Experiment design for batch-to-batch model-based learning control

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***Abstract*—An Experiment Design framework for dynamical systems which execute multiple batches is presented in this paper. After each batch, a model of the system dynamics is refined using the measured data. This model is used to synthesize the controller that will be applied in the next batch. Excitation signals may be injected into the system during each batch. From one hand, perturbing the system worsens the control performance during the current batch. On the other hand, the more informative data set will lead to a better identified model for the following batches. The role of Experiment Design is to choose the proper excitation signals in order to optimize a certain performance criterion defined on the set of batches that is scheduled. A total cost is defined in terms of the excitation and the application cost altogether. The excitation signals are designed by minimizing the total cost in a worst case sense. The Experiment Design is formulated as a Convex Optimization problem which can be solved efficiently using standard algorithms. The applicability of the method is demonstrated in a simulation study.**

I. Introduction

In many practical applications a dynamical system performs a certain operation repetitively over time. Examples are batch processes in the chemical industry and movements of some manipulators in robotics. According to the field, a single operation of the system is called "iteration", "run" , "pass"' or "batch". We will stick to this last term in this paper.

A possible model-based control strategy for this kind of systems involves two steps for each batch. Firstly, data from the previous batch are used to refine a model of the system dynamics (iterative identification step). Secondly, the model is used in order to determine the control action for the next batch (controller design step). There is not a widespread name for this strategy in literature. In a previous contribution [1], the authors referred to it as Iterative Identification Control (IIC) and applied it to the control of a batch crystallization process.

The reasoning of IIC can be extended if we consider that we could have two distinct objectives for each batch. From one hand, we want to follow an application-specific control objective for the batch. In practice, an economical value may be directly linked to this objective. On the other hand, we may want to excite the system dynamics in order to perform an effective identification with the data collected during the batch. Exciting the system normally leads to a decrease in the control performance during the current batch. However, a reward for this price will be gained in the following batches. The more accurate model of the dynamics will be used in order to design a more effective control action. Thus, the application cost will decrease. The user of the system should carefully plan his experiments in order to maximize the profits for the set of batches which is scheduled by weighting the identification and the control objectives in a proper fashion.

The task of designing excitation signals which guarantee some properties of the identified model is known as Experiment Design [2]. Several frameworks for Experiment Design have been considered in literature. The classical approach is to look for an input that minimizes a certain control-oriented measure of the quality of the identified model. Constraints on the excitation signals are often considered in terms of the maximum input/output power allowed during the identification phase [3], [4].

It has been observed that one should spend excitation effort only if he really needs it for his particular application. For this reason, it is usually more sensible to aim for a certain performance level and spend the minimum excitation cost in order to achieve it rather than fixing the excitation cost in advance. While in the classical approach the control performance is minimized under a constraint on the maximum perturbation allowed, in the least costly approach [5] the perturbation is minimized under a constraint on the guaranteed control performance.

In some cases it is difficult even to specify a target performance level. Furthermore, there is not always a clear distinction between the identification and the control experiments. Consider the batch system case: the user is just interested to satisfy a certain control objective as closely as possible for a set of batches. He is allowed to introduce excitation signals in each batch and he should make his choice based on the price that he has to pay during the current batch and the reward that he will gain in the following ones.

In this paper we develop a novel approach to Experiment Design that is suitable for such situations. A total cost is defined in terms of excitation and the application cost altogether. The total cost measures a "distance" between the optimal loop that we would design having a perfect knowledge of the true system and the experimental loop which is achieved in practice, in which the controller design is based on the best model available and an excitation signal may be applied. In fact, the total cost we use is the power of the output difference between these two controlled systems. The excitation signals are designed by minimizing the total cost in a worst-case sense.

Note that our approach goes beyond a multi-objective optimization of the excitation cost and the application cost. The contributions of the two terms are "dimensionally equivalent" and can be added up without any weighting factor in our total cost. As a consequence, there are no tuning parameters to be adjusted in our algorithm.

