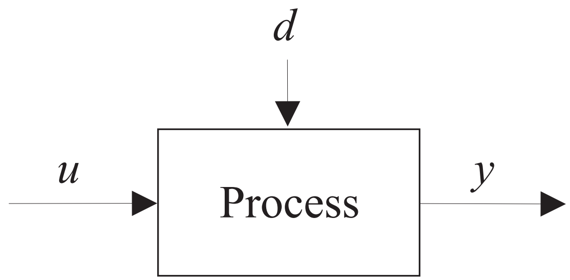
Conventional Control
A Refresher

Process to Be Controlled



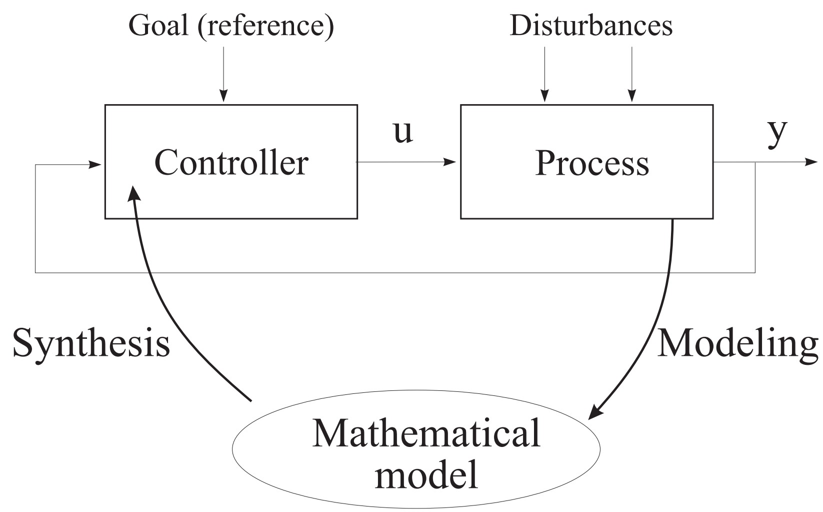
y	:	variable to be controlled (output)
u	:	manipulated variable (control input)
d	:	disturbance (input that cannot be influenced)

dynamic system

Examples of "Processes"

- technical (man-made) system
- natural environment
- organization (company, stock exchange)
- human body
- ...

Classical Control Design



How to Obtain Models?

- physical (mechanistic) modeling
  1. first principles → differential equations (linear or nonlinear)
  2. linearization around an operating point
- system identification
  1. measure input–output data
  2. postulate model structure (linear–nonlinear)
  3. estimate model parameters from data (least squares)

Modeling of Dynamic Systems

$x(t)$ ...state of the system

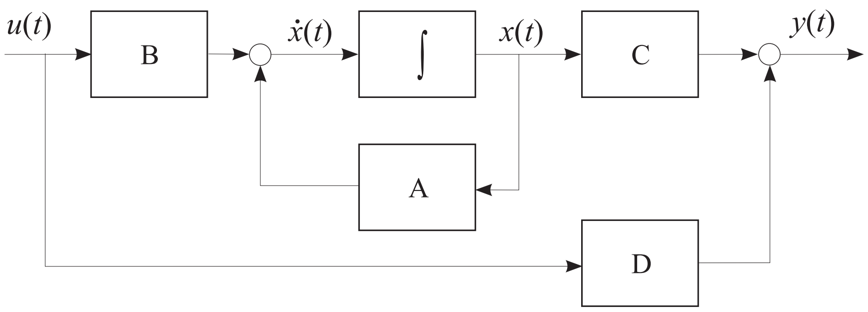
summarizes all history such that if we know $x(t)$ we can predict its development in time, $\dot{x}(t)$, for any input $u(t)$

