Stochastic NMPC of Uncertain Batch Polymerization Reactor

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Main Objectives

- Stochastic NMPC strategy for an uncertain nonlinear system.
- Finite horizon nonlinear optimization problem with chance constraints.
- Sample-based approximation to replace the chance constraints.
- A Markov chain model for the uncertainty.

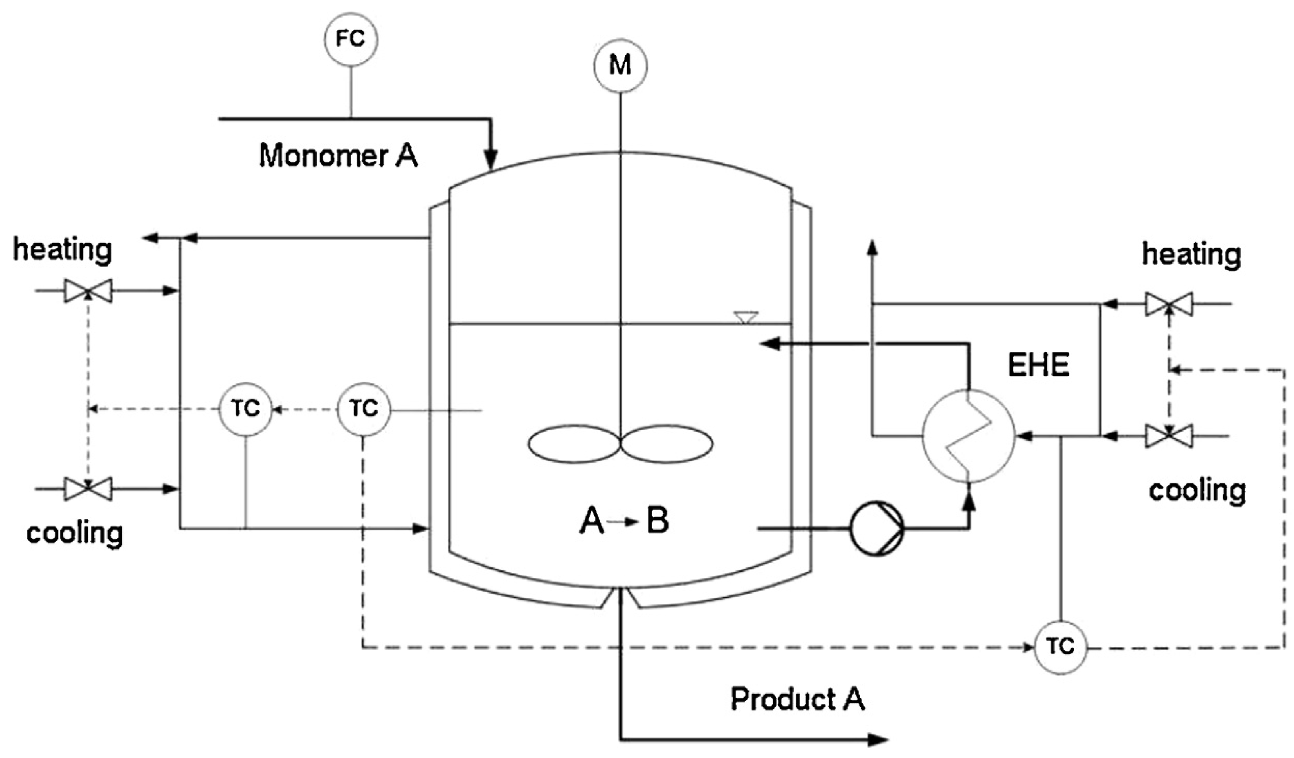

Figure 1: Batch polymerization reactor with an external heat exchanger.