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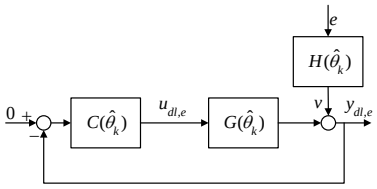

Fig. 1. Design Loop

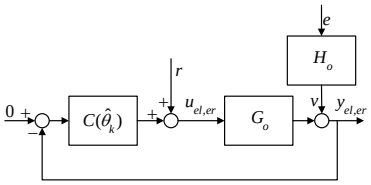

Fig. 2. Experimental Loop

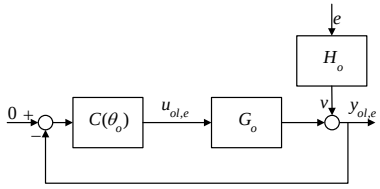

Fig. 3. Optimal Loop

In this work, the Experiment Design framework is applied to a batch control problem. The iterative identification is performed adopting a Bayesian approach and the controller design according to a nominal $\mathcal{H}_2$ criterion.

The rest of this paper is organized as follows. In Section II the framework is discussed in details. In Section III the Experiment Design is posed as a Convex Optimization Problem and transformed to a SemiDefinite Program (SDP) for the numerical solution. The framework is applied to a simulation study in Section IV and conclusions are drawn in Section V.

II. The Framework

A *real system* $\mathcal{S}_o$ is operated over $k = 1, 2, \ldots, n$ batches in a closed-loop configuration. $\mathcal{S}_o$ is the discrete-time linear time-invariant system

$$y = G_o(z)u + H_o(z)e \tag{1}$$

where u is the input, e is white noise with variance σ_e^2 and y is the output. G_o and H_o are stable discrete-time transfer functions; H_o is monic and minimum phase.

$\mathcal{S}_o$ is known to belong to a certain *model set* $\mathcal{M}^*$ parametrized by a *model structure* $\mathcal{M} = \{\mathcal{M}(\theta),\ \theta \in \mathbb{R}^p\}$ where θ is the model parameter. A parametric model $\mathcal{M}(\theta)$ belonging to $\mathcal{M}$ is defined as $y = G(z,\theta)u + H(z,\theta)e$. The model structure satisfies the common properties usually assumed in identification (e.g. Definition 4.3 of [6]). We also assume that the real system $\mathcal{S}_o$ is described in the model structure $\mathcal{M}$ by one and only one parameter vector θ_o, i.e. $\exists! \theta_o \mid \mathcal{S}_o = \mathcal{M}(\theta_o)$.

An initial, possibly rough model $\mathcal{M}_1 = \mathcal{M}(\hat{\theta}_1)$ is known to the user. After each batch k, the input/output data from the plant are collected and used together with the previous model in order to identify an improved model $\mathcal{M}_{k+1}$. We will refer to the selection of $\mathcal{M}_{k+1}$ as the *iterative identification* procedure.

The controller C_k for the batch k is chosen based on a certain specification on the *design loop* $[C_k\ \mathcal{M}_k]$ (Figure 1). C_k is determined based on the model $\mathcal{M}_k$: $C_k = C(\mathcal{M}_k)$ where C is the *controller design* procedure. With a slight notational abuse, we will use also the syntax $C(\hat{\theta}_k)$ instead of $C(\mathcal{M}_k)$.

The controller C_k will be applied in the next batch on the *experimental loop* $[C_k\ \mathcal{S}_o]$. Furthermore, an *excitation signal* r_k may be applied as shown in Figure 2. The excitation signal r_k is determined in the *experiment design procedure*. Let us define at this point the *optimal loop* as $[C_o\ \mathcal{S}_o]$ with $C_o = C(\mathcal{S}_o)$ (Figure 3).

This is the loop that the user would design with the perfect knowledge of the real system $\mathcal{S}_o$ at hand. Achieving the optimal loop is not possible in practice because the perfect knowledge of the real system is never available. A feasible objective is to minimize in some sense a distance between the optimal loop and the experimental loop mesured by a certain function T. The objective J of the Experiment Design is to minimize the sum of T over the n batches

$$J = \sum_{k=1}^{n} T_k \tag{2}$$

and the decision variables are the excitation signals r_k. Different constraints may also be taken into account, as described later in this paper. The objective J is in general a random variable. For this reason, the optimization has to be considered in a stochastic sense. In particular, we will consider the *worst case* performance when the true system belongs to an *uncertainty set* with a specified probability level α.

In the following subsection we will describe in details the iterative identification and controller design procedures, the function T and the worst case settings.

