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Stability and Performance Analysis of Model Predictive Control of Uncertain Linear Systems

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***Abstract*— Model mismatch often poses challenges in model-based controller design. This paper investigates model predictive control (MPC) of constrained uncertain linear systems with input constraints, focusing on stability and closed-loop infinite-horizon performance. The uncertainty arises from a parametric mismatch between the true system and the estimated system, quantified by the matrix Frobenius norm. We examine a simple MPC controller that exclusively uses the estimated system model, and we establish sufficient conditions under which the MPC controller can stabilize the true system. Moreover, we derive a theoretical performance bound based on relaxed dynamic programming (RDP), elucidating the impact of prediction horizon and modeling errors on the suboptimality gap between the proposed controller and the ideal infinite-horizon optimal controller with knowledge of the true system. Simulations of a numerical example demonstrate the theoretical results.**

***Index Terms*— Linear Systems, Model Predictive Control, Performance Analysis, Model Mismatch**

I. Introduction

Model predictive control (MPC) is an optimization-based control strategy that computes inputs to optimize a specific performance metric over a given prediction horizon based on a system model. MPC has found widespread application in various fields, such as chemical processes [1], aerospace vehicles [2], and portfolio optimization [3]. Regardless of the application, the effectiveness of MPC heavily depends on the accuracy of the prediction model. However, obtaining a perfect model is impossible due to inherent modeling errors in practice.

To address the issues that arise from having an imperfect model, adaptive MPC is commonly employed, integrating MPC with a system identification module [4]. Typical approaches use the comparison error [5], neural networks [6], and Bayesian inference [7]. On the other hand, data-driven MPC [8] has also emerged as a promising method for handling model uncertainty by directly using input-output data. However, while much of the literature focuses on the stability, feasibility, and robustness of MPC, studies that provide performance analysis are limited.

Relaxed dynamic programming (RDP) is a notable framework for analyzing the performance of MPC controllers compared to that of the idealized infinite-horizon optimal control problem [9]. RDP quantifies the suboptimality gap by analyzing a general value function that describes the energy-decreasing characteristic along the closed-loop system trajectory [10], [11]. For uncertain systems, recent extensions analyze the effect of disturbances for nonlinear systems [12] and a class of parameterized linear systems where the true matrices of the system lie in a known polytope [13]. However, performance analysis of MPC for general linear systems with modeling errors remains unexplored.

Contributions: This work presents a novel analysis of MPC performance for linear systems with modeling errors. In contrast to the previous work in [13], we do not assume any specific parametric structure of the system nor adapt the system model online, adhering to the framework of model-based control with offline system identification. Moreover, our established bound is a *consistent* extension of the bounds derived in [11], [14], allowing it to recover the case without model mismatch. Using the RDP method, we establish a theoretical performance bound illustrating the impact of modeling errors on the closed-loop performance of the MPC controller. Furthermore, we provide sufficient conditions on the prediction horizon in the presence of modeling errors such that the closed-loop system is stable. This further reveals how closed-loop performance depends on modeling errors as well as the prediction horizon.

The remainder of the paper is organized as follows. After providing the preliminaries in Section II, we formulate the optimal control problems in Section III. Section IV offers stability and performance analysis of the MPC controller, followed by a numerical example in Section V to validate the theoretical results.

II. Preliminaries

A. Notation

Let x be a vector; then its transpose is denoted by $x^\top$, and its vector i-norm by $\|x\|_i$. For a matrix M, $M^\top$, $\|M\|_2$, and $\|M\|_{\mathrm{F}}$ denote its transpose, matrix 2-norm (i.e., spectral norm), and Frobenius norm, respectively. Moreover, $M \geq 0$ ($x \geq 0$) indicates element-wise nonnegativity, while $M \succ (\succeq)\ 0$ indicates positive (semi)definiteness. For a *symmetric* matrix $M \succ 0$, its largest and smallest eigenvalues are, respectively, denoted by $\overline{\sigma}_M$ and $\underline{\sigma}_M$, and we further define $r_M := \frac{\overline{\sigma}_M}{\underline{\sigma}_M}$. For a vector x and a symmetric positive (semi)definite matrix M, $\|x\|_M$ stands for $(x^\top M x)^{1/2}$.

The set of natural numbers is denoted by $\mathbb{N}$, the set of positive integers is denoted by $\mathbb{N}_+$, the set of positive integers

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up to n is denoted by $\mathbb{Z}_n^+$, and the set of real and non-negative real numbers are denoted, respectively, by $\mathbb{R}$ and $\mathbb{R}_+$. We will sometimes use the *bold* letter $\mathbf{x}$ to represent concatenation of a sequence of vectors $\{x_i\}$ as $\mathbf{x} = [x_0^\top, x_1^\top, \dots]^\top$, and $\mathbf{x}[i] := x_i$. For any two vectors (matrices) x and y, $x \otimes y$ stands for their Kronecker product. Moreover, $\mathbf{0}_n$, $\mathbb{1}_n$, and I_n are the zero vector, one vector, and identity matrix of dimension n, respectively.

B. System Description & Definitions

We consider discrete-time linear time-invariant (LTI) systems given by

$$x(t+1) = Ax(t) + Bu(t), \quad t \in \mathbb{N} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state, $u_t \in \mathcal{U} \subseteq \mathbb{R}^m$ is the input with $\mathcal{U}$ being the input constraint set, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the matrices of the *real* system. In this paper, we consider a stabilizable pair (A, B), which is a standard assumption in the field [13], [15]. The set $\mathcal{U}$ is described using a set of linear constraints as

$$\mathcal{U} = \{u \in \mathbb{R}^m \mid F_u u \leq \mathbb{1}_{c_u}\}, \tag{2}$$

where $F_u \in \mathbb{R}^{c_u\times m}$ with c_u the number of input constraints. Note that $0 \in \mathcal{U}$, and hence $\mathcal{U}$ is nonempty. For a general model $x(t + 1) = f(x(t), u(t))$, the *open-loop* predicted state, starting from any state $x \in \mathcal{X}$ and being predicted k steps forward under the control sequence $\mathbf{u}$, is denoted as $\psi_x(k, x, \mathbf{u})$. For linear systems characterized by (A, B), it is commonly known that

$$\psi_x(k, x, \mathbf{u}) = A^k x + \sum_{i=0}^{k-1} A^{k-1-i} B^i \mathbf{u}[i]. \tag{3}$$

On the other hand, given a state-feedback control law $\mu : \mathcal{X} \to \mathcal{U}$, we denote the corresponding *closed-loop* predicted state, starting from any state $x \in \mathcal{X}$ and being predicted k steps forward, as $\phi_x^{[\mu]}(k, x)$. For the system described in (1), the controller only has access to an estimated system governed by the matrices $\hat{A} \in \mathcal{A}(A, \delta_A) \subseteq \mathbb{R}^{n\times n}$ and $\hat{B} \in \mathcal{B}(B, \delta_B) \subseteq \mathbb{R}^{n\times m}$, where the uncertainty sets $\mathcal{A}(A, \delta_A)$ and $\mathcal{B}(B, \delta_B)$ are defined as

$$\mathcal{A}(A, \delta_A) = \{M \in \mathbb{R}^{n\times n} \mid \|A - M\|_{\mathrm{F}} \leq \delta_A\}, \tag{4a}$$
$$\mathcal{B}(B, \delta_B) = \{M \in \mathbb{R}^{n\times m} \mid \|B - M\|_{\mathrm{F}} \leq \delta_B\}, \tag{4b}$$

where the parameters $\delta_A \geq 0$ and $\delta_B \geq 0$ are characterized using system identification or machine learning techniques before initiating the control task. For the open-loop and closed-loop predicted state of the estimated system, we use the notation $\hat{\psi}_x(k, x, \mathbf{u})$ and $\hat{\phi}_x^{[\mu]}(k, x)$, respectively. Although we do not know (A, B) in advance, as indicated above, we assume that (A, B) is stabilizable. This stabilizability condition is also imposed when determining $(\hat{A}, \hat{B})$, leading to the following assumption:

Assumption 1: The pair $(\hat{A}, \hat{B})$ of is stabilizable.

Finally, for convenience in the analysis in Section IV, we provide the following definition:

Definition 1 (Error-consistent function): We call a function $\alpha(\cdot,\cdot) : \mathbb{R}_+^2 \to \mathbb{R}_+$ to be error-consistent if the following conditions hold:

1) $\alpha(\delta_1, \cdot)$ is non-decreasing for any $\delta_1 \in \mathbb{R}^+$,
2) $\alpha(\cdot, \delta_2)$ is non-decreasing for any $\delta_2 \in \mathbb{R}^+$,
3) $\alpha(\delta_1, \delta_2) = 0$ if and only if $\delta_1 = \delta_2 = 0$.

III. Problem Formulation

We consider an infinite-horizon optimal control problem, in which the controller aims to generate an input sequence $\mathbf{u}$ that stabilizes the system (i.e., steers the state to the origin) while minimizing the performance metric

$$J_\infty(x, \mathbf{u}_\infty) := \sum_{t=0}^{\infty} l(x_t, u_t), \tag{5}$$

where $x_0 = x \in \mathcal{X}$. In this work, we consider a quadratic stage cost given by $l(x, u) = \|x\|_Q^2 + \|u\|_R^2$, where $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are symmetric matrices satisfying $Q, R \succ 0$, and we define $l^*(x) = \min_{u\in\mathcal{U}} l(x, u) = \|x\|_Q^2$. In addition, for any linear control law $u = \kappa(x) = Kx$, we define the local region $\Omega_K = \{x \in \mathcal{X} \mid l^*(x) \leq \varepsilon_K\}$ with $\varepsilon_K > 0$ such that $Kx \in \mathcal{U}$ for all $x \in \Omega_K$. The maximum ϵ_K can be computed by solving a min-max optimization problem, the explicit form of which is given as

$$\varepsilon_K = \min_i \frac{1}{\|[F_u K]_{(i,:)}^\top\|_{Q^{-1}}^2}, \tag{6}$$

where $[F_u K]_{(i,:)}$ denotes the i-th row of the matrix $F_u K$. Given the dynamics in (1) and the input constraint set in (2), we consider the following optimization problem:

$$\begin{aligned}
\mathrm{P_{IH-OCP}} : \min_{\{u_t\}_{t=0}^\infty} \; & \sum_{t=0}^{\infty} \|x_t\|_Q^2 + \|u_t\|_R^2 \\
\text{s.t.} \; & x_{t+1} = Ax_t + Bu_t, \forall t \in \mathbb{N}, \\
& u_t \in \mathcal{U}, \forall t \in \mathbb{N}, \\
& x_0 = x(0).
\end{aligned}$$

Starting from $x = x(0)$, we denote the optimal solution to the problem $\mathrm{P_{IH-OCP}}$ by $\mathbf{u}_\infty^*(x)$; the associated optimal value of the cost function is $V_\infty(x) := J_\infty(x, \mathbf{u}_\infty^*(x))$.

Assumption 2: The initial state $x = x(0)$ lies in the region of attraction $\mathcal{X}_{\mathrm{ROA}}$ of the considered system in (1) such that, given the input constraints in (2), $V_\infty(x)$ is *finite* for all $x \in \mathcal{X}_{\mathrm{ROA}}$.

Corollary 1: Under Assumption 2, $\mathcal{X}_{\mathrm{ROA}}$ is a control invariant set, i.e., for all $x \in \mathcal{X}_{\mathrm{ROA}}$, there exists $u \in \mathcal{U}$ such that $Ax + Bu \in \mathcal{X}_{\mathrm{ROA}}$.

