Written Exam Knowledge Based Control Systems (SC 4081)

15 April 2015 from 9:00 until 12:00

Onderstaande aanwijzingen nauwkeurig lezen.

- Schrijf je naam en studienummer duidelijk leesbaar op ieder blad dat je inlevert.
- Lees iedere vraag goed alvorens te antwoorden. Het is **niet** toegestaan om het dictaat, boeken, aantekeningen, en andere hulpmiddelen te gebruiken (behalve rekenmachines).
- Praat nooit met je buurman om welke reden dan ook: het tentamen wordt in dit geval meteen ingenomen.
- Het tentamen bestaat uit 7 vraagstukken. De vraagstelling is in het Engels, antwoorden mogen in het Engels zijn. Motiveer elk antwoord, schrijf je redeneringen op papier. Bij elke vraag staat tussen haakjes het maximaal te behalen aantal punten aangegeven (totaal = 100). Het aantal punten bepaalt het cijfer.
- Succes!

Read carefully the instructions below.

- *Write clearly your name and student number on each sheet you hand in.*
- *Read carefully each question prior to formulating an answer. You are* **not** *allowed to use the course lecture notes, books, your own notes and other aids (except for calculators).*
- *Never talk to your fellow students: violation of this rule will result in an immediate termination of your exam.*
- *The exam consists of 7 questions. The questions are formulated in English, answers preferably in English (use Dutch only if it is really necessary). Motivate each answer, write down your reasoning.*
- *For each question, the maximum number of points you can get is indicated in parentheses (total = 100). The number of points gained will determine the final mark.*
- *Please answer the 7 questions in separated sheet (one sheet per question).*
- *Good luck!*

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1.

a) What is the difference between the membership function of an ordinary set and of a fuzzy set? **(2 p)**

b) Consider fuzzy set A defined in $X \times Y$ with $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$:

$$A = \{a/(x_1, y_1),\ b/(x_1, y_2),\ c/(x_2, y_1), d/(x_2, y_2)\}$$

with a, b, c and d known values in $[0, 1]$. Compute the projections of A onto X and Y. **(4 p)**

c) Define mathematically a convex fuzzy set. Give an example of a convex and non-convex fuzzy set. **(4 p)**

d) Given a fuzzy relation R: $X \times Y \rightarrow [0, 1]$:

$R =$

	y_1	y_2	y_3
x_1	0.1	0.4	0.1
x_2	0.4	0.5	0.2
x_3	0.1	0.2	0.9

and a fuzzy set $A = \{0.2/x_1,\ 0.9/x_2,\ 0.5/x_3\}$. Compute fuzzy set $B = A \circ R$, where '$\circ$' is the max-min composition operator. **(5 p)**

2.

a) Explain the steps of the Mamdani (max-min) inference algorithm for a linguistic fuzzy system with one crisp input and one fuzzy output. Apply these steps to the following rule base:

1) **If** x is A_1 **then** y is B_1,
2) **If** x is A_2 **then** y is B_2,

with

$$A_1 = \{0.2/-1,\ 0.5/0,\ 0.9/1\}, \quad A_2 = \{1/-1,\ 0.4/0,\ 0/1\},$$
$$B_1 = \{1/0,\ 0.9/1,\ 0.1/2\}, \quad B_2 = \{0/0,\ 0.5/1,\ 1/2\},$$

State the inference in terms of equations. Compute the output fuzzy set B' for $x = 1$. **(8 p)**

b) Consider an unknown dynamic system $y(k+1) = f(y(k-3), u(k-1))$. Give an example of a singleton fuzzy model that can be used to approximate this system. Explain all the parameters, operators and discuss/justify any assumption. **(7 p)**

3. a) State the definitions and discuss the differences of fuzzy and non-fuzzy (hard) partitions. Give an example of a fuzzy and non-fuzzy partition matrix. What are the advantages of fuzzy clustering over hard clustering? **(2 p)**

   b) About the Gustafson-Kessel algorithm: explain the algorithm, name all the parameters of the algorithm. Explain the role of the cluster covariance matrices, what represent their eigenvalues and the corresponding eigenvectors. **(8 p)**

4.

   a) There are various ways to parameterize nonlinear models and controllers. Name at least three different parameterizations and explain how they differ from each other. **(3 p)**

   b) Draw a control scheme with a fuzzy PD (proportional-derivative) controller, including the process. Explain the internal structure of the fuzzy PD controller, including the dynamic filter(s), rule base, etc. **(7 p)**

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5. a) What are the steps of the backpropagation algorithm? Give sufficient mathematical details, such that the reader can actually implement the method. **(7 p)**

   b) Consider a dynamic system $y(k+1) = f(y(k), y(k-1), u(k))$, where the function f is unknown. Suppose, we wish to approximate f by a neural network trained by using a sequence of N input–output data samples measured on the unknown system, $\{(u(k), y(k)) | k = 0, 1, \ldots, N\}$. Choose a neural network architecture, draw a scheme of the network and define its inputs and outputs. What are the free parameters that must be trained (optimized) such that the network fits the data well? Define a cost function for the training (by a formula) and name examples of two methods you could use to train the network's parameters. **(8 p)**

6. Consider a first-order affine Takagi–Sugeno model:

$$R_i \qquad \textbf{If } y(k) \text{ is } A_i \textbf{ then } y(k+1) = a_i y(k) + b_i u(k) + c_i$$

a) Derive the formula for the controller based on the inverse of this model, i.e., $u(k) = f(r(k+1), y(k))$, where r is the reference to be followed. **(10 p)**

b) Explain the idea of internal model control (IMC). **(5 p)**

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7. a) In the RL context, explain the terms 1) *environment*, 2) *state*, 3) *Markov property*, 4) *agent*, 5) *policy*, 6) *immediate and cumulative reward*, 7) *discount rate*, 8) *exploration*. Use formulas and equations to make your explanation clear. **(10 p)**

b) Is actor-critic reinforcement learning an on-policy or off-policy method? Explain you answer with details and formulas. **(5 p)**

c) Describe the tabular algorithm for $Q(\lambda)$-learning, using replacing traces to update the eligibility trace. **(5 p)**