A. Iterative Identification Procedure

A Bayesian Approach is followed in order to construct the parameter estimate $\hat{\theta}_{k+1}$. At the end of each batch k, the data $z_k = \{u_k(t), y_k(t),\ t = 1 \ldots N\}$ are collected. Let $\hat{\theta}_k$ be the parameter estimate obtained with the data up to the batch $k-1$. Assuming that $\hat{\theta}_k$ is normally distributed as

$$\hat{\theta}_k \sim \mathcal{N}(\theta_o, R_k^{-1}) \tag{3}$$

we can estimate $\hat{\theta}_{k+1}$ as the maximum *a posteriori* [7] of the probability distribution of the model parameter given the new data z_k. From the Bayes equations we derive

$$\hat{\theta}_{k+1} = \arg\min_{\theta \in \mathbb{R}^p} \frac{1}{\sigma_e^2} \sum_{t=1}^{N} \varepsilon_k^2(t,\theta) + (\theta - \hat{\theta}_k)^\top R_k (\theta - \hat{\theta}_k) \tag{4}$$

where $\varepsilon_k(t,\theta) = H(z,\theta)^{-1}(y_k(t) - G(z,\theta)u_k(t))$.

The parameter vector $\hat{\theta}_{k+1}$ is asymptotically normally distributed around θ_o, i.e.

$$\hat{\theta}_{k+1} \sim \mathcal{N}(\theta_o, R_{k+1}^{-1}) \qquad \text{with } R_{k+1} = R_k + I_k \tag{5}$$

where I_k is the so-called *information matrix*. We will suppose that we do not know the variance of the initial parameter $\hat{\theta}_1$ and set $R_1 = 0$.

The information matrix I_k depends on the particular excitation signal r_k. We will consider excitation signals whose spectrum has structure [5]

$$\Phi_k^r(\omega) = \mathcal{R}_k(0) + 2\sum_{j=1}^{m} \mathcal{R}_k(j)\cos(j\omega) \tag{6}$$

satisfying the positivity constraint

$$\Phi_k^r(\omega) \geq 0 \ \forall\omega. \tag{7}$$

Let us define here for notational convenience the aggregate variable $\mathcal{R}_k \triangleq \{\mathcal{R}_k(0) \ \ldots \ \mathcal{R}_k(m)\}$ containing the coefficients of the excitation spectra for the batch k.Following the reasoning of [5], I_k may be written as a linear function of $\mathcal{R}_k$

$$I_k = \bar{M}(\theta_o) + \sum_{j=1}^{m} M_j(\theta_o, \sigma_e^2)\mathcal{R}_k(j) \tag{8}$$

where the matrix coefficients $\bar{M}$ and M_j are functions of the true parameter θ_o and the noise variance σ_e^2 .

B. Controller Design

Given the parameter estimate $\hat{\theta}_k$, the controller for the batch k is designed according to a *nominal* $\mathcal{H}_2$ criterion

$$C_k = C(\hat{\theta}_k) = \arg\min_K J_K \tag{9}$$

with

$$J_K = \left\| \begin{matrix} \frac{H(\hat{\theta}_k)}{1+KG(\hat{\theta}_k)} \\ \frac{\sqrt{\gamma}KH(\hat{\theta}_k)}{1+KG(\hat{\theta}_k)} \end{matrix} \right\|_{\mathcal{H}_2}^2 . \tag{10}$$

This criterion is equivalent to minimizing the weighted sum of the output power $\overline{E}[y_{dl,e}^2]$ and the input power $\overline{E}[u_{dl,e}^2]$ of the design loop driven by e in batch k (Figure 1).

$$J_K = \overline{E}[y_{dl,e}^2] + \gamma\overline{E}[u_{dl,e}^2]. \tag{11}$$

In the last formula, $\overline{E}$ is the generalized expectation operator defined in [6]. For ease of notation here and in the following we do not indicate the dependence of time signals such as $y_{dl,e}$ and $u_{dl,e}$ on the batch number k.

C. Total Cost, Application Cost, Excitation Cost

In order to measure the distance between the optimal loop and the experimental loop, we define the *total cost* T_k for the batch k as

$$T_k \triangleq \overline{E}[(y_{ol,e} - y_{el,er})^2]. \tag{12}$$

In this formula $y_{ol,e}$ is the output of the optimal loop driven by e and $y_{el,er}$ is the output of the experimental loop driven by both e and r (Figures 3 and 2).