However, due to the infinite nature of $\mathrm{P_{IH-OCP}}$, computing the optimal input is intractable. Therefore, we consider an approximated performance metric as

$$J_N(x(t), \mathbf{u}_N) = \sum_{k=0}^{N} l(x_{k|t}, u_{k|t}), \tag{8}$$

where $N \geq 1$ is the prediction horizon, and $x_{k|t}$ and $u_{k|t}$ are, respectively, the k-step forward predicted state and input with $x_{0|t} = x(t)$ being the true state at time step t. In this work, we consider the formulation without a terminal cost, as also done in [14], [16]. At each time step t, the *ideal* MPC controller that has access to the true system solves the following optimization problem:

$$\begin{aligned}\mathrm{P}_{\mathrm{ID-MPC}}: \min_{\{u_{k|t}\}_{k=0}^{N}} & \sum_{k=0}^{N} \left(\|x_{k|t}\|_Q^2 + \|u_{k|t}\|_R^2\right) \\ \text{s.t. } & x_{k+1|t} = Ax_{k|t} + Bu_{k|t}, \forall k \in \mathbb{Z}_{N-1}^+, \\ & u_{k|t} \in \mathcal{U}, \forall k \in \mathbb{Z}_N^+, \\ & x_{0|t} = x(t).\end{aligned}$$

The optimal solution to the problem $\mathrm{P}_{\mathrm{ID-MPC}}$ is denoted by $\mathbf{u}_N^*(x(t))$, and the corresponding value of the cost function is $V_N(x(t)) := J_N(x(t), \mathbf{u}_N^*(x(t)))$. Moreover, the problem $\mathrm{P}_{\mathrm{ID-MPC}}$ implicitly defines an ideal MPC control law as $\mu_N(x(t)) := \mathbf{u}_N^*(x(t))[0]$. Nonetheless, in reality, the *real* MPC controller can only rely on the *estimated* system, and it instead solves the following optimization problem:

$$\begin{aligned}\mathrm{P}_{\mathrm{RE-MPC}}: \min_{\{u_{k|t}\}_{k=0}^{N}} & \sum_{k=0}^{N} \left(\|x_{k|t}\|_Q^2 + \|u_{k|t}\|_R^2\right) \\ \text{s.t. } & x_{k+1|t} = \hat{A}x_{k|t} + \hat{B}u_{k|t}, \forall k \in \mathbb{Z}_N^+, \\ & u_{k|t} \in \mathcal{U}, \forall k \in \mathbb{Z}_N^+, \\ & x_{0|t} = x(t).\end{aligned}$$

Likewise, we denote the optimal solution to the problem $\mathrm{P}_{\mathrm{RE-MPC}}$ by $\hat{\mathbf{u}}_N^*(x(t))$, the value of the cost function follows as $\hat{V}_N(x(t)) := J_N(x(t), \hat{\mathbf{u}}_N^*(x(t)))$, and the real MPC control law is defined as $\hat{\mu}_N(x(t)) := \hat{\mathbf{u}}_N^*(x(t))[0]$. We apply the real MPC controller recursively, and the resulting performance, starting from $x(0) = x$, is

$$J_\infty^{[\hat{\mu}_N]}(x) = \sum_{t=0}^{\infty} l\left(\phi_x^{[\hat{\mu}_N]}(t, x), \hat{\mu}_N\left(\phi_x^{[\hat{\mu}_N]}(t, x)\right)\right) \quad (11)$$

The main objectives of this paper are twofold: (i) to investigate under which conditions the MPC control law $\hat{\mu}_N$ can stabilize the system and (ii) to quantify the closed-loop performance of the real MPC controller $J_\infty^{[\hat{\mu}_N]}(x)$ relative to the optimal infinite-horizon cost $V_\infty(x)$.

Remark 1 (Nullified input at stage N): For both the optimization problems $\mathrm{P}_{\mathrm{ID-MPC}}$ and $\mathrm{P}_{\mathrm{RE-MPC}}$, the optimal input satisfies $u_{N|t}^* = 0$ due to the positive definiteness of the quadratic cost. We include $u_{N|t}$ in our formulation to be consistent with the one without a terminal cost.

Remark 2 (State constraints & region of attraction): In this work, we do not consider an explicit state constraint set $\mathcal{X}$. However, in the presence of hard input constraints, stabilizing the system may not be possible for any arbitrary initial state $x(0) = x$, especially for unstable systems with $\|A\|_2 \geq 1$. Therefore, we impose Assumption 2 for the validity of our work. Similar assumptions have been made in [17] and [18] using the notion of cost controllability. Characterizing $\mathcal{X}_{\mathrm{ROA}}$ without knowing the true system is an open issue and is out of the scope of this paper.

IV. Theoretical Analysis

In this section, we provide an analysis of the stability and the closed-loop performance of the MPC controller with model mismatch. In Section IV-A, we establish a relation between the MPC value function $\hat{V}_N$ and the infinite-horizon optimal value function V_∞, leveraging the sensitivity analysis of quadratic programs (QPs). In Section IV-B, we theoretically derive a performance bound using the RDP inequality.

A. Evaluation of the MPC value function

We first establish an upper bound of $\hat{V}_N$ in terms of V_∞. It should be noted that these two value functions are constructed using different dynamic models. We first state the main result and highlight some interpretations.

Proposition 1: There exist two error-consistent functions $\alpha_N(\delta_A, \delta_B)$ and $\beta_N(\delta_A, \delta_B)$ such that, for all $x \in \mathcal{X}_{\mathrm{ROA}}$, $\hat{V}_N$ and V_∞ satisfy the following inequality[1]:

$$\hat{V}_N(x) \leq \left(1 + \alpha_N(\delta_A, \delta_B)\right)V_\infty(x) + \beta_N(\delta_A, \delta_B), \quad (12)$$

where functions α_N and β_N relate to the eigenvalues and matrix norms of $\hat{A}$, $\hat{B}$, Q, R, the input constraint set $\mathcal{U}$, but not to quantities derived from A or B.

The upper bound in (12) is consistent with the bound without model mismatch: $V_N(x) \leq V_\infty(x)$. Indeed, if $\delta_A = \delta_B = 0$, $\hat{V}_N$ is identical to V_N, and (12) degenerates to $V_N(x) \leq V_\infty(x)$ since both α_N and β_N are error-consistent. Furthermore, α_N and β_N are *computable* in that they do not depend on the matrix pair (A, B) of the true system. Explicit expressions of α_N and β_N are given in Appendix C. Establishing the upper bound in (12) frequently uses a lemma about the weighted quadratic norms, whose details can be found in (38) in Appendix A.

Due to space limits, we only provide a proof sketch of Prop. 1, highlighting the main steps. More details of the proof are available in the online version [19]. The main course of the proof is composed of the following four steps:

Step 1): Expanding $\hat{V}_N$ in terms of $\hat{\psi}_x(\cdot, x, \hat{\mathbf{u}}_N^*(x))$ and $\hat{\mathbf{u}}_N^*(x)$. By definition, we have

$$\hat{V}_N(x) = \sum_{k=0}^{N} \left(\|\hat{\psi}_x(k, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 + \|\left(\hat{\mathbf{u}}_N^*(x)\right)[k]\|_R^2\right).$$

Step 2): Keeping the input $\hat{\mathbf{u}}_N^*(x)$ unchanged and decomposing $\hat{\psi}_x(\cdot, x, \hat{\mathbf{u}}_N^*(x))$ into $\psi_x(\cdot, x, \hat{\mathbf{u}}_N^*(x))$ (i.e., the open-loop predicted state using the true system model under input $\hat{\mathbf{u}}_N^*(x)$) and $e_\psi(\cdot) := \psi_x(\cdot, x, \hat{\mathbf{u}}_N^*(x)) - \hat{\psi}_x(\cdot, x, \hat{\mathbf{u}}_N^*(x))$ (i.e., the open-loop prediction error under input $\hat{\mathbf{u}}_N^*(x)$). Using Lemma 8, the remaining task is to derive an upper bound for $\|e_\psi(\cdot)\|_Q^2$, which further requires bounding the terms $\|A^k - \hat{A}^k\|_2$ and $\|A^k B - \hat{A}^k \hat{B}\|_2$ for $k \in \mathbb{Z}_N^+$.

Step 3): Decomposing the input $\hat{\mathbf{u}}_N^*(x)$ into $\mathbf{u}_N^*(x)$ (i.e., the optimal solution of the problem $\mathrm{P}_{\mathrm{ID-MPC}}$) and $\delta\mathbf{u}(x) := \mathbf{u}_N^*(x) - \hat{\mathbf{u}}_N^*(x)$. Similar to Step 2, the main difficulty is to derive an upper bound for $\|\delta\mathbf{u}(x)\|_2$. We first transform the

[1]The explicit dependence of α_N and β_N on the prediction horizon is highlighted using the subscript N.

problems $\mathrm{P}_{\mathrm{ID-MPC}}$ and $\mathrm{P}_{\mathrm{RE-MPC}}$, respectively, into their QP formulations using system-level synthesis, then we apply results in sensitivity analysis of QPs [20] to provide an upper bound as a function of δ_A and δ_B.

Step 4): Decomposing $\psi_x(\cdot, x, \hat{\mathbf{u}}_N^*(x))$ into $\psi_x(\cdot, x, \mathbf{u}_N^*(x))$ (i.e., the open-loop predicted state using the true system model under input $\mathbf{u}_N^*(x)$) and $\psi_x(\cdot, 0, \delta\mathbf{u}(x))$ (i.e., the open-loop predicted state, starting from $x = 0$, using the true system model under input $\delta\mathbf{u}(x)$). The core task is to obtain an upper bound for $\|\psi_x(\cdot, 0, \delta\mathbf{u}(x))\|_Q^2$, which further reduces to obtaining an upper bound for $\|\delta\mathbf{u}(x)\|_Q^2$ based on linearity. Therefore, the intermediate results of Step 3 can be used again.

B. Closed-loop Stability and Performance Analysis

Under the MPC control law $\hat{\mu}_N$, at a given state x, the next-step state is computed as $x_{\mathrm{re}}^+ = Ax + B\hat{\mu}_N(x)$. If there exists a so-called energy function $\tilde{V} : \mathbb{R}^n \to \mathbb{R}_+$ such that the RDP inequality

$$\tilde{V}(x_{\mathrm{re}}^+) - \tilde{V}(x) \leq -\epsilon l(x, \hat{\mu}_N(x)) \tag{13}$$

holds for all $x \in \mathcal{X}_{\mathrm{ROA}}$, where $\epsilon \in (0, 1]$, then the controlled system is asymptotically stable [9], [21]. In this paper, we investigate the case where $\tilde{V} = \hat{V}_N$ and theoretically derive the coefficient ϵ as a function of the prediction horizon N and the parameters δ_A and δ_B that quantifies the model mismatch.

We first present a preliminary result that can be applied to both the true system and the estimated system.

Lemma 1: Given a stabilizable linear system, a positive-definite quadratic stage cost $l(x, u) = \|x\|_Q^2 + \|u\|_R^2$, and a local linear stabilizing control law $u = \kappa(x) = Kx$, there exist scalars $\lambda_K > 0$ and $\rho_K \in (0, 1)$ such that for all $x \in \Omega_K$ and $k \in \mathbb{N}$ we have

$$l(\phi_x^{[\kappa]}(k, x), K\phi_x^{[\kappa]}(k, x)) \leq C_K^*(\rho_K)^k l^*(x), \tag{14}$$

where $C_K^* = \left(1 + \underline{\sigma}_Q^{-1}\overline{\sigma}_R\|K\|_2^2\right)\max\{1, r_Q(\lambda_K)^2\}$.