Model and Method

- Uncertain Nonlinear System: $x_{t+1} = F(x_t, u_t, \delta_t)$

$$x_t = [\mathbf{m}_\mathrm{w}, \mathbf{m}_\mathrm{m}, \mathbf{m}_\mathrm{p}, \mathbf{T}_\mathrm{r}, \mathbf{T}_\mathrm{ek}, \mathbf{T}_\mathrm{awt}, \mathbf{T}_\mathrm{j}, \mathbf{T}_\mathrm{s}, \mathbf{T}_\mathrm{adiab}, \mathbf{m}_\mathrm{acc}] \in \mathbb{X} \subseteq \mathbb{R}^{10}$$

$$u_t = [\dot{\mathbf{m}}_\mathrm{f}, \mathbf{T}^\mathrm{in}_\mathrm{awt}, \mathbf{T}^\mathrm{in}_\mathrm{j}] \in \mathbb{U} \subseteq \mathbb{R}^3 \quad \text{and} \quad \delta_t = [k_0, \Delta H_\mathrm{r}] \in \mathbb{R}^2$$

- The main goal is to maximize the amount of polymer and a set-point tracking term for the desired reactor temperature:

$$-\mathbf{m}_{\mathrm{p},t+i|t} + \gamma(\mathbf{T}_{\mathrm{r},t+i|t} - T_\mathrm{set})^2 = \ell\left(x_{t+i|t} = F(x_t, \tilde{u}, \tilde{\delta})\right)$$

- Probabilistically Feasible State: $\mathbb{P}_\delta\left[x_{t+i|t} \in \mathbb{X}\ ,\ \forall\, i\right] \geq 1 - \epsilon$
- Direct multiple-shooting by using CasADi toolbox in Python [1].

Stochastic Nonlinear Model Predictive Control

- The stochastic objective function: $J(x_t, u, \delta) := \sum_{i=1}^{T} \ell\left(x_{t+i|t} = F(x_t, \tilde{u}, \tilde{\delta})\right)$

$$\min_{u \in \mathbb{U}} \ \mathbb{E}\left[J(x_t, u, \delta)\right] \quad \text{subject to:} \quad \mathbb{P}\left[x_{t+i|t} \in \mathbb{X}\ ,\ \forall\, i = \{1, \cdots, T\}\right] \geq 1 - \epsilon$$

Randomized Nonlinear Model Predictive Control

- Tractable formulation is called Randomized NMPC [2]:

$$\min_{u \in \mathbb{U}} \sum_{\delta^k \in W_0} J(x_t, u, \delta^k) \quad \text{subject to:} \quad x_{t+i|t} = F(x_t, u, \delta^l) \in \mathbb{X}\ ,\ \begin{cases} \forall\, i = \{1, \cdots, T\} \\ \forall\, \delta^{(l)} \in W_1 \end{cases}$$

where W_0 and W_1 are related to the set of S_0 and S_1 (scenarios). S_0 and S_1 are tuning variables to approximate the cost function and the chance constraints.

Algorithm 1 Randomized NMPC

1: Fix $S_0 \in [1, \infty)$ to approximate the cost function and $S_1 \in [1, \infty)$. When S_1 goes to infinity, the level of constraint violation 'ϵ' goes to zero.
2: Generate $S = (S_0 + S_1)$ scenarios of $\boldsymbol{\delta}$ (uncertain variables) corresponding to Δ^T.
3: Determine a feasible solution $\boldsymbol{u}^\star$.
4: Apply the first input of solution $u_t := u_0^\star$ to the uncertain real system.
5: Measure state (x_t): **if** $(\mathbf{m}_{p,t})$ is the desired quantity **then** stop.
6: Go to step 2.

Results

- It has been verified a violation level of $T_\mathrm{set} \pm 1.5[^\circ C]$ a posteriori.
- Randomized NMPC resulted in a feasible tracking of the reactor temperature set-point.
- Less reactor temperature variation results in longer batch process time.
- Trade-off between the quality of product and how fast the batch process is.
- Our future work aims at developing a theoretical guarantee for the feasibility of the solution.

