the presence of traffic lights, obtained as the total time spent by each vehicle on each link minus the time it would have taken to cross the link on a free-flow condition without traffic lights.

Finally, the computer used to run these tests had an Intel(R) Core(TM) i7-4510U CPU @ 2.00GHz (4 CPUs), ~2.6GHz with 8192MB RAM, and an OS Windows 10 Home 64-bit. The code was written on MATLAB R2019b.

The test layout is a two-way arterial with three successive signalised junctions (A, B and C), as can be observed in Figure 1. It has five entry/exit links 90 metres long; the distance between junctions A and B is around 810 metres, as is the distance between junctions B and C.

TABLE I
Base Entry - Exit flows of the arterial application.

		Exit [PCU/h]					
		2	7	10	12	14	TOTAL
Entry	**1**		200		100	100	400
[PCU/h]	**8**	400		100	100		600
	9	100	100				200
	11	100	100				200
	13	100	100				200
	TOTAL	700	500	100	200	100	1600

TABLE II
Traffic flow model parameter values.

Parameter	**CTM**	**CA**
Δt time step	1 s	
k_{jam} jam density	200 veh/km	
Cell length	15.00 m	2.50 m
Vehicle length	-	2 cells
v_f free flow speed	15 m/s	6 cells/s
w shock wave speed	5 m/s	-
q_i maximum flow rate	variable	-
p dawdling probability	-	variable
Min speed to apply dawdling	-	variable

The entry/exit flows considered between entry and exit links are displayed in detail in Table I. These values were considered constant over time as uniform traffic flows (see Figure 2), and were then increased to a peak value as a variable traffic flow (see Figure 3).

A. MPC overview: Specification and calibration

This section comprises the following three subsections: i) objective function testing considering deterministic and stochastic vehicle behaviour, ii) analysis of the objective functions with respect to the dawdling rule, iii) application of MPC considering different flow trajectories.

A more detailed description is provided below.

1) Test of the objective function dynamic considering a deterministic and a stochastic vehicle behaviour: The objective function considered by the controller, in accordance with the literature (Lin, 2011; Zegeye, 2011), is the total time spent (TTS). Since the aim of this application is to test the suitability of the hybrid traffic flow model, it was used as a system model and as a prediction model on the MPC controller. In both cases, the hybrid model has the same set of parameters, but due to the stochastic behaviour of the CA sub-model caused by the dawdling rule on each vehicle, which depends on a randomly activated probability, the trend of the system and the plant model may slightly differ. As anticipated in Section III, the model was tested on a two-way arterial with three successive signalised junctions.

Regarding the hybrid traffic flow model, three types of flow were considered depending on the combination between the outflow capacity of the Cell Transmission Model (equal to 2700 or 2000 PCU/h) and the dawdling probability of the Cellular Automata (CA deterministic or stochastic). The model settings are displayed in Table II below.

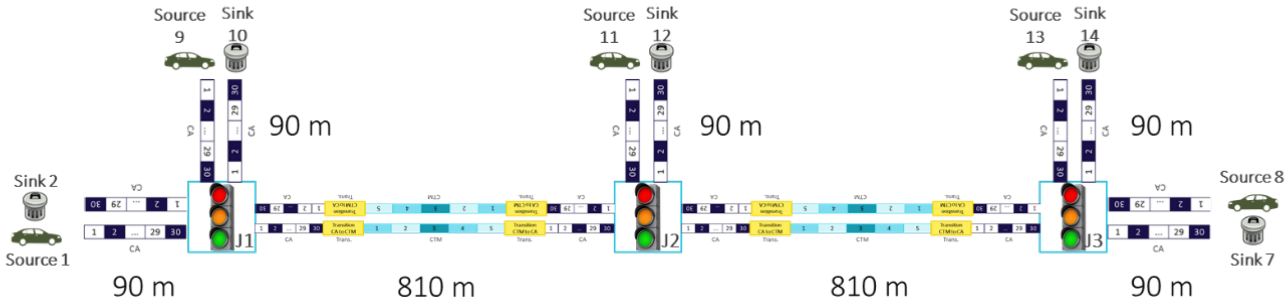


Fig. 1. Layout of the two-way arterial.

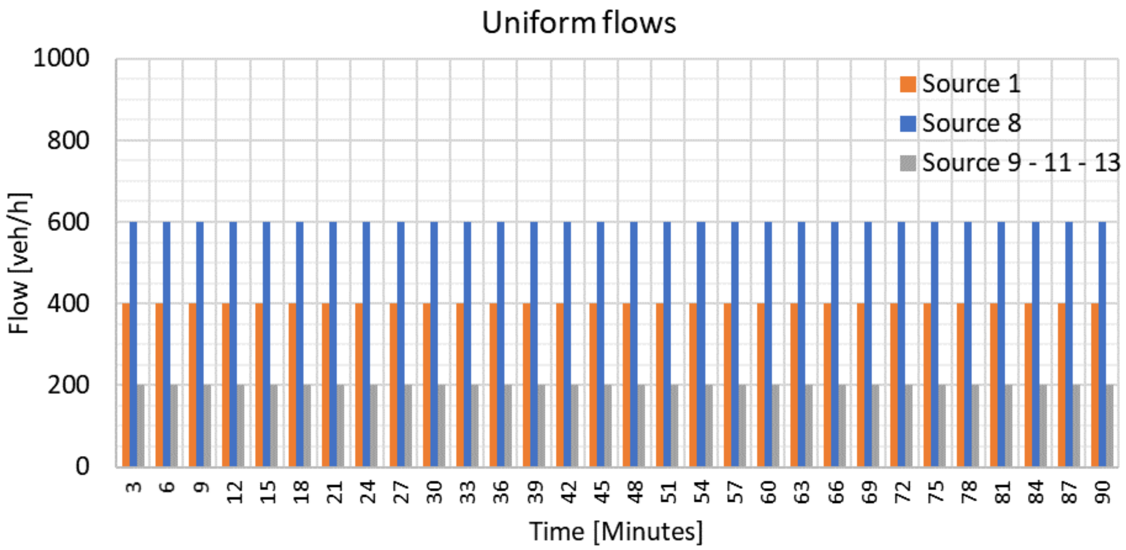


Fig. 2. Uniform flow values over time.