Proof: The proof is given in Appendix B. □

Based on Lemma 1, we have the following result about the properties of the open-loop system:

Lemma 2: Given an upper bound $M_{\hat{V}}$ of $\hat{V}_N$ for all $x = x(0)$ and a stabilizing control law $u = Kx$, there exist constants $L_{\hat{V}} := \max\{\gamma, \frac{M_{\hat{V}}}{\varepsilon_K}\}$ and $N_0 := \left\lceil \max\{0, \frac{M_{\hat{V}} - \gamma\varepsilon_K}{\varepsilon_K}\}\right\rceil$[2] such that for $N \geq N_0$ we have

$$\hat{V}_N(x) \leq L_{\hat{V}} l^*(x) \leq L_{\hat{V}} l(x, \hat{\mu}_N(x)) \tag{15a}$$
$$\|\hat{\psi}_x(N, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 \leq \gamma\rho_\gamma^{N-N_0} l(x, \hat{\mu}_N(x)), \tag{15b}$$

where $\gamma = C_K^*(1 - \rho_K)^{-1}$ and $\rho_\gamma = \gamma^{-1}(\gamma - 1)$ with C_K^* and ρ_K given as in Lemma 1.

Proof: The proof follows a similar procedure as in [17, Theorem 4] and is based on Lemma 1. □

For a stabilizable system, (15a) and (15b) provides an upper bound, respectively, for the value function $\hat{V}_N$ and the cost of the final state, both in terms of the stage cost. Moreover, (15b) implies that, given a sufficiently long horizon, the cost of the final state exponentially decays as the horizon increases. The *critical horizon* N_0 quantifies the required number of steps such that the final state lies in the local region Ω_K.

Proposition 2: There exist a constant η_N and an error-consistent function ξ_N satisfying $\xi_N(\delta_A, \delta_B) + \eta_N < 1$, and for all $x \in \mathcal{X}_{\mathrm{ROA}}$ we have

$$\hat{V}_N(x_{\mathrm{re}}^+) - \hat{V}_N(x) \leq - (1 - \xi_N(\delta_A, \delta_B) - \eta_N) l(x, \hat{\mu}_N(x)), \tag{16}$$

where the function ξ_N and constant η_N relate to the eigenvalues and matrix norms of $\hat{A}$, $\hat{B}$, Q, R, the input constraint set $\mathcal{U}$, but not to quantities derived from A or B.

In Proposition 2, the relation in (16), serving as the RDP inequality for the closed-loop systems (cf. (13)), provides a lower bound of the energy decrease in terms of the stage cost. Moreover, based on $\xi_N(\delta_A, \delta_B) + \eta_N < 1$, a *sufficient* condition on the prediction horizon and modeling error can be derived such that the closed-loop system is stable. The explicit expressions of ξ_N and η_N are given, respectively, in (52) and (51) in Appendix D. The sufficient condition is summarized in the following corollary:

Corollary 2: Given a sufficiently long prediction horizon N satisfying

$$N > N_0 + \frac{\log[(C_{\hat{K}}^* + \|\hat{A}_{\mathrm{cl}}\|_2^2 r_Q - 1)\gamma]}{\log(\rho_\gamma^{-1})},$$

where N_0 is defined as in Lemma 2 using gain K and the other variables can be found in Appendix D, the closed-loop system is asymptotically stable if the modeling error is small enough such that

$$h(\delta_A, \delta_B) < \left\{ \frac{-\omega_{N,(\frac{1}{2})} + [\omega_{N,(\frac{1}{2})}^2 + \omega_{N,(1)}(1 - \eta_N)]^{\frac{1}{2}}}{\omega_{N,(1)}} \right\}^2, \tag{17}$$

where h is defined as in (36) in Lemma 12, η_N is given as in (51), and $\omega_{N,(1)}$ and $\omega_{N,(\frac{1}{2})}$ are given, respectively, as in (54a) and (54b).

Informally, Corollary 2 indicates that the model mismatch should be small enough to ensure the stability of the closed-loop system if the MPC control law is derived from the estimated model instead of the true model. Next, we provide a proof sketch of Proposition 2 due to space limitations. The detailed proof can be found in the online version [19]. The main part of the proof is to establish the upper bound for $\hat{V}_N(x_{\mathrm{re}}^+) - \hat{V}_N(x)$ in terms of $l(x, \hat{\mu}_N(x))$, and it consists of four steps:

Step 1): Expanding $\hat{V}_N(x_{\mathrm{re}}^+)$ in terms of $\hat{\mathbf{u}}_N^*(x_{\mathrm{re}}^+)$ and $\hat{\psi}_x(\cdot, x_{\mathrm{re}}^+, \hat{\mathbf{u}}_N^*(x_{\mathrm{re}}^+))$, and expanding $\hat{V}_N(x)$ in terms of $\hat{\mathbf{u}}_N^*(x)$ and $\hat{\psi}_x(\cdot, x, \hat{\mathbf{u}}_N^*(x))$.

Step 2): Constructing an auxiliary input $\mathbf{v}_N(x)$ satisfying $\left(\mathbf{v}_N(x)\right)[i-1] = \left(\hat{\mathbf{u}}_N^*(x)\right)[i]$ for $i \in \mathbb{Z}_{N-1}^+$, $\left(\mathbf{v}_N(x)\right)[N-1] = K\hat{\psi}_x(N-1, x_{\mathrm{re}}^+, \mathbf{v}_{N-1}(x))$, and $\left(\mathbf{v}_N(x)\right)[N] = 0$,

[2] $\lceil\cdot\rceil$ is the standard ceiling operator (i.e., the least integer operator).

and then relaxing $\hat{V}_N(x_{\text{re}}^+)$ as $J_N(x_{\text{re}}^+, \mathbf{v}_N(x))$. This auxiliary input has two benefits when computing the difference $J_N(x_{\text{re}}^+, \mathbf{v}_N(x)) - \hat{V}_N(x)$:

1) The input-incurred cost $\|(\mathbf{v}_N(x))[i-1]\|_R^2$ and $\|(\hat{\mathbf{u}}_N^*(x))[i]\|_R^2$ cancel each other;
2) We can prove that $\hat{\psi}_x(i-1, x_{\text{re}}^+, \mathbf{v}_N(x)) = \hat{A}^{i-1}\Delta x + \hat{\psi}_x(i, x, \hat{\mathbf{u}}_N^*(x))$ for all $i \in \mathbb{Z}_N^+$, where $\Delta x = (A - \hat{A})x + (B - \hat{B})\hat{\mu}_N(x)$ is the one-step-ahead prediction error under the MPC control law $\hat{\mu}_N$.

Step 3): Reorganizing the terms obtained from Step 2 and applying Lemma 8, we build a relation between $\|\hat{A}^{i-1}\Delta x\|_Q^2$ and $l(x, \hat{\mu}_N(x))$.

Step 4): Using the property given in Remark 1, the remaining term is $\|\hat{\psi}_x(N-1, x_{\text{re}}^+, \mathbf{v}_N(x))\|_Q^2 + \|(\mathbf{v}_N(x))[N-1]\|_R^2 + \|\hat{\psi}_x(N, x_{\text{re}}^+, \mathbf{v}_N(x))\|_Q^2 - \|\hat{\psi}_x(N, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2$, which can be further bounded using Lemma 1 and Lemma 2.

The final infinite-horizon performance guarantee is stated in the following theorem:

Theorem 1: Given α_N and β_N as in Proposition 1, and ξ_N and η_N as in Proposition 2, we have

$$J_\infty^{[\hat{\mu}_N]}(x) \leq \frac{1 + \alpha_N}{1 - \xi_N - \eta_N} V_\infty(x) + \frac{\beta_N}{1 - \xi_N - \eta_N}, \quad (18)$$

where $J_\infty^{[\hat{\mu}_N]}(x)$ is defined as in (11).

Proof: The proof resembles that in [11, Prop. 2.2]. By performing a telescopic sum of (16), for any $T \in \mathbb{N}$, we have

$$(1 - \xi_N - \eta_N) \sum_{t=0}^{T-1} l\left(\phi_x^{[\hat{\mu}_N]}(t, x), \hat{\mu}_N(\phi_x^{[\hat{\mu}_N]}(t, x))\right) \leq \hat{V}_N(x) - \hat{V}_N(\phi_x^{[\hat{\mu}_N]}(T, x)). \quad (19)$$

Taking $T \to \infty$, using the performance definition in (11) and $0 \leq \hat{V}_N(\phi_x^{[\hat{\mu}_N]}(\infty, x))$, (19) yields

$$(1 - \xi_N - \eta_N) J_\infty^{[\hat{\mu}_N]}(x) \leq \hat{V}_N(x). \quad (20)$$

Then, substituting the bound in (12) into (20) leads to

$$(1 - \xi_N - \eta_N) J_\infty^{[\hat{\mu}_N]}(x) \leq \alpha_N V_\infty(x) + \beta_N, \quad (21)$$

which gives the final bound in (18) after dividing both sides by the constant $1 - \xi_N - \eta_N$. □

Given that α_N, β_N, and ξ_N are all error-consistent, the worst-case performance bound as in (18) will increase if the modeling error is in general larger (cf. the non-decreasing property of the error-consistent functions in Def. 1). Besides, if the model is perfect, the final performance bound will degenerate to

$$J_\infty^{[\mu_N]}(x) \leq \frac{1}{1 - \eta_N} V_\infty(x). \quad (22)$$

Note that (22) is a variant of the bounds given in [11], [17] without modeling error.

V. Numerical Example

Consider a linear system of the form (1) specified by

$$A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

and the input constraint set in (2) specified by $F_u = [1, -1]^\top$. We choose the parameters $\delta_A = \delta_B = 0.01$ to quantify the uncertainty level in the model. In addition, the matrices Q and R in the quadratic cost are given by

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = 1.$$

We consider an estimated model with

$$\hat{A} = \begin{bmatrix} 0.99 & 0.1 \\ 0 & 1 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0 \\ 0.11 \end{bmatrix}$$

We simulate four cases with different initial states $x(0)$, and the prediction horizon of the MPC controller is chosen as $N = 10$. Relevant quantities of the final performance bound as in (18) are summarized in TABLE I.

TABLE I

Performance bound validation with $N = 10$.

$x(0)$	$J_\infty^{[\hat{\mu}_N]}(x(0))$	$V_\infty(x(0))$	$\frac{1+\alpha_N}{1-\xi_N-\eta_N}$	$\frac{\beta_N}{1-\xi_N-\eta_N}$
$[1, 1]^\top$	91.7019	76.1933	9.2935	103.3456
$[1, -1]^\top$	16.9365	15.4439	6.2107	53.7721
$[-2, 2]^\top$	70.5044	64.7976	8.2256	90.2059
$[-2, -2]^\top$	669.1805	634.1123	23.4417	389.0821

Based on the numerical data provided in TABLE I, the derived performance bound is valid but conservative. This conservatism primarily arises from repeatedly applying inequalities, leading to accumulated relaxation. Additionally, opting for the Frobenius norm to quantify the bound on input difference through sensitivity analysis of QPs introduces extra conservatism. Exploring alternative methods to offer a tighter bound is left for future work.