linear state-space model:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

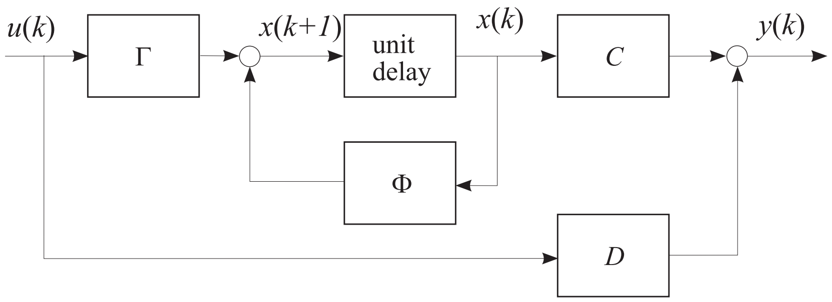
Continuous-Time State-Space Model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}$$



Discrete-Time State-Space Model

$$\begin{aligned}x(k+1) &= \Phi x(k) + \Gamma u(k)\\ y(k) &= Cx(k) + Du(k)\end{aligned}$$



Input–Output Models

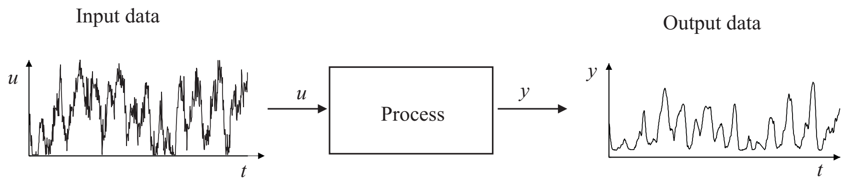
Continuous time:

$$y^{(n)}(t) = f\left(y^{(n-1)}(t), \ldots, y^{(1)}(t), y(t), u^{(n-1)}(t), \ldots, u^{(1)}(t), u(t)\right)$$

Discrete time:

$$\begin{aligned} y(k+1) &= f\Big(y(k), y(k-1), \ldots, y(k-n_y+1), \ldots, \\ &\qquad u(k), u(k-1), \ldots, u(k-n_u+1)\Big) \end{aligned}$$

System Identification



$u(1), u(2), \ldots, u(N)$ $y(1), y(2), \ldots, y(N)$

System Identification

Given data set
$\{(u(k), y(k)) \mid k = 1, 2, \ldots, N\}$:

1. Postulate model structure, e.g.:

$$\hat{y}(k+1) = ay(k) + bu(k)$$

2. Form regression equations:

$$\begin{aligned} y(2) &= ay(1) + bu(1) \\ y(3) &= ay(2) + bu(2) \\ &\vdots \\ y(N) &= ay(N-1) + bu(N-1) \end{aligned}$$