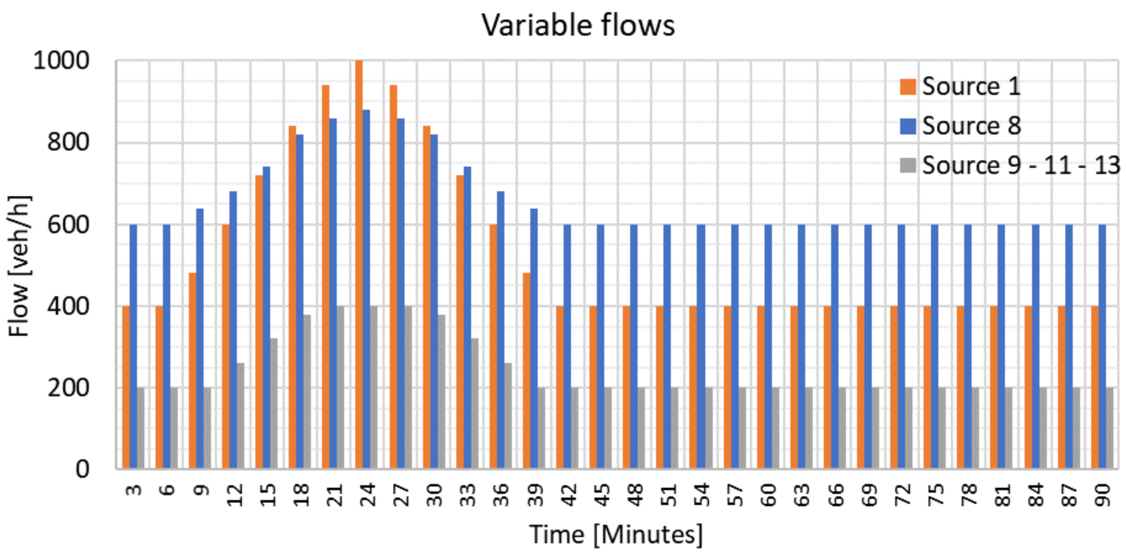


Fig. 3. Variable flow values over time.

TABLE III
OBJECTIVE FUNCTION MEAN VALUES.

Control time interval [s]	Type 1	Type 2	Type 3
90	5236.5	5471.6	5279.5
180	10473.0	10895.1	10559.0

The maximum flow rate for the CTM model, and the dawdling probability and minimum speed to apply the dawdling rule on the CA model were varied to consider three types of flows, producing a different flow-density relationship. Such types of flows are summarised in Figure 4 which displays the flow-density relationships and the value set for these parameters.

A first implementation of MPC was considered, with a uniform entry flow for each of the three flow types. The results are displayed for two combinations of MPC settings (control time interval, control horizon and prediction horizon) in the figures below (see Figure 5) in which the landscape of the objective function (Total Time Spent) against the control step is displayed. In both cases, the cycle length of the junctions is equal to 90 s, and the total simulation period of the system is set at 90 minutes (i.e., 5400 s equal to 60 cycles). For this implementation, as optimisation variables, three green times (the same for the three junctions) and two independent absolute offset values were selected.

Furthermore, the mean values of the objective function after a warm-up period of 360 seconds are displayed in the following table (see Table III).

Since the considered entry flow is constant over time, a constant trajectory of the objective function (i.e., the TTS) is expected. However, it is possible to see a difference between the flow types:

- Flow types 1 and 3 have the same behaviour, since the dawdling rule in the CA sub-model is not applied (set to 0). Therefore, the value of the objective function is constant or periodic across several control steps. When the control time interval is set to 90 seconds, the value of the objective function (Total Time Spent) oscillates around 5250, while it is constant for a control time interval of 180 seconds close to 10500. The oscillation in the first case is caused by a difference in the number of vehicles being inserted into the artery at each control step, varying the objective function.
- Flow type 2 behaves differently since in this case the dawdling rule is applied, giving, as a result, a stochastic behaviour. Each vehicle can therefore reduce its speed due to the dawdling rule even if the gap is sufficient to maintain its speed or accelerate. Such stochasticity between vehicles thus produces higher values of the objective function compared to the deterministic scenarios (around 5400 for a control time interval of 90 seconds, and 10900 for a control time interval of 180 seconds, vs 5250 and 10500 for flows 1 and 3), varying across different control steps, even under the same traffic demand.

2) Analysis of the objective function dynamic: generating the dawdling rule: Since at every control step, different control inputs must be tested to find the most suitable one, it is necessary to consider the stochasticity of the traffic flow models to compare different results. Therefore, to study the stochastic nature of the traffic flow models and how it may affect the objective function of the MPC, a further analysis was made. The results with the Krauß traffic flow model (see Krauß, 1998) were compared with the deterministic and stochastic CA. Finally, regarding the stochastic CA, three further applications were considered: i) without setting the seed at each control step to generate the random number to apply the dawdling rule (random – no seed scenario), ii) by setting the seed at each control step (random – seed fixed scenario, so that different control inputs start with the same seed) and iii) by applying the rule every 1/(dawdling probability) vehicle (for instance if $p = 0.266$ the rule is applied every $3.759 = 1/0.266$ vehicles) considering a fixed formula to apply dawdling. The results are displayed in Figure 6 in which two different clustered behaviours may be observed: the first of a highly stochastic nature consists of the Krauß traffic flow model and the 1/(dawdling probability) implementation, while the second consists of the deterministic scenario and the two "seed/no seed" scenarios.

In the same Figure 6 the values of the mean and standard deviation (St. Dev.) are also displayed for each of the simulations. It is evident that the random – seed fixed scenario must be considered.

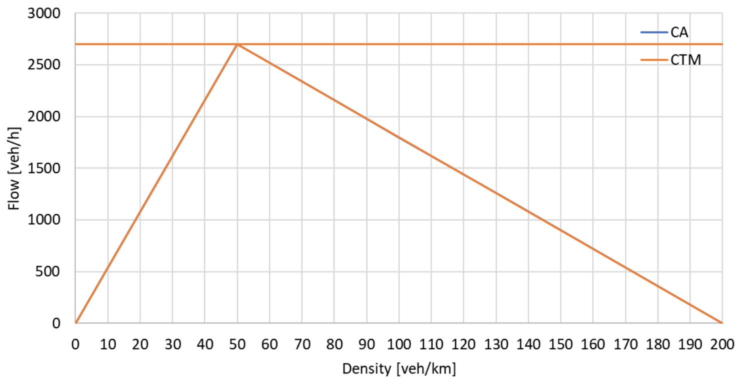
In particular, the random – seed fixed scenario is the most appropriate approach to deal with the stochasticity of the model, since it allows the same result to be obtained within a control step, given the same conditions (same initial state and equal set of control inputs) when the controller is testing several control inputs to find the optimum set.

3) Applying the MPC with different flow trajectories: After choosing the fixed seed strategy considering flow type 2 (stochastic behaviour of the CA), four scenarios (S1, S2, S3 and S4) were specified, as displayed in Figure 7 below. The first two scenarios have as decision variables nine green times and two absolute offsets on one control horizon, considering the uniform and variable flow values, as seen in Figure 2 and Figure 3. The third scenario considers two control horizons, therefore improving the objective function for the variable flow value. Finally, the fourth scenario evaluates the objective function considering a fixed timing strategy with a variable flow.

As expected, the objective function (Total Time Spent) grows as the flow increases for scenario S3, but the controller adjusts the green timings and the offsets, reacting to this variation. If the green timings and offsets are fixed (scenario S4), the total time spent would increase since some initially uncongested links would become congested.