VI. Conclusions

This paper has provided stability and closed-loop performance analysis of MPC for uncertain linear systems. We have proposed a performance bound quantifying the suboptimality gap between MPC controllers using the estimated model and the theoretically expert infinite-horizon optimal controller with access to the true system model. Additionally, we have established a sufficient condition on the prediction horizon and the model mismatch for ensuring stability in the closed-loop system. Furthermore, our analysis reveals how the prediction horizon and model mismatch jointly influence the performance. These insights offer valuable guidance for designing and implementing MPC controllers for uncertain linear systems. Potential future works include extending the existing analysis framework to MPC with terminal costs and analyzing the performance of learning-based MPC that learns the model on the fly.

Appendix

A. Matrix Inequalities

Lemma 3: Given matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$ and any well-posed matrix norm $\|\cdot\|$ that possesses sub-additivity and sub-multiplicativity, we have

$$\|M_1^i - M_2^i\| \le (\delta_M + \|M_2\|)^i - \|M_2\|^i, \tag{23}$$

where $i \in \mathbb{N}$ and $\|M_1 - M_2\| \le \delta_M$.

Proof: For $i = 0$, we have $\|M_1^i - M_2^i\| = \|I_n - I_n\| = 0$ and $(\delta_M + \|M_2\|)^i - \|M_2\|^i = 1 - 1 = 0$, indicating $\|M_1^i - M_2^i\| = (\delta_M + \|M_2\|)^i - \|M_2\|^i$. On the other hand, for $i \ge 1$, we have

$$\begin{aligned}
&\|M_1^i - M_2^i\| \\
=&\|(M_1 - M_2)(M_1^{i-1} + M_1^{i-2}M_2 + \cdots + M_2^{i-1})\| \\
=&\|M_1 - M_2\| \sum_{j=0}^{i-1} \|M_1^{i-j-1}M_2^j\| \\
=&\|M_1 - M_2\| \sum_{j=0}^{i-1} \|(M_1 - M_2) + M_2\|^{i-j-1}\|M_2\|^j \\
\le&\delta_M \sum_{j=0}^{i-1} (\delta_M + \|M_2\|)^{i-j-1}\|M_2\|^j \\
\le&\big[(\delta_M + \|M_2\|) - \|M_2\|\big] \sum_{j=0}^{i-1} (\delta_M + \|M_2\|)^{i-j-1}\|M_2\|^j \\
\le&(\delta_M + \|M_2\|)^i - \|M_2\|^i,
\end{aligned}$$

which completes the proof. □

Lemma 4: Given matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$, $N_1, N_2 \in \mathbb{R}^{n\times m}$, and any well-posed matrix norm $\|\cdot\|$ that possesses sub-additivity and sub-multiplicativity, we have

$$\begin{aligned}
\|M_1^iN_1 - M_2^iN_2\| \le \delta_N\|M_2\|^i + \\
+ (\delta_N + \|N_2\|)\big[(\delta_M + \|M_2\|)^i - \|M_2\|^i\big],
\end{aligned} \tag{24}$$

where $i \in \mathbb{N}$, $\|M_1 - M_2\| \le \delta_M$, and $\|N_1 - N_2\| \le \delta_N$.

Proof:

$$\begin{aligned}
&\|M_1^iN_1 - M_2^iN_2\| \\
=&\|M_1^iN_1 - M_2^iN_1 + M_2^iN_1 - M_2^iN_2\| \\
=&\|M_2^i(N_1 - N_2) + (M_1^i - M_2^i)N_1\| \\
\le&\|M_2\|^i\|N_1 - N_2\| + \|M_1^i - M_2^i\|\|(N_1 - N_2) + N_2\| \\
\le&\delta_N\|M_2\|^i + (\delta_N + \|N_2\|)\|M_1^i - M_2^i\| \\
\le&\delta_N\|M_2\|^i + (\delta_N + \|N_2\|)\big[(\delta_M + \|M_2\|)^i - \|M_2\|^i\big],
\end{aligned}$$

where the last inequality is due to Lemma 3. □

Lemma 5: Given the matrices $\hat{A}$ and $\hat{B}$ satisfying $\hat{A} \in \mathcal{A}(A, \delta_A) \subseteq \mathbb{R}^{n\times n}$ and $\hat{B} \in \mathcal{B}(B, \delta_B) \subseteq \mathbb{R}^{n\times m}$, where $\mathcal{A}(A, \delta_A)$ and $\mathcal{B}(B, \delta_B)$ are defined as in (4), we have

$$\|A^i - \hat{A}^i\|_2 \le (\delta_A + \|\hat{A}\|_2)^i - (\|\hat{A}\|_2)^i, \tag{25a}$$

$$\begin{aligned}
\|A^iB - \hat{A}^i\hat{B}\|_2 \le \delta_B(\|\hat{A}\|_2)^i + \\
+ (\delta_B + \|\hat{B}\|_2)\big[(\delta_A + \|\hat{A}\|_2)^i - (\|\hat{A}\|_2)^i\big],
\end{aligned} \tag{25b}$$

where $i \in \mathbb{N}$.

Proof: Applying Lemma 3 to A and $\hat{A}$ directly leads to (25a). Likewise, (25b) is a direct consequence of applying Lemma 4 to the set of matrices A, B, $\hat{A}$ and $\hat{B}$. □

Based on (25a) and (25b), we define two error-consistent functions $g_{i,(x)}^{(n)}$ and $g_{i,(u)}^{(n)}$ for all $n \in \mathbb{N}_+$ as

$$g_{i,(x)}^{(n)}(\delta_A, \cdot) = \big[(\delta_A + \|\hat{A}\|_2)^i - (\|\hat{A}\|_2)^i\big]^n, \tag{26a}$$

$$\begin{aligned}
g_{i,(u)}^{(n)}(\delta_A, \delta_B) = \Big\{(\delta_B + \|\hat{B}\|_2)g_{i,(x)}^{(1)}(\delta_A, \cdot) + \\
+ \delta_B(\|\hat{A}\|_2)^i\Big\}^n.
\end{aligned} \tag{26b}$$

Lemma 6: Given matrices $M_1, M_2 \in \mathbb{R}^{p\times q}$, $\Xi \in \mathbb{R}^{q\times n}$, $N_1, N_2 \in \mathbb{R}^{n\times m}$, and any well-posed matrix norm $\|\cdot\|$ that possesses sub-additivity and sub-multiplicativity, we have

$$\begin{aligned}
\|M_1\Xi N_1 - M_2\Xi N_2\| \le \|\Xi\|\|M_1 - M_2\|\|N_1 - N_2\| + \\
+ \|M_1\Xi\|\|N_1 - N_2\| + \|\Xi N_1\|\|M_1 - M_2\|
\end{aligned} \tag{27}$$

Proof:

$$\begin{aligned}
&\|M_1\Xi N_1 - M_2\Xi N_2\| \\
=&\|M_1\Xi N_1 - M_1\Xi N_2 + M_1\Xi N_2 - M_2\Xi N_2\| \\
\le&\|M_1\Xi(N_1 - N_2)\| + \|(M_1 - M_2)\Xi N_2\| \\
\le&\|M_1\Xi\|\|N_1 - N_2\| + \|M_1 - M_2\|\|\Xi N_1\| + \\
&+ \|M_1 - M_2\|\|\Xi(N_2 - N_1)\| \\
\le&\|M_1\Xi\|\|N_1 - N_2\| + \|M_1 - M_2\|\|\Xi N_1\| + \\
&+ \|\Xi\|\|M_1 - M_2\|\|N_1 - N_2\|
\end{aligned}$$

□

Corollary 3: Given matrices $M_1, M_2 \in \mathbb{R}^{p\times q}$, $\Xi \in \mathbb{R}^{p\times p}$, and any well-posed matrix norm $\|\cdot\|$ that possesses sub-additivity and sub-multiplicativity, we have

$$\begin{aligned}
\|M_1^\top\Xi M_1 - M_2^\top\Xi M_2\| \le \|\Xi\|\|M_1 - M_2\|^2 + \\
+ (\|M_1^\top\Xi\| + \|\Xi M_1\|)\|M_1 - M_2\|.
\end{aligned} \tag{28}$$

Lemma 7: Given matrices A and B as in (1) and their estimated counterparts $\hat{A}$ and $\hat{B}$, we define $\overline{Q}_N := I_N \otimes Q$,

$$\Phi_N := \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, \Gamma_N := \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix},$$

$$\hat{\Phi}_N := \begin{bmatrix} I \\ \hat{A} \\ \hat{A}^2 \\ \vdots \\ \hat{A}^N \end{bmatrix}, \hat{\Gamma}_N := \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \hat{B} & 0 & \cdots & 0 \\ \hat{A}\hat{B} & \hat{B} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \hat{A}^{N-1}\hat{B} & \hat{A}^{N-2}\hat{B} & \cdots & \hat{B} \end{bmatrix}.$$

The following results hold:

$$\begin{aligned}
\|\hat{\Gamma}_N^\top\overline{Q}_{N+1}\hat{\Phi}_N - \Gamma_N^\top\overline{Q}_{N+1}\Phi_N\|_2 \le \\
\overline{\sigma}_Q\big(\|\hat{\Gamma}_N\|_2\bar{g}_{(x)} + \|\hat{\Phi}_N\|_2\bar{g}_{(u)} + \bar{g}_{(x)}\bar{g}_{(u)}\big),
\end{aligned} \tag{29a}$$

$$\|\hat{\Gamma}_N^\top \overline{Q}_{N+1}\hat{\Gamma}_N - \Gamma_N^\top \overline{Q}_{N+1}\Gamma_N\|_2 \leq \overline{\sigma}_Q\big(2\|\hat{\Gamma}_N\|_2\bar{g}_{(x)} + \bar{g}^2_{(x)}\big), \quad (29b)$$

where $\bar{g}_{(x)}$ and $\bar{g}_{(u)}$ are, respectively, defined as $\bar{g}_{(x)} := \sum_{i=1}^N g^{(1)}_{i,(x)}$ and $\bar{g}_{(u)} := \sum_{i=1}^N \sum_{j=0}^{i-1} g^{(1)}_{j,(u)}$ with $g^{(1)}_{i,(x)}$ and $g^{(1)}_{i,(u)}$ given as in (26),

Proof: We first provide the proof of (29a). Due to Lemma 6, we have

$$\|\hat{\Gamma}_N^\top \overline{Q}_{N+1}\hat{\Phi}_N - \Gamma_N^\top \overline{Q}_{N+1}\Phi_N\|_2 \leq \|\overline{Q}_{N+1}\|_2\bigg(\|\hat{\Gamma}_N\|_2\|\hat{\Phi}_N - \Phi_N\|_2 + \\ + \|\hat{\Phi}_N\|_2\|\hat{\Gamma}_N - \Gamma_N\|_2 + \|\hat{\Phi}_N - \Phi_N\|_2\|\hat{\Gamma}_N - \Gamma_N\|_2\bigg).$$

In addition, due to the lemma 5 and the property of the spectral norm of block matrix [22], we have $\|\hat{\Phi}_N - \Phi_N\|_2 \leq \sum_{i=1}^N \|A^i - \hat{A}^i\|_2 \leq \sum_{i=1}^N g^{(1)}_{i,(x)}$ and $\|\hat{\Gamma}_N - \Gamma_N\|_2 \leq \sum_{i=1}^N \sum_{j=0}^{i-1} \|A^j B - \hat{A}^j\hat{B}\|_2 \leq \sum_{i=1}^N \sum_{j=0}^{i-1} g^{(1)}_{j,(u)}$. The final inequality as in (29a) is thus established by substituting the above results.