Since r and e are independent, we can split T_k in two parts

$$\overbrace{\overline{E}[(y_{ol,e} - y_{el,er})^2]}^{T_k} = \overbrace{\overline{E}[(y_{ol,e} - y_{el,e})^2]}^{V_k} + \overbrace{\overline{E}[y_{el,r}^2]}^{E_k} \tag{13}$$

where $y_{el,e}$ is the output of the experimental loop driven by the noise term e only and $y_{el,r}$ is the output of the same loop driven by the excitation term r only:

$$y_{el,e} = \frac{H_o}{1 + C(\hat{\theta}_k)G_o}e, \qquad y_{el,r} = \frac{G_o}{1 + C(\hat{\theta}_k)G_o}r. \tag{14}$$

The term V_k in (13) is called *application cost* and represents the cost that is paid due to the use of the controller $C(\hat{\theta}_k)$ instead of the optimal controller C_o. The term E_k is called *excitation cost* and is paid due to the introduction of the excitation signal r_k.

Using Parseval relation we can write V_k as

$$V_k = \left\| \frac{H(\theta_o)}{1 + C(\hat{\theta}_k)G(\theta_o)} - \frac{H(\theta_o)}{1 + C(\theta_o)G(\theta_o)} \right\|_{\mathcal{H}_2}^2 \sigma_e^2 \tag{15}$$

This is a nonlinear function of the true parameter θ_o, the parameter estimate $\hat{\theta}_k$ and the noise variance σ_e^2. Note that V_k seen as a function of $\hat{\theta}_k$ has a global minimum in $\hat{\theta}_k = \theta_o$. Expanding V_k in the variable $\hat{\theta}_k$ locally around θ_o we get

$$V_k = \frac{1}{2}(\hat{\theta}_k - \theta_o)^\top V''(\hat{\theta}_k - \theta_o) + O\left(\left\|\theta_o - \hat{\theta}_k\right\|^3\right). \tag{16}$$

The Hessian V'' of the application cost will be here computed numerically using finite differences. A second order approximation of V_k obtained by ignoring the last term in (16) will be used to formulate the Experiment Design Problem.

Using the Parseval relation we can also write E_k as

$$E_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left| \frac{G(e^{i\omega}, \theta_o)}{1 + C(e^{i\omega}, \hat{\theta}_k)G(e^{i\omega}, \theta_o)} \right|^2 \Phi_k^r(\omega)\, d\omega \ . \tag{17}$$

When the spectrum of r_k has the structure (6), E_k can be written as a linear function of $\mathcal{R}_k$ [5]

$$E_k = c_0(\theta_o)\mathcal{R}_k(0) + \sum_{j=1}^{m} c_i(\theta_o)\mathcal{R}_k(j). \tag{18}$$

D. Worst Case Settings

Let us consider the parameter set

$$\mathcal{D}_k = \left\{\theta \mid (\theta - \theta_o)^\top R_k(\theta - \theta_o) \leq \chi^2_{\alpha(p)}\right\} \tag{19}$$

where $\chi^2_\alpha(p)$ is the α-percentile of the χ^2 distribution with p degrees of freedom. Due to (5), we have that

$$\hat{\theta}_k \in \mathcal{D}_k \qquad \text{with probability } \alpha. \tag{20}$$

We define the worst case application cost V_k^{wc} with probability $\geq \alpha$ for the batch k as

$$V_k^{\text{wc}} \triangleq \max_{\hat{\theta}_k \in \mathcal{D}_k} V_k. \tag{21}$$

This is the maximum application cost on an uncertainty ellipsoid in which $\hat{\theta}_k$ lays with probability α. Accepting the

second order approximation for the V_k and following the reasoning of [8], we can compute V_k^{wc} as

$$V_k^{\text{wc}} = \min_{\lambda} \frac{1}{\lambda} \text{ such that } R_k \geq \lambda \frac{V'' \chi_\alpha^2(p)}{2}. \tag{22}$$

Note that R_k depends on true parameter θ_o through the coefficients $\bar{M}, M_j$. Since θ_o is unknown, the estimate $\hat{\theta}_1$ will be used in order to set the Experiment Design problem.