Figure 8 shows the results obtained for the green timings of junction J1[2] , while Figure 9 shows the results for the absolute offsets, both considering the four scenarios described. The first observation to be made is that the traffic light decision variables are more stable in the case of uniform flows than of variable flows, as expected. However, since a metaheuristic

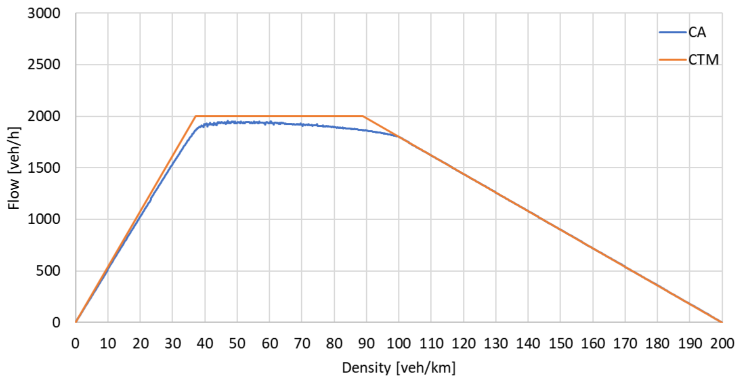
[2]For sake of brevity only the results for J1 and J2 are shown.



Type of flow 1
CTM:
q_i maximum flow rate = 2700 [veh/h]

CA
p dawdling probability = 0
Min speed to apply dawdling = 0

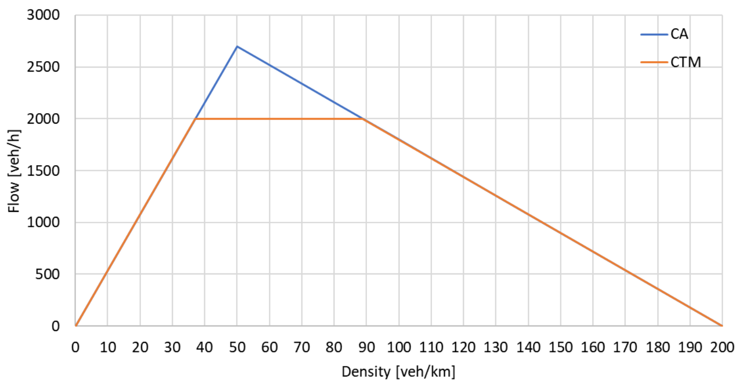
Deterministic behaviour of the CA- (no dawdling)



Type of flow 2
CTM:
q_i maximum flow rate = 2000 [veh/h]

CA
p dawdling probability = 0.266
Min speed to apply dawdling = 2

Stochastic behaviour of the CA- (dawdling applied)



Type of flow 3
CTM:
q_i maximum flow rate = 2000 [veh/h]

CA
p dawdling probability = 0
Min speed to apply dawdling = 0

Deterministic behaviour of the CA- (No dawdling)

Fig. 4. Flow classification.

procedure is used, a variation in green times may also be seen for uniform entry flow (uniform flow of Figure 2, scenario S1 of Figure 8 and Figure 9).

B. MPC vs fixed /adaptive strategy #CTM

This section provides further analysis, using the artery layout modelled with a CTM to compare several indicators obtained with the green times and offsets from the MPC traffic control strategy, a fixed[3] time strategy and the adaptive SCOOT strategy.

The analyses regard internal and external links, identified as in the following figure (see Figure 10), considering uniform flows (Unif. Flows) and variable flows (Var. Flows).

The following Table IV provides the values of the mean maximum queue (MMQ) and mean standard deviation (SD) of the different strategies for the internal links, as well as the total time spent (TTS) of the whole artery considering the simulation horizon of 5400s.

As in the previous case, the value of the indicators varies across several control steps for uniform flows under the MPC strategy. The other indicators not considered in the optimisation procedure of the MPC strategy (that is MMQ and SD) are higher in the scenario with variable entry flow than in uniform entry flow. The TTS indicator (objective function for the MPC optimisation procedure) is 76% lower when applying the green times and offsets obtained with the MPC than to the case of fixed strategy and 65% lower than obtained with the SCOOT approach, confirming the effectiveness of the MPC.

Figure 11 displays the results of the indicators considering uniform and variable flows, differentiating between internal and external links, across each control step (of 360 s, i.e., four cycles) for the simulation horizon.

[3]The fixed time strategy is based on a synchronisation method (Cantarella et al., 2015) aiming to minimise network total delay.