The proof of (29b) follows a similar procedure using Corollary 3. □

Based on (29a) and (29b), we further define

$$\theta_{N,(x)} = \overline{\sigma}_Q\big(2\|\hat{\Gamma}_N\|_2\bar{g}_{(x)} + \bar{g}^2_{(x)}\big), \quad (30a)$$

$$\theta_{N,(x,u)} = \overline{\sigma}_Q\big(\|\hat{\Gamma}_N\|_2\bar{g}_{(x)} + \|\hat{\Phi}_N\|_2\bar{g}_{(u)} + \bar{g}_{(x)}\bar{g}_{(u)}\big). \quad (30b)$$

Lemma 8: Given two sequences of vectors $\{a_i\}_{i=1}^N$ and $\{b_i\}_{i=1}^N$ where $a_i, b_i \in \mathbb{R}^n, \forall i \in \mathbb{Z}_N^+$ and a symmetric positive definite matrix $Q \in \mathbb{R}^{n\times n}$, we have

$$\sum_{i=1}^N \|a_i \pm b_i\|_Q^2 \leq \sum_{i=1}^N \Big[\|a_i\|_Q^2 + \|b_i\|_Q^2 + 2(\|a_i\|_Q^2\|b_i\|_Q^2)^{\frac{1}{2}}\Big] \\ \leq \sum_{i=1}^N(\|a_i\|_Q^2 + \|b_i\|_Q^2) + 2\left[(\sum_{i=1}^N \|a_i\|_Q^2)(\sum_{i=1}^N \|b_i\|_Q^2)\right]^{\frac{1}{2}} \quad (31)$$

Proof: We denote the Cholesky decomposition of the matrix Q by $Q = \Gamma_Q^\top\Gamma_Q$, then we can proceed as follows:

$$\sum_{i=1}^N \|a_i \pm b_i\|_Q^2 \\ \leq \sum_{i=1}^N \big[\|a_i\|_Q^2 + \|b_i\|_Q^2 + 2|a_i^\top Q b_i|\big] \\ \leq \sum_{i=1}^N \Big[\|a_i\|_Q^2 + \|b_i\|_Q^2 + 2(\|\Gamma_Q a_i\|_2^2\|\Gamma_Q b_i\|_2^2)^{\frac{1}{2}}\Big] \\ \leq \sum_{i=1}^N \Big[\|a_i\|_Q^2 + \|b_i\|_Q^2 + 2(\|a_i\|_Q^2\|b_i\|_Q^2)^{\frac{1}{2}}\Big],$$

where the second inequality holds due to Cauchy–Schwarz inequality. We thus proved the first inequality in (31), and further applying Cauchy–Schwarz inequality to the product $(\|a_i\|_Q^2\|b_i\|_Q^2)^{\frac{1}{2}}$ in the above result leads to

$$\sum_{i=1}^N \Big[\|a_i\|_Q^2 + \|b_i\|_Q^2 + 2(\|a_i\|_Q^2\|b_i\|_Q^2)^{\frac{1}{2}}\Big] \\ \leq \sum_{i=1}^N(\|a_i\|_Q^2 + \|b_i\|_Q^2) + 2\left[(\sum_{i=1}^N \|a_i\|_Q^2)(\sum_{i=1}^N \|b_i\|_Q^2)\right]^{\frac{1}{2}},$$

which is the second inequality in (31). □

B. Intermediate results

1) Proof of Lemma 1: Given a stabilizable matrix pair (A, B), by definition there exists a matrix K such that $\|A_{\mathrm{cl}}\|_2 := \|A + BK\|_2 < 1$. Due to Gelfand's formula [23, Lemma IX.1.8], we know that there exist scalars λ_K and $\rho_K \in (0, 1)$ s.t. $\|(A_{\mathrm{cl}})^k\|_2 \leq \lambda_K(\sqrt{\rho_K})^k$, $\forall k \in \mathbb{N}$. On the other hand, for any vector $x \in \mathbb{R}^n$, we have

$$\|x\|^2_{Q+K^\top RK} = \|x\|_Q^2 + \|Kx\|_R^2 \leq \|x\|_Q^2 + \overline{\sigma}_R\|Kx\|_2^2 \\ \leq \|x\|_Q^2 + \overline{\sigma}_R\|K\|_2^2\|x\|_2^2 \\ \leq (1 + \underline{\sigma}_Q^{-1}\overline{\sigma}_R\|K\|_2^2)\|x\|_Q^2.$$

Thus, $Q + K^\top RK \preceq (1 + \underline{\sigma}_Q^{-1}\overline{\sigma}_R\|K\|_2^2)Q$. As such, for $k \in \mathbb{N}_+$ and for the linear control law $u = \kappa(x) = Kx$, the closed-loop stage cost can be bounded as

$$l(\phi_x^{[\kappa]}(k, x), K\phi_x^{[\kappa]}(k, x)) \\ = x^\top\left\{[(A_{\mathrm{cl}})^k]^\top(Q + K^\top RK)[(A_{\mathrm{cl}})^k]\right\}x \\ \leq (1 + \underline{\sigma}_Q^{-1}\overline{\sigma}_R\|K\|_2^2)r_Q\|[(A_{\mathrm{cl}})^k]x\|_Q^2 \\ \leq (1 + \underline{\sigma}_Q^{-1}\overline{\sigma}_R\|K\|_2^2)r_Q(\lambda_K)^2(\rho_K)^k\|x\|_Q^2.$$

Taking $C_K^* = \big(1 + \underline{\sigma}_Q^{-1}\overline{\sigma}_R\|K\|_2^2\big)\max\{1, r_Q\lambda^2\}$, we arrive at (14) by combining the above results.

2) Linear bound on square root function: We provide a trivial lemma about the square root function.

Lemma 9: Given $x \in \mathbb{R}_+$, we have $\sqrt{x} \leq px + q$ where $4pq = 1$ and $p > 0$.

Proof: By the simple AM-GM inequality, we know $px + q \geq 2(pqx)^{\frac{1}{2}} = \sqrt{x}$. □

3) Difference between the optimal inputs: As in Section IV-A, we define the difference between the two optimal inputs as $\delta\mathbf{u}(x) := \mathbf{u}_N^*(x) - \hat{\mathbf{u}}_N^*(x)$, the following lemma provides an upper bound on $\|\delta\mathbf{u}(x)\|_2$.

Lemma 10: The input difference $\delta\mathbf{u}(x)$ satisfies

$$\|\delta\mathbf{u}(x)\|_2 \leq \overline{\theta}_N\left(\underline{\sigma}_{\hat{H}_N} - \overline{\theta}_N\right)^{-1}\left(1 + (N\overline{u})^{\frac{1}{2}}\right), \quad (32)$$

where $\overline{\theta}_N = \max\{\theta_{N,(x)}, \|x(0)\|_2\theta_{N,(x,u)}\}$ with $\theta_{N,(x)}$ and $\theta_{N,(x,u)}$ defined as in (30), $\hat{H}_N = \overline{R}_N + \hat{\Gamma}_N^\top\overline{Q}_{N+1}\hat{\Gamma}_N$ with $\overline{R}_N = I_N \otimes R$, and $\overline{u} := \max_{F_u u \leq \mathbb{1}} \|u\|_2^2$.

Proof: For the problem $\mathrm{P_{ID-MPC}}$, we form decision variable vectors $\mathbf{x}_{N,t} = [x_{0|t}^\top, x_{1|t}^\top, \ldots, x_{N|t}^\top]^\top$ and $\mathbf{u}_{N-1,t} = [u_{0|t}^\top, u_{1|t}^\top, \ldots, u_{N-1|t}^\top]^\top$. We exclude $u_{N|t}$ since its optimal value is 0, which does not influence the following reasoning. By (3), we obtain

$$\mathbf{x}_{N,t} = \Phi_N x(0) + \Gamma_N\mathbf{u}_{N-1,t},$$

where Φ_N and Γ_N are defined as in Lemma 7. By noting that the objective function of $\mathrm{P}_{\mathrm{ID-MPC}}$ can be rewritten as $\mathbf{x}_{N,t}^\top \overline{Q}_{N+1}\mathbf{x}_{N,t} + \mathbf{u}_{N-1,t}^\top \overline{R}_N \mathbf{u}_{N-1,t}$, an QP reformulation of the problem $\mathrm{P}_{\mathrm{ID-MPC}}$ can be obtained as

$$\mathrm{QP}_{\mathrm{ID-MPC}} : \min_{\mathbf{u}_{N-1,t}} \frac{1}{2}\mathbf{u}_{N-1,t}^\top H_N \mathbf{u}_{N-1,t} + \mathbf{u}_{N-1,t}^\top b_N$$

$$\text{s.t. } (I_N \otimes F_u)\mathbf{u}_{N-1,t} \leq \mathbb{1}_{Nc_u},$$

where $H_N = \overline{R}_N + \Gamma_N^\top \overline{Q}_{N+1}\Gamma_N$ and $b_N = \Gamma_N^\top \overline{Q}_{N+1}\Phi_N x(0)$. Similarly, we reformulate the problem $\mathrm{P}_{\mathrm{RE-MPC}}$ as

$$\mathrm{QP}_{\mathrm{RE-MPC}} : \min_{\mathbf{u}_{N-1,t}} \frac{1}{2}\mathbf{u}_{N-1,t}^\top \hat{H}_N \mathbf{u}_{N-1,t} + \mathbf{u}_{N-1,t}^\top \hat{b}_N$$

$$\text{s.t. } (I_N \otimes F_u)\mathbf{u}_{N-1,t} \leq \mathbb{1}_{Nc_u},$$

where $\hat{b}_N = \hat{\Gamma}_N^\top \overline{Q}_{N+1}\hat{\Phi}_N x(0)$. By treating $\mathrm{QP}_{\mathrm{RE-MPC}}$ as the original optimization problem, $\mathrm{QP}_{\mathrm{ID-MPC}}$ can be viewed as its perturbed counterpart. Therefore, leveraging the bound given in [20], we have

$$\|\delta\mathbf{u}(x)\|_2 \leq \tilde{\theta}\left(\underline{\sigma}_{\hat{H}_N} - \tilde{\theta}\right)^{-1}(1 + \|\hat{\mathbf{u}}_N^*(x)\|_2),$$

where $\tilde{\theta} = \max\left\{\|\hat{H}_N - H_N\|_2, \|\hat{b}_N - b_N\|_2\right\}$. Besides, we further have

$$\begin{aligned}
\tilde{\theta} &= \max\left\{\|\hat{H}_N - H_N\|_2, \|\hat{b}_N - b_N\|_2\right\} \\
&\leq \max\Big\{\|\hat{\Gamma}_N^\top \overline{Q}_{N+1}\hat{\Gamma}_N - \Gamma_N^\top \overline{Q}_{N+1}\Gamma_N\|_2, \\
&\qquad \|x(0)\|_2\|\hat{\Gamma}_N^\top \overline{Q}_{N+1}\hat{\Phi}_N - \Gamma_N^\top \overline{Q}_{N+1}\Phi_N\|_2\Big\} \\
&\leq \max\left\{\theta_{N,(x)}, \|x(0)\|_2\theta_{N,(x,u)}\right\},
\end{aligned}$$

where the last inequality is due to (29). Thus, we finally obtain

$$\begin{aligned}
\|\delta\mathbf{u}(x)\|_2 &\leq \tilde{\theta}\left(\underline{\sigma}_{\hat{H}_N} - \tilde{\theta}\right)^{-1}(1 + \|\hat{\mathbf{u}}_N^*(x)\|_2) \\
&\leq \overline{\theta}_N\left(\underline{\sigma}_{\hat{H}_N} - \overline{\theta}_N\right)^{-1}(1 + \|\hat{\mathbf{u}}_N^*(x)\|_2) \\
&\leq \overline{\theta}_N\left(\underline{\sigma}_{\hat{H}_N} - \overline{\theta}_N\right)^{-1}\left(1 + (N\overline{u})^{\frac{1}{2}}\right),
\end{aligned}$$

which completes the proof. □

4) State evolution with zero initial state: The following lemma provides a bound about the open-loop state trajectory starting from $x = 0$.