The excitation cost is considered in our settings in a nominal sense (i.e. not in a worst-case sense) in order to limit the problem complexity. We consider the excitation cost on the design loop available in batch 1, which is a function of $\hat{\theta}_1$

$$\tilde{E}_k = c_0(\hat{\theta}_1)\mathcal{R}_k(0) + \sum_{j=1}^{m} c_i(\hat{\theta}_1)\mathcal{R}_k(j). \tag{23}$$

We define the worst case total cost as

$$T_k^{\text{wc}} \triangleq V_k^{\text{wc}} + \tilde{E}_k. \tag{24}$$

For the first batch V_1^{wc} cannot be computed in our settings since the covariance of $\hat{\theta}_1$ is not specified. Note that even if this covariance were given, we could not influence V_1^{wc} in the Experiment Design.

For these reasons, we define the objective to be minimized as

$$J^{\text{wc}} \triangleq \tilde{E}_1 + \sum_{i=2}^{n} T_i^{\text{wc}} = \sum_{i=2}^{n} V_i^{\text{wc}} + \sum_{i=1}^{n} \tilde{E}_i. \tag{25}$$

Let us define the aggregate variable $\mathcal{R} \triangleq \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_n\}$ containing the coefficients describing the spectra of all the excitation signals. Our Experiment Design problem is finally

$$\mathcal{R}^o = \arg\min_{\mathcal{R}} J^{\text{wc}} \tag{26}$$

III. Experiment Design Problem

Using the results from Section II, we are able to formulate the Experiment Design Problem (26) as the following Convex Optimization Problem

$$\mathcal{R}^o, \lambda^o, Q^o = \arg\min_{\mathcal{R},\lambda,Q} \overbrace{\sum_{k=2}^{n} \frac{1}{\lambda_k}}^{\text{w.c. application cost}} + \overbrace{\sum_{k=1}^{n} \tilde{E}_k}^{\text{excitation cost}} \quad \text{s.t.} \tag{27}$$

$$\tilde{E}_k = 2c_0(\hat{\theta}_1)\mathcal{R}_k(0) + \sum_{j=1}^{m} c_i(\hat{\theta}_1)\mathcal{R}_k(j) \tag{28}$$

$$\overbrace{R_{k-1} + \bar{M}(\hat{\theta}_1) + \sum_{j=0}^{m} M_j(\hat{\theta}_1, \sigma_e^2)\mathcal{R}_{k-1}(j)}^{R_k} \geq \lambda_k \frac{\chi_\alpha^2(n) V''}{2}$$

$$\text{for } k = 2, \ldots, n \tag{29}$$

$$\lambda_k \geq 0 \qquad \text{for } k = 2, \ldots, n \tag{30}$$

$$\begin{bmatrix} Q_k - A^\top Q_k A & C_k^\top - A^\top Q_k B \\ C_k - B^\top Q_k A & D_k + D_k^\top - B^\top Q_k B \end{bmatrix} \geq 0$$

$$\text{for } k = 1, \ldots, n \tag{31}$$

with

$$A = \begin{bmatrix} 0 & 0 \\ I_{m-1} & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}$$
$$C_k = \begin{bmatrix} \mathcal{R}_k(1) & \mathcal{R}_k(2) & \ldots & \mathcal{R}_k(m) \end{bmatrix}, D_k = \frac{\mathcal{R}_k(0)}{2}, \tag{32}$$

and $Q_k = Q_k^\top$ symmetric matrix variables of proper size. For notational convenience we also defined the aggregate variables $\lambda \triangleq \{\lambda_2, \lambda_3, \ldots, \lambda_n\}$ and $Q \triangleq \{Q_1, Q_2, \ldots, Q_n\}$.

The objective function (27) is the sum of the worst case application costs $\frac{1}{\lambda_k}$ and the excitation costs $\tilde{E}_k$. Equation (29) is used to compute the worst case application cost $\frac{1}{\lambda_k}$ for each batch. Equation (31) guarantees the positivity constraint (7) as an application of the positive-real lemma as shown for instance in [9]. The problem (27-31) is a minimization of a convex function in the variables $\mathcal{R}, \lambda, Q$ subject to Linear Matrix Inequality (LMI) constraints.