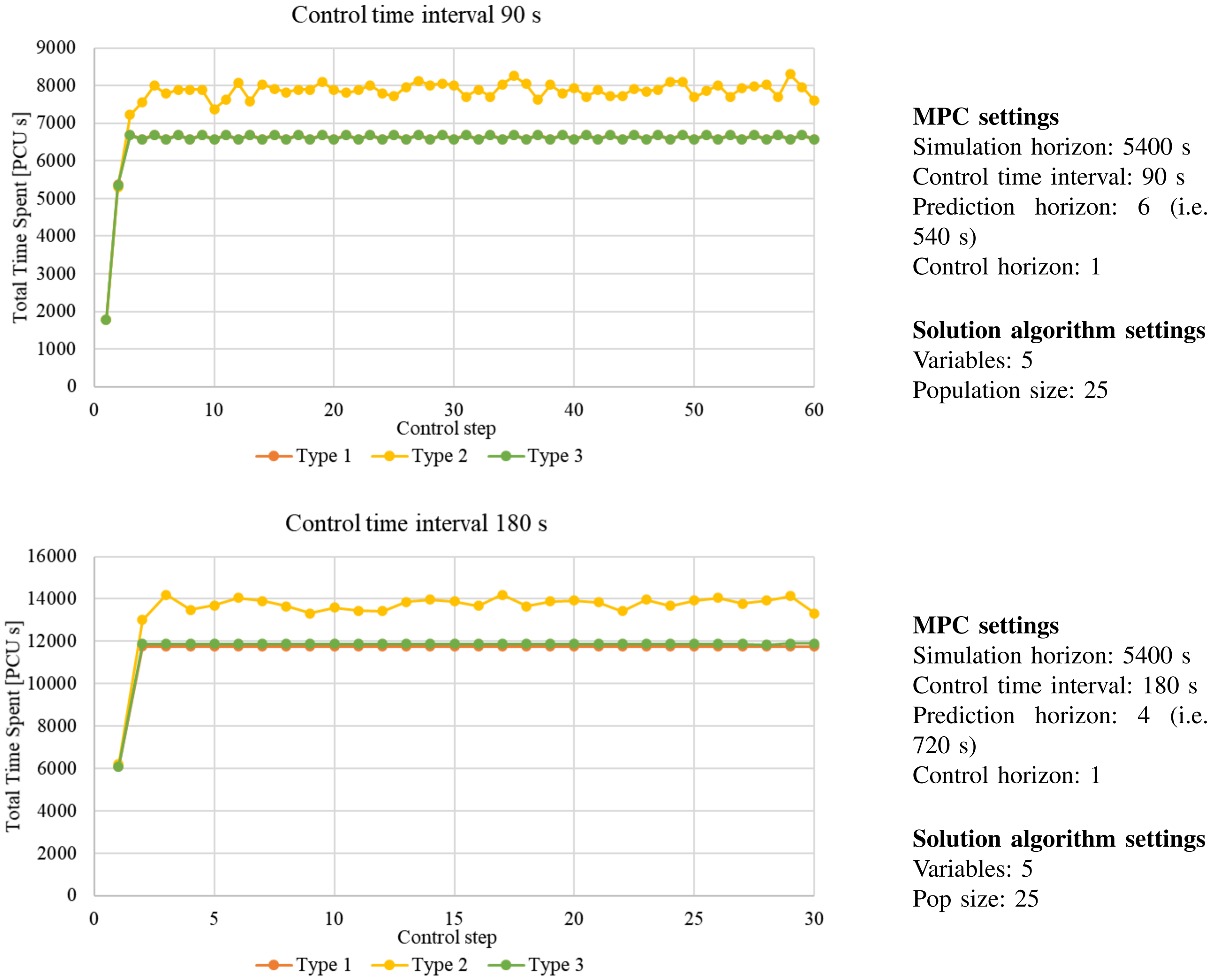


Fig. 5. Values of the objective function (Total Time Spent) with respect to two different MPC settings.

C. CTM vs H-CTM&CA #MPC strategy

This section details an evaluation based on the computational effort between using CTM and the proposed hybrid traffic flow model H-CTM&CA, applying an MPC approach. The first analysis compares the traffic flow models considering a deterministic and stochastic behaviour of the hybrid model (changing the value of the dawdling probability, deterministic if it is set to 0, or stochastic if it is greater than 0) and the variation of the outflow capacity. Our results point out that a comparable and lower value of the elapsed time is observed. In particular, the results (see tables below) for the running time over 50 control steps show that the time for 1000 executions is still comparable, irrespective of the values of the dawdling probability.

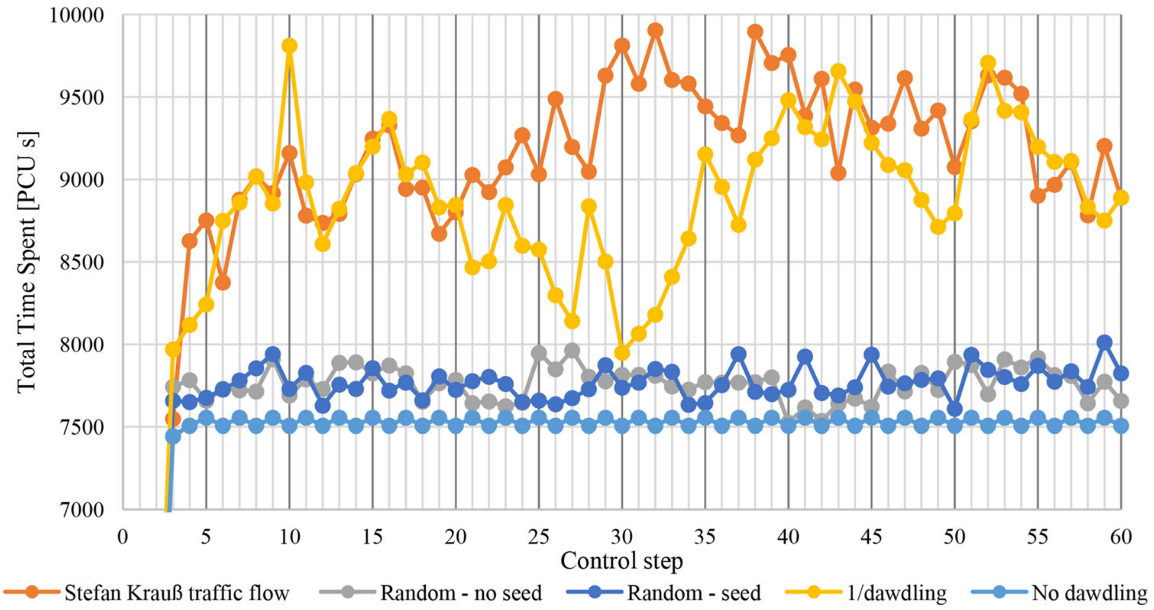
With reference to Table VI, concerning the evaluation of the H-CTM&CA in terms of execution times considering dawdling deterministic/stochastic and a different population size on the Differential Evolution method, the results show a lower total elapsed time compared to the other tests.

Finally, the network throughput (vehicle in-out) was also analysed for the different demand profiles and signal setting design strategies, as well as the number of vehicles waiting to enter at the end of the simulation. The results are summarised in Table VII which confirm that in the case of uniform flow, fixed time and MPC provide similar results, whereas in the case of variable flows the MPC is able to guarantee a higher value of vehicle throughput and then a lower number of vehicles waiting to enter at the end of the simulation.

D. Green light optimized speed advisory (GLOSA)

Several issues may be found in the field of connected and automated vehicles. One of them is investigation of cooperative junction management, with speed optimisation as the main purpose to ensure vehicle arrivals at the green light while satisfying some constraints. This procedure is consistent with the European research programme GLOSA (Green Light Optimized Speed Advisory), where the speed of an approaching vehicle to the junction contributes to fuel consumption reduction based on signal timing (Katsaros et al., 2011).

This section discusses the results of GLOSA application to the same layout in mixed traffic flow conditions (especially in



	SUMO	stochastic			deterministic
	Stefan Krauß traffic flow	*rand – no seed*	***rand – seed fixed***	*1/(dawdling probability)*	*no dawdling*
Mean	9214.82	7762.79	**7768.28**	8902.30	7530.58
St. Dev.	354.40	100.73	**91.62**	424.56	24.21

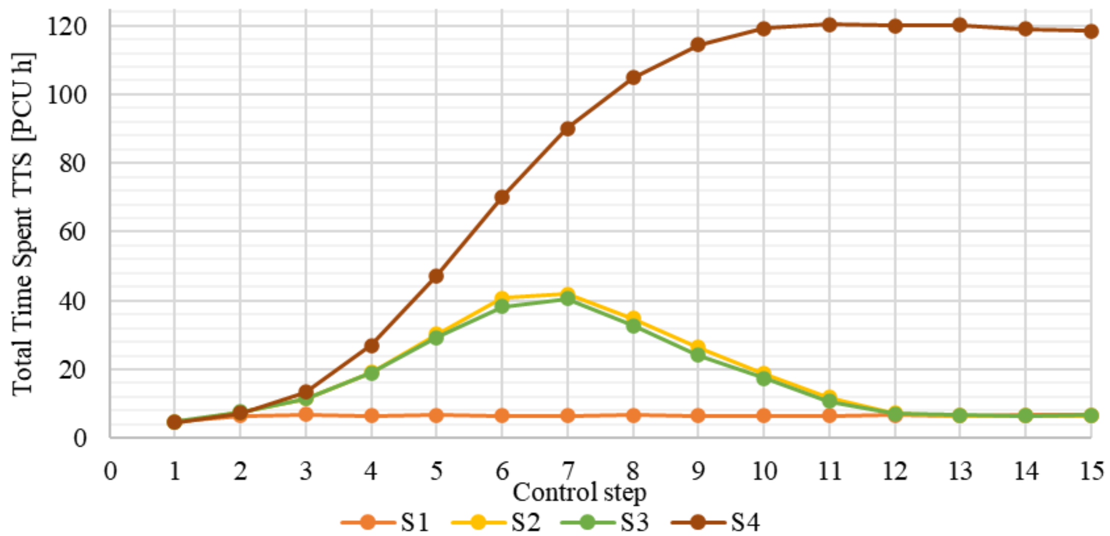
Fig. 6. Values of the objective function (Total Time Spent) considering different settings of the H-CTM&CA model, and considering the Krauß traffic flow model for the comparison.