in a matrix form: $\boldsymbol{y} = \boldsymbol{\varphi}[a\ b]^T$

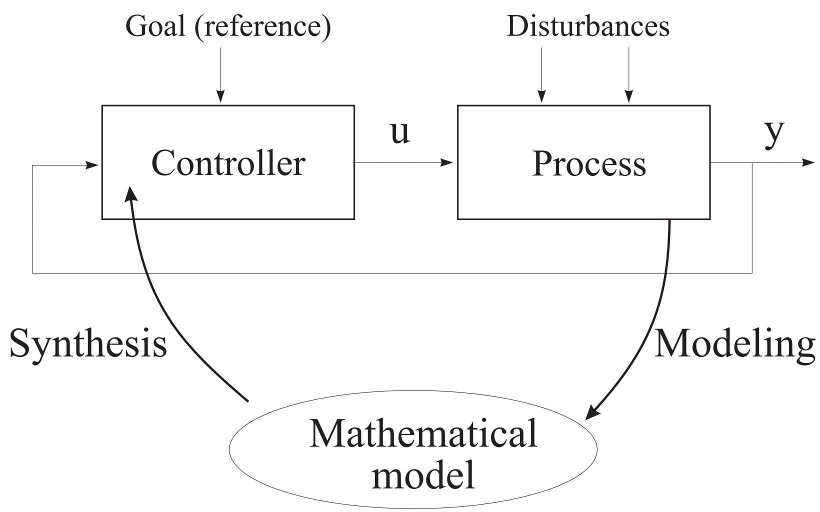
System Identification

3. Solve the equations for $[a\ b]$ (least-squares solution):

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{\varphi}[a\ b]^T \\ \boldsymbol{\varphi}^T\boldsymbol{y} &= \boldsymbol{\varphi}^T\boldsymbol{\varphi}[a\ b]^T \\ [a\ b]^T &= [\boldsymbol{\varphi}^T\boldsymbol{\varphi}]^{-1}\boldsymbol{\varphi}^T\boldsymbol{y} \end{aligned}$$

Numerically better methods are available
(in MATLAB `[a b] = φ \ y`).

Classical Control Design



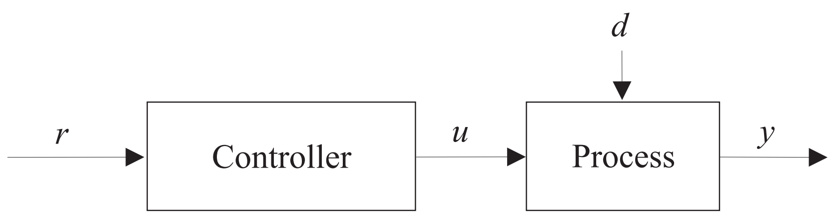
Design Procedure

- Criterion (goal)
  - stabilize an unstable process
  - suppress influence of disturbances
  - improve performance (e.g., speed of response)
- Structure of the controller
- Parameters of the controller (tuning)

Taxonomy of Controllers

- Presence of feedback: feedforward, feedback, 2-DOF
- Type of feedback: output, state
- Presence of dynamics: static, dynamic
- Dependence on time: fixed, adaptive
- Use of models: model-free, model-based

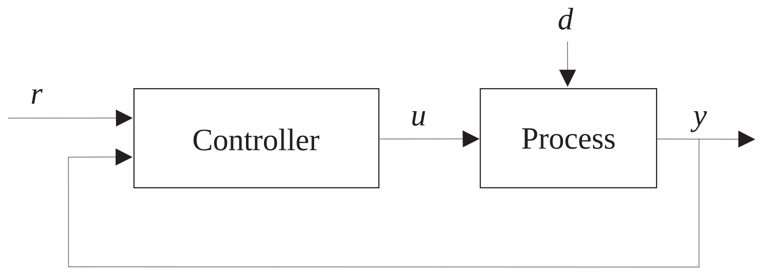
Feedforward Control



Controller:

- (dynamic) inverse of process model
- cannot stabilize unstable processes
- cannot suppress the effect of d
- sensitive to uncertainty in the model

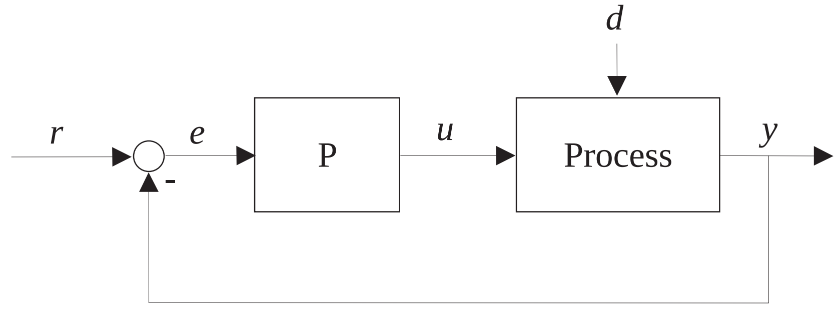
Feedback Control



Controller:

- dynamic or static ($\neq$ inverse of process)
- can stabilize unstable processes (destabilize stable ones!)
- can suppress the effect of d