We can proof that there exists an optimal solution to this problem which has all the excitation concentrated in the first batch, i.e. $\mathcal{R}_k^o = 0$ for $k > 1$.

Theorem 1: There is an optimal solution $\{\mathcal{R}^o, \lambda^o, Q^o\}$ to the problem (27-31) such that $\mathcal{R}_k = 0$ for $k > 1$.

Proof: The proof is performed *ad absurdum*. Let us assume that there exists an optimal solution with $\{\mathcal{R}^o, \lambda^o, Q^o\}$ such that $\mathcal{R}_k^o \neq 0$ for some $k > 1$. Let J^o be the objective value computed on this solution. Then, there exists another feasible solution $\{\mathcal{R}', \lambda', Q'\}$ with $\mathcal{R}' = \{\mathcal{R}_1^o + \mathcal{R}_2^o + \ldots \mathcal{R}_n^o, 0, \ldots, 0\}$ which has objective value $J' \leq J^o$. Thus, $\{\mathcal{R}^o, \lambda^o, Q^o\}$ cannot be the only optimal solution to the problem.

Feasibility of $\{\mathcal{R}', \lambda', Q'\}$ is obvious. Indeed, the spectrum corresponding to $\mathcal{R}_1'$ is the sum the of the positive spectra corresponding to $\mathcal{R}_1^o, \mathcal{R}_2^o, \ldots, \mathcal{R}_n^o$. Thus, it is also positive and feasible. As a consequence, there exists a feasible solution $\{\mathcal{R}', \lambda, Q'\}$ for some λ.

Proving that $J' \leq J^o$ is a bit more involved. We need to consider the two terms constituing the objective funciton (27) separately. The excitation cost is clearly the same in the two cases due to the linearity of Equation (28).

In order to show that the worst case application cost is lower for $\mathcal{R}'$ it is sufficient to write the matrices R_k in both situation and observe that $R_k' \geq R_k^o$. Since $\sum_{j=0}^{m} M_j \mathcal{R}_i^o(j) \geq 0$, $\forall i$, we have indeed that $R_k' = (k-1)\bar{M} + \sum_{i=1}^{n-1}\sum_{j=0}^{m} M_j \mathcal{R}_i^o(j) \geq R_k^o = (k-1)\bar{M} + \sum_{i=1}^{k-1}\sum_{j=0}^{m} M_j \mathcal{R}_i^o(j)$. This implies that there exists λ' such that $\{\mathcal{R}', \lambda', Q'\}$ is feasible and $\lambda_i' \geq \lambda_i^o$, $\forall i$. ∎

The interpretation of Theorem 1 is that when it is possible to excite the system, it is convenient to do it already during the first batch. Indeed, the price to be paid for the excitation is the same, while the benefit of having a better model will be enjoyed for all the following batches.

However, Theorem 1 does not hold if we consider interesting modifications of our problem formulation, for instance by giving different weights to the batches or by setting constraints to some variables. We will present relevant constraints for the Experiment Design problem in the following subsection.

A. Additional Constraints

It may be desirable to consider some constraint in the Experiment Design problem. For instance, an upper bound on the excitation cost $\tilde{E}_k$ can be set with

$$\tilde{E}_k \leq \bar{E},\ k = 1 \dots n. \tag{33}$$

The reasons for doing so may be to satisfy safety limits and avoid to operate in regions far away from the usual working point. Similar constraints may be set on the application and/or on the total cost as well, according to the need of the user. Including this kind of constraints the optimal excitation is not necessarily concentrated in the first batch as it was the unconstrained case.

B. Formulation as SemiDefinite Program

It is possible to transform the problem (27-31) to the SDP form, i.e. a *linear* objective function subject to a set of LMI constraints. For this class of problems, a number of efficient algorithms has been developed and is implemented in commonly available software packages [10], [11].