TABLE IV
INDICATORS FOR A FIXED GREEN TIMES AND OFFSETS STRATEGY ON A CTM MODEL.

Uniform entry flow								
Fixed strategy			MPC strategy			SCOOT strategy		
MMQ [PCU]	SD [%]	TTS [PCU h]	MMQ [PCU]	SD [%]	TTS [PCU h]	MMQ [PCU]	SD [%]	TTS [PCU h]
4.5	77.58	85.72	3.17	77.57	60.56	7.87	92	91.75
Variable entry flow								
Fixed strategy			**MPC strategy**			**SCOOT strategy**		
MMQ [PCU]	SD [%]	TTS [PCU h]	MMQ [PCU]	SD [%]	TTS [PCU h]	MMQ [PCU]	SD [%]	TTS [PCU h]
4.88	82.17	484.97	13.38	90.28	118.29	10.24	98	332.24

TABLE V
CTM VS H-CTM&CA#EXECUTION TIMES CONSIDERING DAWDLING DETERMINISTIC/STOCHASTIC, AND PARAMETER SETTINGS.

	CTM	H-CTM&CA	CTM	H-CTM&CA
Flow model	Det.	Det.	Det.	Det.
Scenario	A1	B1	A2	B2
Simulation horizon [s]	4.500	4.500	4.500	4.500
Control time interval [s]	90	90	90	90
Prediction horizon	6	6	6	6
Control horizon	1	1	1	1
Outflow capacity [PCU/h]	**3.600**	**3.600**	**1.800**	**1.800**
N variables	4	4	4	4
Population size	22	22	22	22
Total number of iterations	4847	4429	3802	4389
Total number of executions	106634	97438	83644	96558
Total elapsed time [s]	11363.8	8759.6	9824.8	9733.5
Time for 1000 executions [s]	106.568	89.899	117.459	100.804



Scenario	**S1**	**S2**	**S3**	**S4**
Flow entry	Uniform	Variable	Variable	Variable
Simulation horizon	5400 s			
Control time interval	360 s			
Prediction horizon	2 (i.e. 720 s)			Fixed
Control horizon	1	1	2	
Variables	11	11	22	
Pop size	110	110	220	

Fig. 7. Values of the objective function (Total Time Spent) considering different scenarios (varying the flow, the parameters of the MPC controller, and considering a fixed value of the green times and offsets).

TABLE VI
H-CTM&CA#EXECUTION TIMES CONSIDERING DAWDLING DETERMINISTIC/STOCHASTIC AND DIFFERENT POPULATION SIZE.

	H-CTM&CA			
Flow model	Det.	Stoch.	Det.	Stoch.
Scenario	B2	C1	B3	C2
Simulation horizon [s]	4.500	4.500	4.500	4.500
Control time interval [s]	90	90	90	90
Prediction horizon	6	6	6	6
Control horizon	1	1	1	1
Outflow capacity [PCU/h]	1.800	1.800	1.800	1.800
N variables	4	4	4	4
Population size	**22**	**22**	**11**	**11**
Total number of iterations	4389	6703	2869	3622
Total number of executions	96558	147466	31559	39842
Total elapsed time [s]	9733.5	13872	3890.1	4739.3
Time for 1000 executions [s]	100.804	94.070	123.264	118.951

TABLE VII
THROUGHPUT ANALYSIS FOR THE ARTERIAL CASE STUDY MODELLED WITH THE H-CTM&CA MODEL.

ARTERIAL With a H-CTM&CA model		Total vehicles trying to enter	Total vehicles that have entered	Total vehicles that have exited	Number of vehicles waiting to enter at the end of the simulation	Number of vehicles in the network at the end of the simulation
Uniform flows	fixed time	2400	2385	2343	15	42
Uniform flows	with MPC	2400	2385	2339	15	46
Variable flow	fixed time	2874	2523	2480	351	43
Variable flow	with MPC	2874	2859	2816	15	43