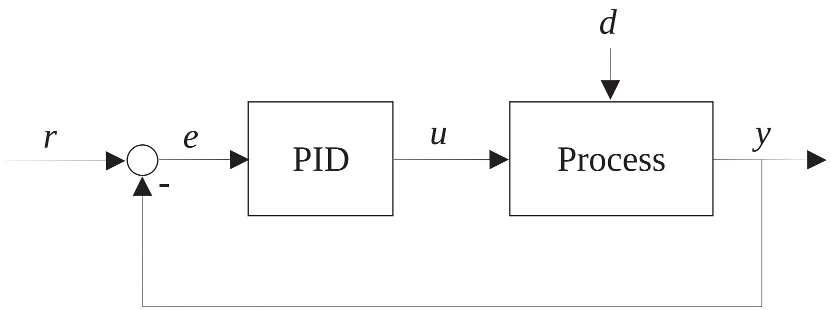
Proportional Control



Controller:

- static gain P: $u(t) = Pe(t)$

PID Control

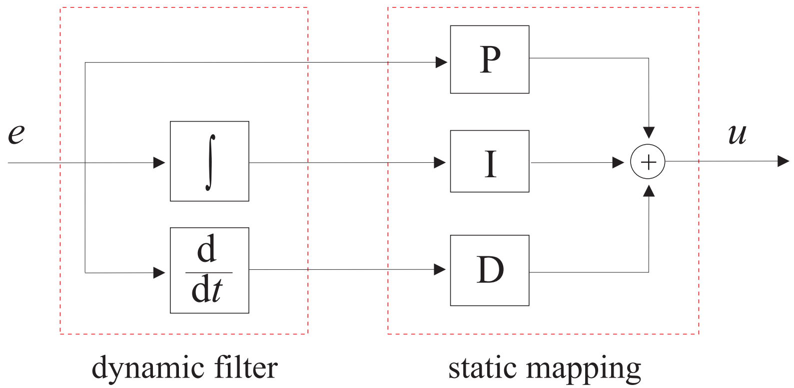


Controller:

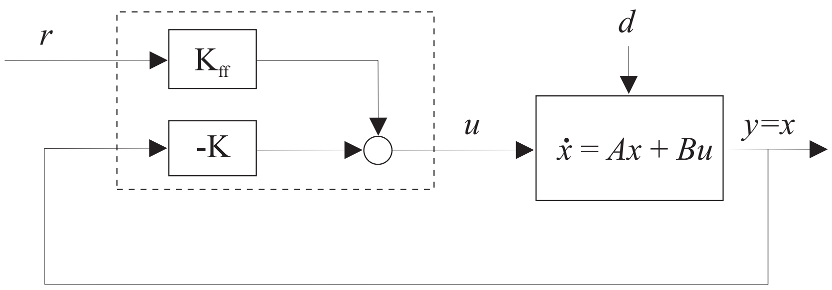
- dynamic: $u(t) = Pe(t) + I \int_0^t e(\tau)d\tau + D\frac{de(t)}{dt}$
- P, I and D are the proportional, integral and derivative gains, respectively

PID Control: Internal View

$$u(t) = Pe(t) + I \int_0^t e(\tau)d\tau + D\frac{de(t)}{dt}$$



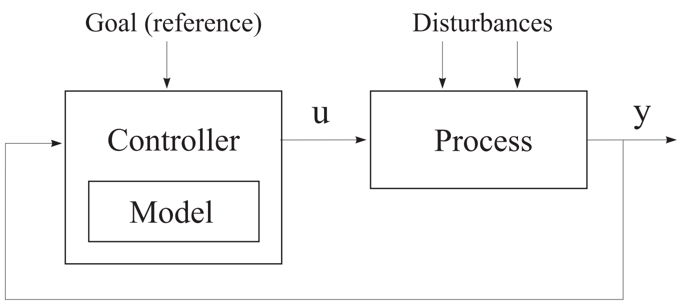
State Feedback



Controller:

- static: $u(t) = Kx(t)$
- K can be computed such that $(A + BK)$ is stable
- K_{ff} takes care of the (unity) gain from r to y

Model-Based Control



- state observer
- model-based predictive control
- adaptive control