Introducing the additional slack variables $t_k \geq 0$ we can write the equivalent SDP problem

$$\mathcal{R}^o, \lambda^o, Q^o, t^o = \arg \min_{R,\lambda,Q,t} \quad \sum_{k=2}^{n} t_k + \sum_{k=1}^{n} \tilde{E}_k \quad \text{s.t.} \tag{34}$$

(28), (29), (31)

$$\begin{bmatrix} \lambda_k & 1 \\ 1 & t_k \end{bmatrix} \geq 0 \qquad \text{for } k = 2, \dots, n \tag{35}$$

where we defined the aggregate variable $t \triangleq \{t_2, t_3, \dots, t_n\}$. Note that the additional set of LMIs (35) guarantees the conditions $\lambda_k \geq 0$, $t_k \geq 0$ and $t_k \geq \frac{1}{\lambda_k}$ simultaneously.

IV. Simulation Study

The Experiment Design framework is applied to a simulation study. We consider the true system $\mathcal{S}_o$

$$y = \overbrace{\frac{0.8z^{-1}}{1 - 0.9854z^{-1} + 0.8187z^{-2}}}^{G_o(z)} u + \overbrace{\frac{1}{1 - 0.6z^{-1}}}^{H_o(z)} e \tag{36}$$

with noise variance $\sigma_e^2 = 1$. A Box-Jenkins (BJ) model structure $\mathcal{M} = \{\mathcal{M}(\theta),\ \theta \in \mathbb{R}^6\}$ is assumed. A model $\mathcal{M}(\theta)$ in the structure is defined as

$$y = \overbrace{\frac{\theta_1 z^{-1} + \theta_2 z^{-2}}{1 + \theta_5 z^{-1} + \theta_6 z^{-2}}}^{G(z,\theta)} u(t) + \overbrace{\frac{1 + \theta_3 z^{-1}}{1 + \theta_4 z^{-1}}}^{H(z,\theta)} e. \tag{37}$$

The initial model is $\mathcal{M}(\hat{\theta}_1)$ with

$$\hat{\theta}_1 = [0.6489\ \ -0.67\ \ -0.0103\ \ -0.6008\ \ -1.332\ 0.847]^\top.$$

The parameter γ of the controller design (10) is 0.1, the confidence level α in (19) is 0.99 and the FIR structure (6) for the excitation signal is chosen with $m = 5$. The total number of batches is $n = 10$ and each batch has length $N = 1000$. The constraints (33) on the excitation cost is used with $\bar{E} = 0.03$.

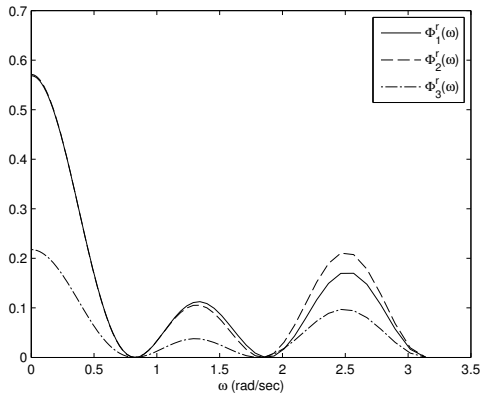

Fig. 4. Optimal spectra $\Phi_1^r, \Phi_2^r, \Phi_3^r$

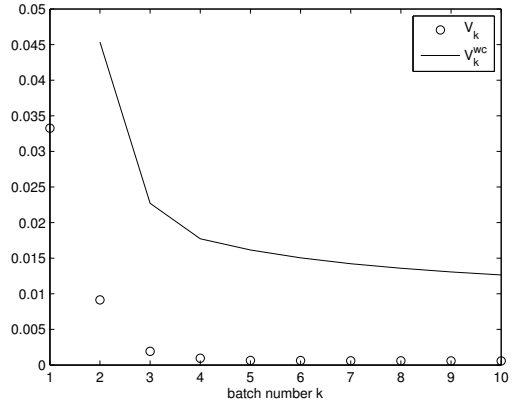

Fig. 5. Application cost V_k vs. batch number k

The LMI Lab toolbox [10] is used to obtain the numerical solution of the Experiment Design problem formulated in the SDP form.

The optimal spectra of the excitation signal for the first three batches $\Phi_1^r, \Phi_2^r, \Phi_3^r$ are reported in Figure 4. These spectra have approximately the same shape. However, Φ_3^r has a smaller amplitude compared to Φ_1^r and Φ_2^r. In fact, the amplitude of the first two spectra are limited by the constraint on the excitation power. This constraint is not active on the third spectrum. The optimal spectra for the following batches are zero up to numerical precision.