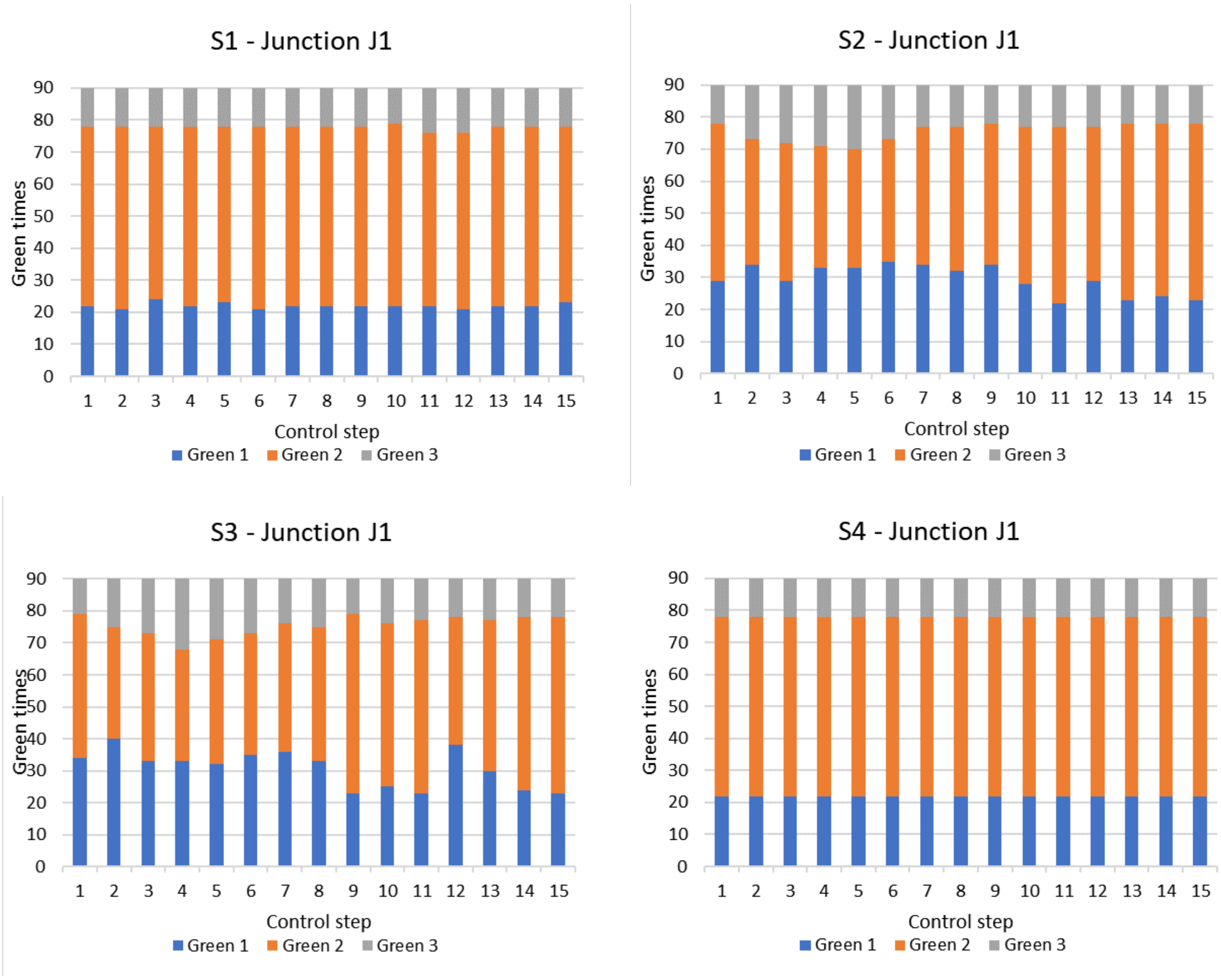


Fig. 8. Green timings for junction J1 for each scenario considered.

the case of fully connected and automated vehicle penetration rates). The application focuses on the junction's optimisation; to this end and to provide an overview of the model suitability for the application, the traffic lights decision variables (i.e., green lights and offsets) are optimised considering the fixed time and MPC control, whereas the speed of each vehicle approaching a junction is considered as a decision variable applying the procedure[4] displayed in Table VIII.

To carry out the application, the following settings were considered:

- 100% CAV penetration rate
- $\text{optimisation}_{\text{distance}}$: 210 metres - distance to optimise speed
- $\text{base}_{\text{speed}}$: 6 cells/s – base desired speed near the junction
- $\text{max}_{\text{speed}}$: 9 cells/s - maximum desired speed near the junction
- $\text{min}_{\text{speed}}$: 1 cells/s - minimum desired speed near the junction

[4]To carry out the application, a simplified procedure was considered. Indeed, optimisation is not the focus of the paper, and the application aims to highlight model suitability in the presence of connected and automated vehicles and human-driven vehicles. Therefore, it was not necessary, regarding the context, to investigate other more sophisticated approaches.

Table IX provides the results highlighting the suitability of the proposed approach and its effectiveness in terms of performance indicators. In particular, very similar results may be observed for uniform flows, whereas a lower value of the TTS indicator is still shown for the MPC variable flows scenario.

Furthermore, bearing in mind that the number of CAVs on the streets will increase in the years to come, in the short term, traffic flow conditions are expected to remain mixed given the major role played by human-driven vehicles (Levin and Boyles 2016a, 2016b). Therefore, several studies have investigated the application of control strategies to non-connected vehicles since their interactions with connected vehicles are worth analysing. In this paper, different CAV penetration rates are considered, and the results are in general slightly better with respect to overall indicators (see Table X and Table XI).

TABLE VIII
PROCEDURE OF THE SPEED ADVISORY AT THE NETWORK JUNCTIONS.

```
cell_length %cell length equal to 2.5 m
base_speed %desired speed equal to 15 m/s
optimisation_distance %distance to the junction equal to 200 metres
if road_XY_CA(1,cell,1) == 2 and cell >= last_cell - optimisation_distance / cell_length % the condition occurs that the vehicle is type 2 (connected) and the cell where it is positioned is on the last 210 metres (cell >= last_cell of the road – 210[m]/2.5[m/cell] )
    distance_to_traffic_light = last_cell - (cell)
    time_with_desired_speed = distance_to_traffic_light/ base_speed
    stage_when_arriving = active_stage(time + time_with_desired_speed)
    %speed optimisation procedure
    if stage_road <> stage_when_arriving % the defined stage of the road is not equal to the stage of the traffic light when the vehicle arrives at the junction with the desired base speed.
        % faster/slower_speed definition
        for k = 1 : cycle_length
            time_step = time + time_with_desired_speed + k;
            if active_stage(time_step) == stage_road
                slower_delta_time = time_with_desired_speed + k;
                faster_delta_time = slower_delta_time - cycle_length + green_time - 1
                break
            end
        end
        faster_speed = distance_to_traffic_light / faster_delta_time
        slower_speed = distance_to_traffic_light / slower_delta_time
        if faster_speed <= max_speed and faster_speed > 1 % an upper limit of the desired base speed is imposed, and negative values are avoided when subtracting the cycle length
            v_max = faster_speed
        elseif slower_speed >= min_speed % a lower limit is imposed on the slower speed
            v_max = slower_speed
        else
            v_max = base_speed %desired speed of 15 m/s
        end
    end
else
    v_max = base_speed %desired speed of 15 m/s if it is not a connected vehicle
end
```

TABLE IX
SCENARIO WITH CAV - H-CTM&CA MODEL - INDICATORS FOR A FIXED GREEN TIMES/MPC AND OFFSETS STRATEGY + SPEED ADVISORY.

Fixed strategy			MPC strategy		
Uniform entry flow			Uniform entry flow		
MMQ [PCU]	SD [%]	TTS [PCU h]	MMQ [PCU]	SD [%]	TTS [PCU h]
2.10	77.28	90.37	1.93	77.21	91.69
Variable entry flow			**Variable entry flow**		
MMQ [PCU]	SD [%]	TTS [PCU h]	MMQ [PCU]	SD [%]	TTS [PCU h]
2.38	79.96	493.09	3.05	88.35	220.31

TABLE X
RESULTS OF THE NUMERICAL APPLICATIONS CONSIDERING DIFFERENT CAV PENETRATION RATES [H-CTM&CA MODEL – FIXED STRATEGY].