Lemma 11: Given an admissible input sequence $\mathbf{u}_{N-1}$ such that $\mathbf{u}_{N-1}[i] \in \mathcal{U}, i = 0, 1, \dots, N-1$, the open-loop state that starts from $x = 0$ is given by $\psi_x(k, 0, \mathbf{u}_{N-1}) = \sum_{i=0}^{k-1} A^{k-1-i}B^i\mathbf{u}_{N-1}[i]$. Denote $\boldsymbol{\psi}_x(0, \mathbf{u}_{N-1}) = [\psi_x^\top(0, 0, \mathbf{u}_{N-1}), \psi_x^\top(1, 0, \mathbf{u}_{N-1}), \dots, \psi_x^\top(N, 0, \mathbf{u}_{N-1})]^\top$, we have

$$\|\boldsymbol{\psi}_x(0, \mathbf{u}_{N-1})\|_2 \leq \left(\|\hat{\Gamma}_N\|_2 + \overline{g}_{(u)}\right)\|\mathbf{u}_{N-1}\|_2, \tag{35}$$

where $\hat{\Gamma}_N$ and $\overline{g}_{(u)}$ are defined as in Lemma 7.

Proof: Similar to the proof of Lemma 10, through system-level synthesis, we have $\boldsymbol{\psi}_x(0, \mathbf{u}_{N-1}) = \Gamma_N \mathbf{u}_{N-1}$. Then, we can proceed as

$$\begin{aligned}
\|\boldsymbol{\psi}_x(0, \mathbf{u}_{N-1})\|_2 &= \|\Gamma_N \mathbf{u}_{N-1}\|_2 \\
&\leq \|\Gamma_N\|_2\|\mathbf{u}_{N-1}\|_2 \\
&\leq \left(\|\hat{\Gamma}_N\|_2 + \|\Gamma_N - \hat{\Gamma}_N\|_2\right)\|\mathbf{u}_{N-1}\|_2 \\
&\leq \left(\|\hat{\Gamma}_N\|_2 + \overline{g}_{(u)}\right)\|\mathbf{u}_{N-1}\|_2,
\end{aligned}$$

which completes the proof. □

5) One-step prediction error: As in the proof sketch in Section IV-B, we define the one-step prediction error as $\Delta x := (A - \hat{A})x + (B - \hat{B})\hat{\mu}_N(x)$. The following lemma provides a bound on Δx in terms of $l(x, \hat{\mu}_N(x))$.

Lemma 12: The one-step prediction error Δx satisfies

$$\|\Delta x\|_2^2 \leq h(\delta_A, \delta_B)l(x, \hat{\mu}_N(x)), \tag{36}$$

where $h(\delta_A, \delta_B) = \underline{\sigma}_Q^{-1}\delta_A^2 + \underline{\sigma}_R^{-1}\delta_B^2$ is error-consistent.

Proof: By the Cauchy-Schwarz and Young's inequalities, when $\delta_A, \delta_B > 0$, we can proceed as

$$\begin{aligned}
\|\Delta x\|_2^2 &= \|(A - \hat{A})x + (B - \hat{B})\hat{\mu}_N(x)\|_2^2 \\
&\leq \left(1 + \frac{\delta_B^2\underline{\sigma}_Q}{\delta_A^2\underline{\sigma}_R}\right)\|(A - \hat{A})x\|_2^2 + \\
&\qquad + \left(1 + \frac{\delta_A^2\underline{\sigma}_R}{\delta_B^2\underline{\sigma}_Q}\right)\|(B - \hat{B})\hat{\mu}_N(x)\|_2^2 \\
&\leq \left(\delta_A^2 + \frac{\delta_B^2\underline{\sigma}_Q}{\underline{\sigma}_R}\right)\|x\|_2^2 + \\
&\qquad + \left(\delta_B^2 + \frac{\delta_A^2\underline{\sigma}_R}{\underline{\sigma}_Q}\right)\|\hat{\mu}_N(x)\|_2^2 \\
&\leq \left(\frac{\delta_A^2}{\underline{\sigma}_Q} + \frac{\delta_B^2}{\underline{\sigma}_R}\right)\left(\|x\|_Q^2 + \|\hat{\mu}_N(x)\|_R^2\right) \\
&\leq \left(\frac{\delta_A^2}{\underline{\sigma}_Q} + \frac{\delta_B^2}{\underline{\sigma}_R}\right)l(x, \hat{\mu}_N(x)).
\end{aligned}$$

When $\delta_A = 0$ and $\delta_B > 0$, we have

$$\begin{aligned}
\|\Delta x\|_2^2 &= \|(B - \hat{B})\hat{\mu}_N(x)\|_2^2 \\
&\leq \frac{\delta_B^2}{\underline{\sigma}_R}\|\hat{\mu}_N(x)\|_R^2 \leq \frac{\delta_B^2}{\underline{\sigma}_R}l(x, \hat{\mu}_N(x)).
\end{aligned}$$

Likewise, when $\delta_B = 0$ and $\delta_A > 0$, we have

$$\begin{aligned}
\|\Delta x\|_2^2 &= \|(A - \hat{A})x\|_2^2 \\
&\leq \frac{\delta_A^2}{\underline{\sigma}_Q}\|x\|_Q^2 \leq \frac{\delta_A^2}{\underline{\sigma}_Q}l(x, \hat{\mu}_N(x)).
\end{aligned}$$

Finally, when $\delta_A = \delta_B = 0$, we have $\|\Delta x\|_2^2 = 0$ and $h(\delta_A, \delta_B) = 0$, meaning (36) is also valid. The proof is completed. □

6) Multi-step prediction error: As in Step 3 of the proof sketch in Section IV-A, the k-step open-loop prediction error under control input $\hat{\mathbf{u}}_N^*(x))$ is defined as $e_\psi(k) := \psi_x(k, x, \hat{\mathbf{u}}_N^*(x)) - \hat{\psi}_x(k, x, \hat{\mathbf{u}}_N^*(x))$, the following lemma, provides an upper bound for $e_\psi(k)$.

Lemma 13: The concatenated prediction error $\mathbf{e}_{\psi,N} = [e_\psi^\top(0), e_\psi^\top(1), \ldots, e_\psi^\top(N)]^\top$ satisfies the following upper bound:

$$\|\mathbf{e}_{\psi,N}\|_2^2 \leq \sum_{k=0}^{N} \left[\left(g_{k,(x)}^{(2)} + \sum_{i=0}^{k-1} g_{k-i-1,(u)}^{(2)} \right) (\|x\|_2^2 + k\overline{u}) \right] \quad (37)$$

where $g_{k,(x)}^{(2)}$ and $g_{k-i-1,(u)}^{(2)}$ are defined as in (26).

Proof: For the prediction error $e_\psi(k)$, we have

$$\begin{aligned} e_\psi(k) = & \underbrace{(A^k - \hat{A}^k)}_{:=D_0} x + \\ & + \sum_{i=0}^{k-1} \underbrace{(A^{k-i-1}B - \hat{A}^{k-i-1}\hat{B})}_{:=D_{i+1}} (\hat{\mathbf{u}}_N^*(x))[i]. \end{aligned}$$

When $\|D_i\| > 0, i = 0, 1, \ldots, k$, we select $\epsilon_{i,j} = \|D_j\|_2^2 \|D_i\|_2^{-2}$. By the Cauchy-Schwarz and Young's inequalities, we can obtain

$$\begin{aligned} \|e_\psi(k)\|_2^2 &= \|D_0 x + \sum_{i=0}^{k-1} D_{i+1} (\hat{\mathbf{u}}_N^*(x))[i]\|_2^2 \\ &\leq \left(1 + \sum_{j=1}^{k} \epsilon_{0,j} \right) \|D_0 x\|_2^2 + \\ &\quad + \sum_{i=0}^{k-1} \left(1 + \sum_{\substack{j \neq i+1 \\ 0 \leq j \leq k}} \epsilon_{i+1,j} \right) \|D_{i+1} (\hat{\mathbf{u}}_N^*(x))[i]\|_2^2 \\ &\leq \left(\sum_{i=0}^{k} \|D_i\|_2^2 \right) (\|x\|_2^2 + k\overline{u}) \\ &\leq \left(g_{k,(x)}^{(2)} + \sum_{i=0}^{k-1} g_{k-i-1,(u)}^{(2)} \right) (\|x\|_2^2 + k\overline{u}) . \end{aligned}$$

In cases where $\|D_i\|_2 = 0$ for some i's, we know $D_i = 0$, then proceeding the above procedure without those terms with $D_i = 0$ will reach the same upper bound. The final upper bound as in (37) is established by summing up the above result over the index k. □

C. More on Proposition 1

Proposition 1: There exist two error-consistent functions $\alpha_N(\delta_A, \delta_B)$ and $\beta_N(\delta_A, \delta_B)$ such that, for all $x \in \mathcal{X}_{\mathrm{ROA}}$, $\hat{V}_N$ and V_∞ satisfy the following inequality:

$$\hat{V}_N(x) \leq \big(1 + \alpha_N(\delta_A, \delta_B)\big) V_\infty(x) + \beta_N(\delta_A, \delta_B),$$

where functions α_N and β_N relate to the eigenvalues and matrix norms of $\hat{A}$, $\hat{B}$, Q, R, the input constraint set $\mathcal{U}$, but not to quantities derived from A or B. The detailed expressions of α_N and β_N are

$$\begin{aligned} \alpha_N = \max \Big\{ & p_1 E_{N,(\psi)}^{\frac{1}{2}} + p_2 E_{N,(\psi,u)}^{\frac{1}{2}} + \\ & + p_1 p_2 (E_{N,(\psi)} E_{N,(\psi,u)})^{\frac{1}{2}}, \ p_3 E_{N,(u)}^{\frac{1}{2}} \Big\} \end{aligned} \quad (38a)$$

$$\begin{aligned} \beta_N = & (1 + p_1 E_{N,(\psi)}^{\frac{1}{2}})(q_2 E_{N,(\psi,u)}^{\frac{1}{2}} + E_{N,(\psi,u)}) + \\ & + q_3 E_{N,(u)}^{\frac{1}{2}} + E_{N,(u)} + q_1 E_{N,(\psi)}^{\frac{1}{2}} + E_{N,(\psi)}^{\frac{1}{2}}, \end{aligned} \quad (38b)$$

where the pairs $(p_i, q_i) \in \mathbb{R}_+^2$ satisfy $p_i q_i = 1$ and the details of the terms $E_{N,(\psi)}$, $E_{N,(u)}$, and $E_{N,(\psi,u)}$ are given, respectively, in (39), (40), and (41).