Excitation signal r_1, r_2, r_3 with spectra $\Phi_1^r, \Phi_2^r, \Phi_3^r$ are generated and $n = 10$ batches are simulated applying the iterative identification/controller design framework. The excitation signal r_1, r_2, r_3 are applied during the batches $1, 2, 3$ respectively.

In Figure 5 we present the worst case application cost V_k^{wc} computed in the Experiment Design together with the application cost V_k that is achieved in the simulation (as computed from (15)). As expected, V_k decreases significantly in the initial batches owing to the improving parameter estimates obtained with the iterative identification procedure. In the following batches, V_k remains very small. V_k is always smaller than the worst case performance V_k^{wc}. Note that the sequence of V_k obtained is just a single realization of a stochastic

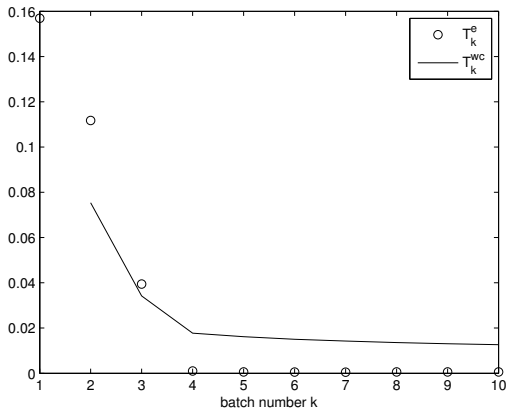

Fig. 6. Experimental total cost T_k^e vs. batch number k

process which depends on the particular noise realizations. If we repeat the experiment many times, we expect however the realizations of V_k to be lower than the bound V_k^{wc} in the 99% of the cases.

In Figure 6 the *experimental total cost* T_k^e defined as

$$T_k^e \triangleq \frac{1}{N}\sum_{t=1}^{N}(y_{ol,e}(t) - y_{el,er}(t))^2 \tag{38}$$

is reported. Note that T_k^e is a finite-time approximation of the actual total cost defined in (12). T_k^e decreases from batch to batch and is close to zero in batch 4. However, the constraint on the maximum excitation power is not satisfied in the simulation. Note for instance that the total experimental cost for the first batch is $T_1^e = 0.16$, while the application cost is $V_1 = 0.035$. This means that the actual excitation power was 0.125. Due to the violation of the constraint on the excitation power, T_k^e is also larger than the theoretical worst case T_k^{wc} for the batches 2 and 3.

The violation is possible because the excitation power is computed based on the initial model estimate in our Experiment Design formulation. Therefore, the constraint is not guaranteed to be satisfied when the excitation signals are applied on experimental loop. Possible techniques to insure robustness of the constraints with respect to the uncertainty of the initial model estimate are discussed in the conclusions.

V. Conclusion

We have presented an Experiment Design framework for systems performing multiple batches. After each batch, experimental data are collected and used to refine a model of the system dynamics. Subsequently, the model is used in order to design the controller for the next batch. The role of the Experiment Design is to generate the excitation signals for each batch in order to minimize a total cost defined over the set of batches that is scheduled.

The Experiment Design problem has been formulated as a Convex Optimization problem which can be solved efficiently. The applicability of the method has been verified in a simulation study.

A limitation of our approach is that the Experiment Design is based on an initial model estimate. This may give poor results when the initial estimate is far from the true system. A possible way to alleviate this problem is to perform the Experiment Design in an adaptive fashion. Before the execution of the first batch, we design the excitation signals for all the following batches. However, we only apply the first excitation signal in the first batch. When the first batch is completed, we perform the iterative identification procedure and we use the model obtained in order to formulate a new Experiment Design problem considering the remaining batches. Again only the first excitation signal computed is applied in the next batch, and so on and so forth up to the last batch. One can appreciate the analogy with the concept of Receding Horizon which is common in Model Predictive Control algorithms.

Another limitation is that the excitation cost is considered only in a nominal sense in our formulation. Ideally we would like to evaluate this term in a robust sense, as it is done for the application cost. A strategy to achieve this could be to grid the parameter space and consider the nominal problem on a finite number of points. Randomized algorithms could be applied in order draw the grid points, avoiding the curse of dimensionality typical of the deterministic methods.

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