Uniform entry flow						Variable entry flow				
CAV penetration %	MMQ [PCU]	Mean queue [PCU]	SD [%]	TD [PCU h/h]	TTS [PCU h]	MMQ [PCU]	Mean queue [PCU]	SD [%]	TD [PCU h/h]	TTS [PCU h]
0%	3.65	1.31	78.27%	20.72	90.27	3.65	1.89	89.70%	694.68	490.58
25%	3.51	1.26	78.24%	21.84	91.78	3.54	1.80	89.66%	698.79	493.05
50%	3.27	1.09	78.26%	20.64	90.22	3.47	1.74	89.82%	678.65	485.01
75%	3.09	1.06	78.24%	20.75	90.32	3.24	1.69	89.88%	673.42	483.76
100%	2.96	1.00	78.26%	20.84	90.37	3.08	1.68	89.70%	697.51	493.09

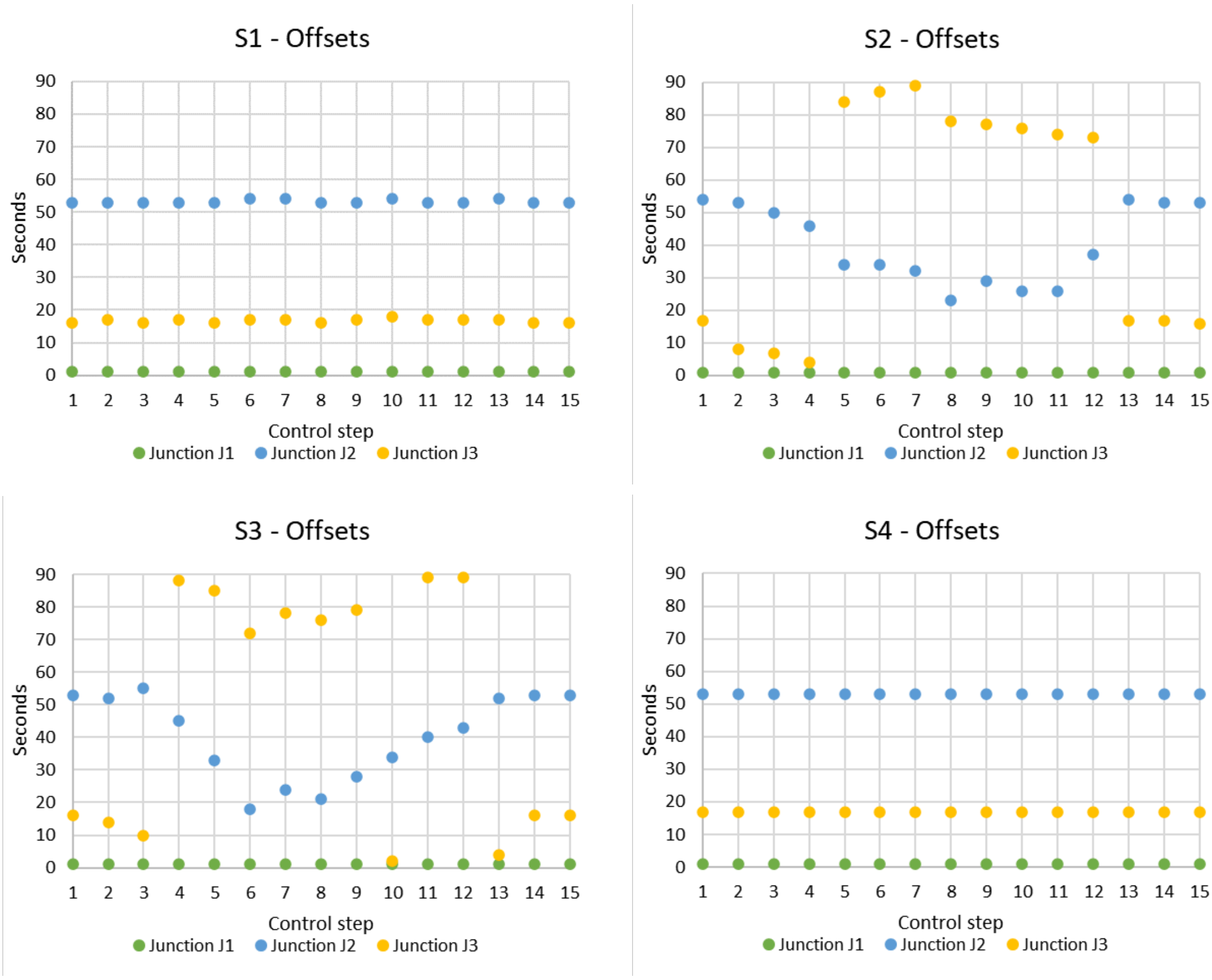


Fig. 9. Absolute offsets for the three junctions, for each scenario considered.

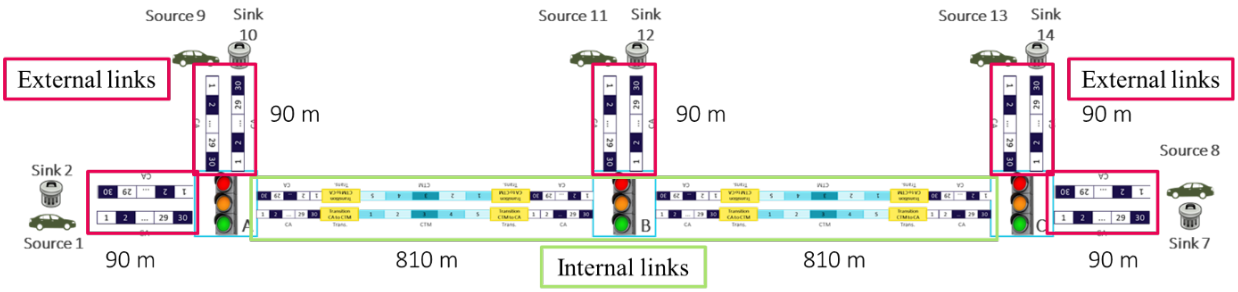


Fig. 10. Internal/external link identification wrt the whole layout.

TABLE XI
RESULTS OF THE NUMERICAL APPLICATIONS CONSIDERING DIFFERENT CAV PENETRATION RATES [H-CTM&CA MODEL – MPC STRATEGY].

Uniform entry flow						Variable entry flow				
CAV penetration %	MMQ [PCU]	Mean queue [PCU]	SD [%]	TD [PCU h/h]	TTS [PCU h]	MMQ [PCU]	Mean queue [PCU]	SD [%]	TD [PCU h/h]	TTS [PCU h]
0%	3.66	1.38	78.68%	21.92	91.63	3.65	1.72	85.52%	123.84	210.41
25%	3.56	1.20	78.70%	20.95	90.38	3.54	1.68	85.52%	119.38	205.46
50%	3.28	1.10	78.69%	20.68	90.06	3.45	1.64	85.52%	125.12	212.80
75%	3.18	1.09	78.70%	21.67	91.36	3.35	1.63	85.52%	132.70	220.31
100%	2.89	1.08	78.67%	21.97	91.69	3.65	1.72	85.52%	123.84	210.41