(1) Details of $E_{N,(\psi)}$: Following Lemma 13, the explicit expression of $E_{N,(\psi)}$ is

$$E_{N,(\psi)} = \overline{\sigma}_Q \sum_{k=0}^{N} \left[\left(g_{k,(x)}^{(2)} + \sum_{i=0}^{k-1} g_{k-i-1,(u)}^{(2)} \right) (\|x\|_2^2 + k\overline{u}) \right] \quad (39)$$

(2) Details of $E_{N,(u)}$ **and** $E_{N,(\psi,u)}$: Based on Lemma 10 and Lemma 11, the explicit form of $E_{N,(u)}$ is

$$E_{N,(u)} = \overline{\sigma}_R \overline{\theta}_N^2 \left(\underline{\sigma}_{\hat{H}_N} - \overline{\theta}_N \right)^{-2} \left(1 + (N\overline{u})^{\frac{1}{2}} \right)^2, \quad (40)$$

and the expression of $E_{N,(\psi,u)}$ follows as

$$E_{N,(\psi,u)} = \frac{\overline{\sigma}_Q}{\overline{\sigma}_R} \left(\|\hat{\Gamma}_N\|_2 + \bar{g}_{(u)} \right)^2 E_{N,(u)}. \quad (41)$$

Proof: By definition, $\hat{V}_N(x)$ can be expanded as

$$\hat{V}_N(x) = \sum_{k=0}^{N} \left(\|\hat{\psi}_x(k, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 + \|\big(\hat{\mathbf{u}}_N^*(x)\big)[k]\|_R^2 \right), \quad (42)$$

which, by incorporating the prediction error $e_\psi(k)$, can be rewritten as

$$\begin{aligned} \hat{V}_N(x) = \sum_{k=0}^{N} \big(& \|\psi_x(k, x, \hat{\mathbf{u}}_N^*(x)) - e_\psi(k)\|_Q^2 + \\ & + \|\big(\hat{\mathbf{u}}_N^*(x)\big)[k]\|_R^2 \big). \end{aligned} \quad (43)$$

Applying Lemma 8 to (43), we obtain

$$\begin{aligned} \hat{V}_N(x) \leq & \sum_{k=0}^{N} \big(\|\psi_x(k, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 + \|\big(\hat{\mathbf{u}}_N^*(x)\big)[k]\|_R^2 \big) + \\ & + 2 \left[\left(\sum_{k=0}^{N} \|\psi_x(k, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 \right) \left(\sum_{k=0}^{N} \|e_\psi(k)\|_Q^2 \right) \right]^{\frac{1}{2}} + \\ & + \sum_{k=0}^{N} \|e_\psi(k)\|_Q^2, \end{aligned} \quad (44)$$

which can be further relaxed as

$$\begin{aligned} \hat{V}_N(x) \leq & \sum_{k=0}^{N} \big(\|\psi_x(k, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 + \|\big(\hat{\mathbf{u}}_N^*(x)\big)[k]\|_R^2 \big) + \\ & + 2 \left[\left(\sum_{k=0}^{N} \|\psi_x(k, x, \hat{\mathbf{u}}_N^*(x))\|_Q^2 \right) \left(\overline{\sigma}_Q \|\mathbf{e}_{\psi,N}\|_2^2 \right) \right]^{\frac{1}{2}} + \\ & + \overline{\sigma}_Q \|\mathbf{e}_{\psi,N}\|_2^2, \end{aligned} \quad (45)$$

where $\mathbf{e}_{\psi,N} = [e_{\psi}^{\top}(0), e_{\psi}^{\top}(1), \dots, e_{\psi}^{\top}(N)]^{\top}$. Using Lemma 13 and recalling the notation in (39), we have

$$\hat{V}_N(x) \leq \sum_{k=0}^{N} \left(\|\psi_x(k,x,\hat{\mathbf{u}}_N^*(x))\|_Q^2 + \|\big(\hat{\mathbf{u}}_N^*(x)\big)[k]\|_R^2 \right) + \\ + 2\left[\left(\sum_{k=0}^{N} \|\psi_x(k,x,\hat{\mathbf{u}}_N^*(x))\|_Q^2\right) E_{N,(\psi)}\right]^{\frac{1}{2}} + \\ + E_{N,(\psi)}. \quad (46)$$

Then, due to Lemma 9, we can further obtain

$$\hat{V}_N(x) \leq \left(1 + p_1 E_{N,(\psi)}^{\frac{1}{2}}\right) \sum_{k=0}^{N} \|\psi_x(k,x,\hat{\mathbf{u}}_N^*(x))\|_Q^2 + \\ + \sum_{k=0}^{N} \|\big(\hat{\mathbf{u}}_N^*(x)\big)[k]\|_R^2 + q_1 E_{N,(\psi)}^{\frac{1}{2}} + E_{N,(\psi)}, \quad (47)$$

where $p_1 q_1 = 1, p_1 > 0$.

Next, by leveraging the input difference $\big(\delta\mathbf{u}(x)\big)[k] = \big(\mathbf{u}_N^*(x)\big)[k] - \big(\hat{\mathbf{u}}_N^*(x)\big)[k]$, we have

$$\hat{V}_N(x) \leq \left(1 + p_1 E_{N,(\psi)}^{\frac{1}{2}}\right) \sum_{k=0}^{N} \|\psi_x(k,x,\hat{\mathbf{u}}_N^*(x))\|_Q^2 + \\ + \sum_{k=0}^{N} \|\big(\mathbf{u}_N^*(x)\big)[k] - \big(\delta\mathbf{u}(x)\big)[k]\|_R^2 + \\ + q_1 E_{N,(\psi)}^{\frac{1}{2}} + E_{N,(\psi)}, \quad (48)$$

In (48), the term $\sum_{k=0}^{N} \|\big(\mathbf{u}_N^*(x)\big)[k] - \big(\delta\mathbf{u}(x)\big)[k]\|_R^2$ can be proceeded as

$$\sum_{k=0}^{N} \|\big(\mathbf{u}_N^*(x)\big)[k] - \big(\delta\mathbf{u}(x)\big)[k]\|_R^2 \\ \leq \sum_{k=0}^{N} \left(\|\big(\mathbf{u}_N^*(x)\big)[k]\|_R^2 + \|\big(\delta\mathbf{u}(x)\big)[k]\|_R^2 \right) + \\ + 2\left[\left(\sum_{k=0}^{N} \|\big(\mathbf{u}_N^*(x)\big)[k]\|_R^2\right)\left(\sum_{k=0}^{N} \|\big(\delta\mathbf{u}(x)\big)[k]\|_R^2\right)\right]^{\frac{1}{2}} \\ \leq (1 + p_2 E_{N,(u)}^{\frac{1}{2}}) \sum_{k=0}^{N} \|\big(\mathbf{u}_N^*(x)\big)[k]\|_R^2 + \\ + q_2 E_{N,(u)}^{\frac{1}{2}} + E_{N,(u)}, \quad (49)$$

where the first inequality is due to Lemma 8 and the second inequality is due to Lemma 9, Lemma 10, and the definition as in (40). In addition, due to linearity, we have $\psi_x(k,x,\hat{\mathbf{u}}_N^*(x)) = \psi_x(k,x,\mathbf{u}_N^*(x)) - \psi_x(\cdot,0,\delta\mathbf{u}(x))$, and the term $\sum_{k=0}^{N} \|\psi_x(k,x,\hat{\mathbf{u}}_N^*(x))\|_Q^2$ in (48) can be further proceeded as

$$\sum_{k=0}^{N} \|\psi_x(k,x,\hat{\mathbf{u}}_N^*(x))\|_Q^2 \\ = \sum_{k=0}^{N} \|\psi_x(k,x,\mathbf{u}_N^*(x)) - \psi_x(k,0,\delta\mathbf{u}(x))\|_Q^2 \\ \leq \sum_{k=0}^{N} \left(\|\psi_x(k,x,\mathbf{u}_N^*(x))\|_Q^2 + \|\psi_x(k,0,\delta\mathbf{u}(x))\|_Q^2 \right) + \\ + 2\left[\left(\sum_{k=0}^{N} \|\psi_x(k,x,\mathbf{u}_N^*(x))\|_Q^2\right)\left(\sum_{k=0}^{N} \|\psi_x(k,0,\delta\mathbf{u}(x))\|_Q^2\right)\right]^{\frac{1}{2}} \\ \leq (1 + p_3 E_{N,(\psi,u)}^{\frac{1}{2}}) \sum_{k=0}^{N} \|\psi_x(k,x,\mathbf{u}_N^*(x))\|_Q^2 + \\ + q_3 E_{N,(\psi,u)}^{\frac{1}{2}} + E_{N,(\psi,u)}, \quad (50)$$

where the first inequality is due to Lemma 8 and the second inequality is due to Lemma 9, Lemma 11, and the definition as in (41).

Finally, substituting (49) and (50) into (48) leads to

$$\hat{V}_N(x) \leq (1+\alpha_N)V_N(x) + \beta_N \leq (1+\alpha_N)V_\infty(x) + \beta_N,$$

where α_N and β_N are given as in (38). □

D. More on Proposition 2

Proposition 2: There exist a constant η_N and an error-consistent function ξ_N satisfying $\xi_N(\delta_A, \delta_B) + \eta_N < 1$, and for all $x \in \mathcal{X}_{\text{ROA}}$ we have

$$\hat{V}_N(x_{\text{re}}^+) - \hat{V}_N(x) \leq \\ - (1 - \xi_N(\delta_A, \delta_B) - \eta_N) l(x, \hat{\mu}_N(x)),$$

where the function ξ_N and constant η_N relate to the eigenvalues and matrix norms of $\hat{A}$, $\hat{B}$, Q, R, the input constraint set $\mathcal{U}$, but not to quantities derived from A or B. The detailed expression of η_N is

$$\eta_N = (C_{\hat{K}}^* + \|\hat{A}_{\text{cl}}\|_2^2 r_Q - 1)\gamma \rho_\gamma^{N-N_0}, \quad (51)$$

where $C_{\hat{K}}^*$ is defined as in Lemma 1 with linear feedback gain $\hat{K}$, the closed-loop gain $\hat{A}_{\text{cl}} = \hat{A} + \hat{B}\hat{K}$, and the scalars γ and ρ_γ as defined in Lemma 2 using a different gain K. Finally, the detailed expression of ξ_N is

$$\xi_N = \omega_{N,(1)} h(\delta_A, \delta_B) + 2\omega_{N,(\frac{1}{2})} h^{\frac{1}{2}}(\delta_A, \delta_B), \quad (52)$$

where the function h is given as in (36) in Lemma 12, and the terms $\omega_{N,(1)}$ and $\omega_{N,(\frac{1}{2})}$ are given, respectively, in (54a) and (54b).

Details of $\omega_{N,(1)}$ and $\omega_{N,(\frac{1}{2})}$: We define $G_N(\hat{A}) = \sum_{i=1}^{N-1} \|\hat{A}\|_2^{2(i-1)}$ as

$$G_N(\hat{A}) = \begin{cases} N-1 & \text{if } \|\hat{A}\|_2 = 1 \\ \frac{1-(\|\hat{A}\|_2^2)^{N-1}}{1-\|\hat{A}\|_2^2} & \text{if } \|\hat{A}\|_2 \neq 1, \end{cases} \quad (53)$$

and $\omega_{N,(1)}$ and $\omega_{N,(\frac{1}{2})}$ follows as

$$\omega_{N,(1)}:=\overline{\sigma}_Q\Big[(C^*_{\hat{K}}+\|\hat{A}_{\mathrm{cl}}\|_2^2 r_Q)(\|\hat{A}\|_2^2)^{N-1}+G_N(\hat{A})\Big], \tag{54a}$$

$$\omega_{N,(\frac{1}{2})}:=\Big[\overline{\sigma}_Q(L_{\hat{V}}-1)G_N(\hat{A})\Big]^{\frac{1}{2}}+ + \frac{C^*_{\hat{K}}+\|\hat{A}_{\mathrm{cl}}\|_2^2 r_Q}{2}\Big[\overline{\sigma}_Q(\|\hat{A}\|_2^2)^{N-1}\gamma\rho_\gamma^{N-N_0}\Big]^{\frac{1}{2}}, \tag{54b}$$

where $L_{\hat{V}}$ is given as in Lemma 2 and $G_N(\hat{A})$ is given as in (53).