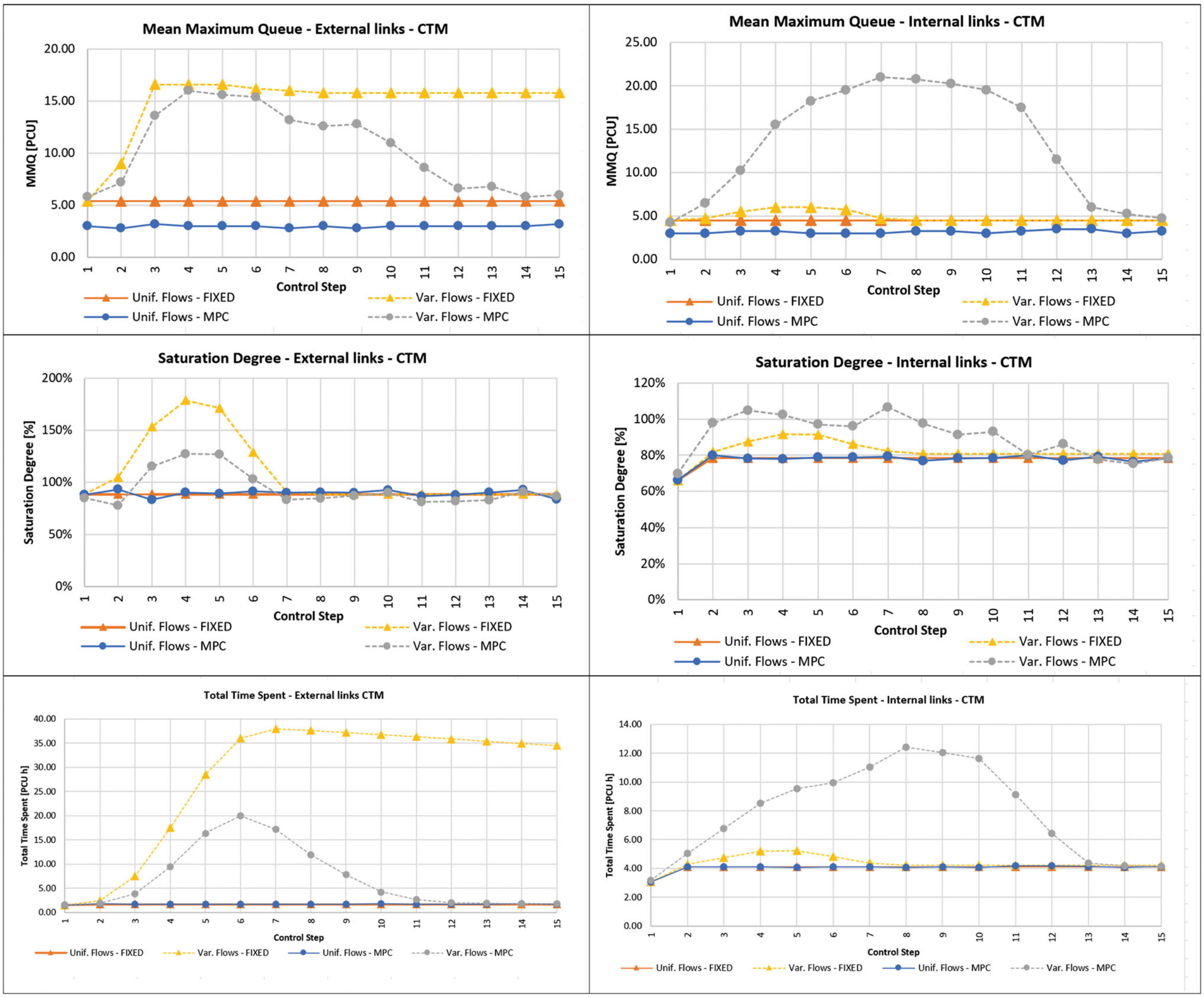


Fig. 11. Fixed vs MPC strategy, using CTM - Performance indicators vs control steps.

IV. Conclusions

The purpose of this paper was to develop guidelines for Model Predictive Control based on a hybrid traffic flow model which operates as a prediction and plant model. The hybrid traffic flow model (H-CTM&CA)[5] in question was based on the combination between the microscopic-disaggregated Cellular Automata model (CA; Nagel and Schreckenberg, 1992) and the macroscopic-aggregated Cell Transmission Model (CTM; Daganzo, 1994). The main contribution of our study concerned the integration of the traffic flow model with the Model Predictive Control approach. We sought to develop a centralised traffic control framework based on the adoption of MPC, and, in particular, specify its integration with a hybrid traffic flow model (H-CTM&CA; Storani et al., 2022), which is considered as a prediction and plant model unlike previous approaches described elsewhere.

[5]The model is discussed in terms of specification, calibration and validation and comparison with other benchmark models in Storani et al. (2021, 2022).

For this purpose, the numerical results of application to a two-way arterial with three junctions were displayed and analysed. The above layout was used to test different applications.

First of all, the model was appropriately calibrated, and then the green times and offsets were optimised: the first application was a fixed time strategy, and the second an enhanced strategy based on Model Predictive Control (MPC; Lin, 2011; Zegeye, 2011) considering the proposed traffic flow model H-CTM&CA. Our results show that in the case of uniform flows the indicators are slightly different, whilst in the case of variable flows, the MPC indicators, which are not considered in the optimisation procedure, namely MMQ and SD, are slightly higher in the variable entry flow scenario than in uniform entry flow. By contrast, the TTS indicator considered for the MPC optimisation procedure is lower than 57%, confirming the effectiveness of the adopted strategy. Furthermore, the suitability of the traffic flow model in the case of more complex traffic signal optimisation and the effectiveness of the adopted MPC also clearly emerged.

A further analysis was carried out considering the same arterial layout modelled with a CTM to compare several indicators obtained with the green times and offsets from the MPC traffic control strategy, a fixed[6] time strategy and the adaptive SCOOT strategy. As in the previous case, the results confirm that the value of the indicators varies across several control steps for uniform flows under the MPC strategy. The TTS indicator (objective function for the MPC optimisation procedure) is 76% lower when applying the green times and offsets obtained with the MPC than to the case of fixed strategy and 65% lower than obtained with the SCOOT approach, confirming the effectiveness of the MPC.

Other analyses were performed in terms of computational effort and throughput (vehicle in-out) by comparing the CTM and the H-CTM&CA, considering an MPC strategy. It was shown that a comparable and lower value of the elapsed time is observed. In particular, results for the running time over 50 control steps show that the time for 1000 executions is still comparable, irrespective of the dawdling probability. Furthermore, the network throughput (vehicle in-out) was also analysed for different demand profiles and signal setting design strategies. The results confirm the model's effectiveness for the throughput analysis in both cases: an increase in demand and the impact of a more complex traffic control strategy.

Finally, to highlight the suitability of the traffic flow model in the presence of connected and automated vehicles, an additional application aimed at integrating traffic lights and vehicle speed optimisation was introduced. The results highlight the suitability of the proposed approach and its effectiveness in terms of performance indicators.

In terms of future research, two main perspectives will be considered:

i) The proposed framework will be tested in greater detail on the application in the presence of connected and automated vehicles by including the impact of different penetration rates and the impact of different powertrains (e.g., electric vehicles; Fiori et al., 2021), taking advantage of the information they provide for the optimisation procedure of the network;
ii) The proposed centralised framework will be tested at a larger scale; the simplified hybrid model is expected to support the suitability of the framework in the case of more complex networks, such as the microscopic modelling usually adopted.

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[6]The fixed time strategy is based on a synchronisation method (Cantarella et al., 2015) aiming to minimise network total delay.

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