Before presenting the main proof of Proposition 2, we need to make some preparations and provide some additional lemmas. By definition, we have $\hat{V}_N(x) = J_N\big(x,\hat{\mathbf{u}}^*_N(x)\big)$ with

$$\hat{\mathbf{u}}^*_N(x)=\Big[\big(\hat{\mathbf{u}}^*_N(x)\big)[0]^\top,\big(\hat{\mathbf{u}}^*_N(x)\big)[1]^\top,\dots,\big(\hat{\mathbf{u}}^*_N(x)\big)[N-1]^\top,0\Big]^\top.$$

Then, we form an auxiliary input sequence $\mathbf{v}_N(x)$ as

$$\mathbf{v}_N(x)=\Big[\big(\hat{\mathbf{u}}^*_N(x)\big)[1]^\top,\big(\hat{\mathbf{u}}^*_N(x)\big)[2]^\top,\dots,\big(\hat{\mathbf{u}}^*_N(x)\big)[N-1]^\top,u_f,0\Big]^\top,$$

where $u_f = \hat{K}\hat{\psi}_x\big(N-1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\big)$ with $x^+_{\mathrm{re}} = Ax + B\hat{\mu}_N(x)$ defined as in Section IV-B and $\hat{K}$ being a stabilizable linear control gain. Note that u_f is not recursively defined since $\hat{\psi}_x\big(N-1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\big)$ only requires the first $N-1$ inputs of $\mathbf{v}_N(x)$.

Lemma 14: For $k=0,1,\dots,N-1$, the following relation holds:

$$\hat{\psi}_x\left(k,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)=\hat{\psi}_x\big(k+1,x,\hat{\mathbf{u}}^*_N(x)\big)+\hat{A}^k\Delta x, \tag{55}$$

where $\Delta x = (A-\hat{A})x + (B-\hat{B})\hat{\mu}_N(x)$ is defined as in part (5) in Appendix B

Proof: The proof mainly relies on the formula of open-loop state given as in (3), the shifting property of $\mathbf{v}_N(x)$ with respect to $\hat{\mathbf{u}}^*_N(x)$, and the definitions of x^+_{re}, Δx, as well as $\hat{\mu}_N(x)$.

$$\begin{aligned}
&\hat{\psi}_x\left(k,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\\
=&\hat{A}^k x^+_{\mathrm{re}}+\sum_{i=0}^{k-1}\hat{A}^{k-i-1}\hat{B}\big(\mathbf{v}_N(x)\big)[i]\\
=&\hat{A}^k\big(Ax+B\hat{\mu}_N(x)\big)+\sum_{i=0}^{k-1}\hat{A}^{k-i-1}\hat{B}\big(\hat{\mathbf{u}}^*_N(x)\big)[i+1]\\
=&\hat{A}^kAx+\hat{A}^kB\hat{\mu}_N(x)+\sum_{i=1}^{k}\hat{A}^{k-i}\hat{B}\big(\hat{\mathbf{u}}^*_N(x)\big)[i]\\
=&\hat{A}^k\left[(A-\hat{A})x+(B-\hat{B})\hat{\mu}_N(x)\right]+\\
&+\hat{A}^{k+1}x+\sum_{i=0}^{k}\hat{A}^{k-i}\hat{B}\big(\hat{\mathbf{u}}^*_N(x)\big)[i]\\
=&\hat{A}^k\Delta x+\hat{\psi}_x\big(k+1,x,\hat{\mathbf{u}}^*_N(x)\big),
\end{aligned}$$

which builds the relation as in (55). □

To ensure that u_f is admissible, we further impose an additional assumption.

Assumption 3: There exists a linear feedback gain $\hat{K}$ and its associated local region $\Omega_{\hat{K}} = \{x\in\mathcal{X} \mid l^*(x)\le\varepsilon_{\hat{K}}\}$ with $\varepsilon_{\hat{K}} > \varepsilon_K$ such that, when $\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}^*_N(x)\big)\in\Omega_K$, $\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}^*_N(x)\big)+\hat{A}^{N-1}\Delta x\in\Omega_{\hat{K}}$.

The proof of Proposition 2 is then presented as follows.

Proof: By definition of $\hat{V}_N$, we have

$$\begin{aligned}
&\hat{V}_N(x^+_{\mathrm{re}})-\hat{V}_N(x)\\
=&J_N\big(x^+_{\mathrm{re}},\hat{\mathbf{u}}^*_N(x^+_{\mathrm{re}})\big)-J_N\big(x,\hat{\mathbf{u}}^*_N(x)\big)\\
\le&J_N\big(x^+_{\mathrm{re}},\mathbf{v}_N(x)\big)-J_N\big(x,\hat{\mathbf{u}}^*_N(x)\big)\\
\le&\sum_{k=0}^{N}\left(\|\hat{\psi}_x\left(k,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2+\|\big(\mathbf{v}_N(x)\big)[k]\|_R^2\right)-\\
&-\sum_{k=0}^{N}\left(\|\hat{\psi}_x\big(k,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2+\|\big(\hat{\mathbf{u}}^*_N(x)\big)[k]\|_R^2\right)\\
\le&-l\big(x,\hat{\mu}_N(x)\big)+\|\hat{\psi}_x\left(N-1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2+\|u_f\|_R^2+\\
&+\underbrace{\sum_{k=0}^{N-2}\left(\|\big(\mathbf{v}_N(x)\big)[k]\|_R^2-\|\big(\hat{\mathbf{u}}^*_N(x)\big)[k+1]\|_R^2\right)}_{=0}+\\
&+\sum_{k=0}^{N-2}\left(\|\hat{\psi}_x\left(k,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2-\|\hat{\psi}_x\big(k{+}1,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2\right)+\\
&+\|\hat{\psi}_x\left(N,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2-\|\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2,
\end{aligned} \tag{56}$$

where the first inequality is due to the optimality of the input sequence $\hat{\mathbf{u}}^*_N(x^+_{\mathrm{re}})$. Since $u_f = \hat{K}\hat{\psi}_x\big(N{-}1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\big)$, we further have the following relations:

$$\|\hat{\psi}_x\left(N{-}1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2+\|u_f\|_R^2\le C^*_{\hat{K}}\|\hat{\psi}_x\left(N{-}1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2 \tag{57a}$$

$$\|\hat{\psi}_x\left(N,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2\le r_Q\|\hat{A}_{\mathrm{cl}}\|_2^2\|\hat{\psi}_x\left(N{-}1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2 \tag{57b}$$

Substituting (57a) and (57b) into (56) yields

$$\begin{aligned}
&\hat{V}_N(x^+_{\mathrm{re}})-\hat{V}_N(x)\\
\le&-l\big(x,\hat{\mu}_N(x)\big)-\|\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2+\\
&+\sum_{k=0}^{N-2}\left(\|\hat{\psi}_x\left(k,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2-\|\hat{\psi}_x\big(k{+}1,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2\right)+\\
&+\left(C^*_{\hat{K}}+r_Q\|\hat{A}_{\mathrm{cl}}\|_2^2\right)\|\hat{\psi}_x\left(N{-}1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2.
\end{aligned} \tag{58}$$

Using Lemma 8 and Lemma 14, for $k=N-1$, we have

$$\begin{aligned}
&\|\hat{\psi}_x\left(N-1,x^+_{\mathrm{re}},\mathbf{v}_N(x)\right)\|_Q^2\\
\le&\|\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2+\|\hat{A}^{N-1}\Delta x\|_Q^2+\\
&+2\left(\|\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}^*_N(x)\big)\|_Q^2\|\hat{A}^{N-1}\Delta x\|_Q^2\right)^{\frac{1}{2}},
\end{aligned} \tag{59}$$

and for $k=0,1,\dots,N-2$, we have

$$\sum_{k=0}^{N-2}\left(\|\hat{\psi}_x\left(k,x_{\mathrm{re}}^{+},\mathbf{v}_N(x)\right)\|_Q^2-\|\hat{\psi}_x\big(k+1,x,\hat{\mathbf{u}}_N^{*}(x)\big)\|_Q^2\right)$$
$$\leq 2\left[\left(\sum_{k=1}^{N-1}\|\hat{\psi}_x\big(k,x,\hat{\mathbf{u}}_N^{*}(x)\big)\|_Q^2\right)\left(\sum_{k=1}^{N-1}\|\hat{A}^{k-1}\Delta x\|_Q^2\right)\right]^{\frac{1}{2}}+$$
$$+\sum_{k=1}^{N-1}\|\hat{A}^{k-1}\Delta x\|_Q^2. \tag{60}$$

In addition, due to Lemma 12, for $k=0,1,\dots,N-1$, we can obtain

$$\|\hat{A}^k\Delta x\|_Q^2\leq\overline{\sigma}_Q\|\hat{A}\|_2^{2k}hl\big(x,\hat{\mu}_N(x)\big). \tag{61}$$

Leveraging (61) and (15b), we can further relax (59) as

$$\|\hat{\psi}_x\left(N-1,x_{\mathrm{re}}^{+},\mathbf{v}_N(x)\right)\|_Q^2$$
$$\leq\|\hat{\psi}_x\big(N,x,\hat{\mathbf{u}}_N^{*}(x)\big)\|_Q^2+\overline{\sigma}_Q\|\hat{A}\|_2^{2(N-1)}hl\big(x,\hat{\mu}_N(x)\big)+$$
$$+\left[\overline{\sigma}_Q\|\hat{A}\|_2^{2(N-1)}\gamma\rho_\gamma^{N-N_0}\right]^{\frac{1}{2}}h^{\frac{1}{2}}l\big(x,\hat{\mu}_N(x)\big). \tag{62}$$

Likewise, using (61) and (15a), (59) can also be relaxed as

$$\sum_{k=0}^{N-2}\left(\|\hat{\psi}_x\left(k,x_{\mathrm{re}}^{+},\mathbf{v}_N(x)\right)\|_Q^2-\|\hat{\psi}_x\big(k+1,x,\hat{\mathbf{u}}_N^{*}(x)\big)\|_Q^2\right)$$
$$\leq 2\left[\overline{\sigma}_Q(L_{\hat{V}}-1)G_N(\hat{A})\right]^{\frac{1}{2}}h^{\frac{1}{2}}l\big(x,\hat{\mu}_N(x)\big)+$$
$$+\overline{\sigma}_Q G_N(\hat{A})hl\big(x,\hat{\mu}_N(x)\big), \tag{63}$$

where $G_N(\hat{A})$ is given as in (53).

Finally, by substituting (62) and (63) into (58), rearranging the terms and applying (15b) once again, we obtain

$$\hat{V}_N(x_{\mathrm{re}}^{+})-\hat{V}_N(x)$$
$$\leq-(1-\omega_{N,(1)}h-2\omega_{N,(\frac{1}{2})}h^{\frac{1}{2}}-\eta_N)l\big(x,\hat{\mu}_N(x)\big)$$
$$\leq-(1-\xi_N-\eta_N)l\big(x,\hat{\mu}_N(x)\big),$$

where $\omega_{N,(1)}$ and $\omega_{N,(\frac{1}{2})}$ are given as in (54a) and (54b), respectively; ξ_N and η_N are given as in (52) and (51), respectively. The proof is completed. □

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