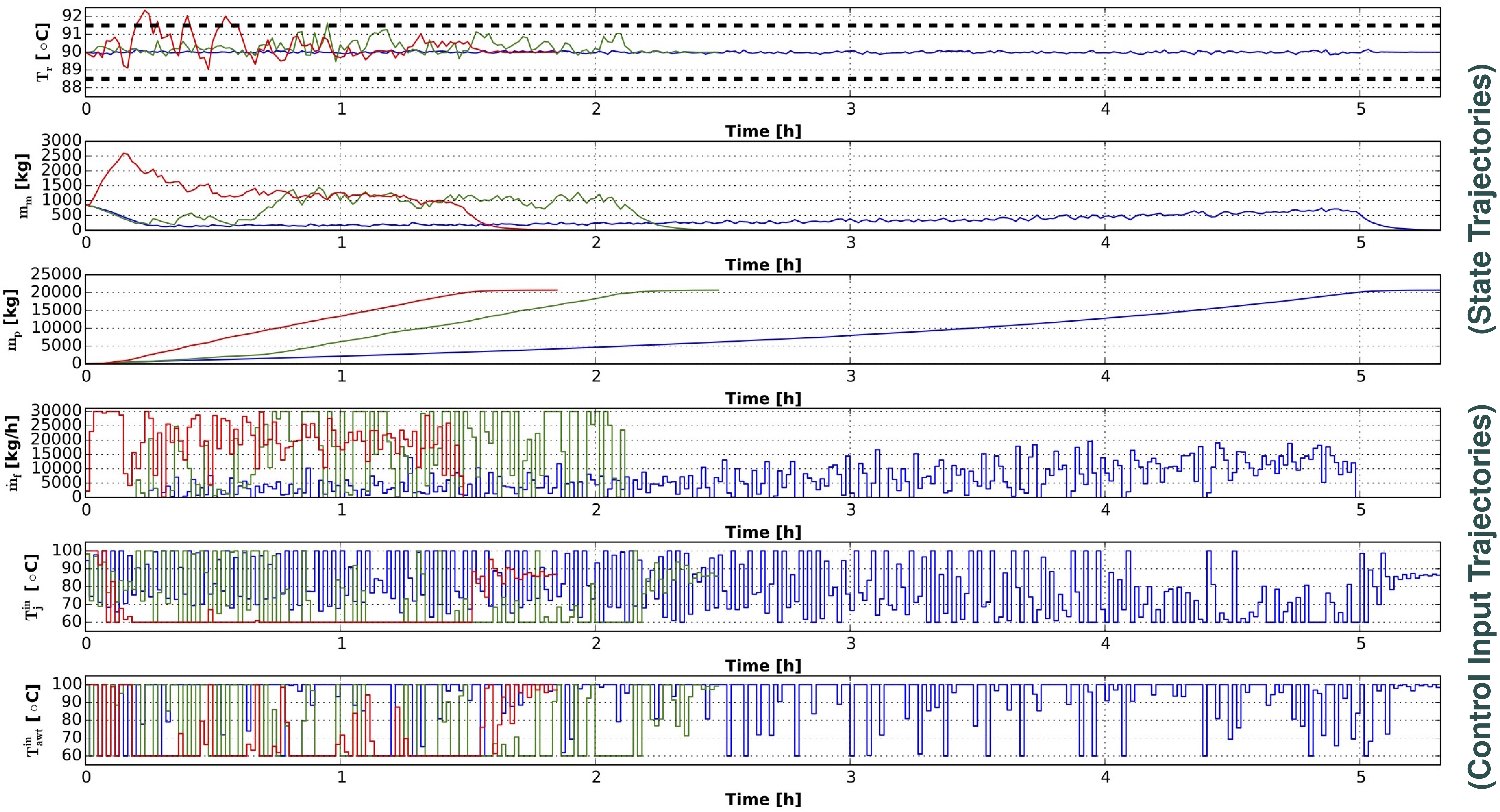

Figure 2: Red color denotes the results of implementation of deterministic NMPC. Green color represents the results obtained via randomized NMPC with just four different scenarios for uncertainties and blue color represents the results obtained via randomized NMPC with a hundred of different scenarios for uncertainties using Algorithm 1.

[1] Joel Andersson et al., CasADi: a symbolic package for automatic differentiation and optimal control, Recent Advances in Algorithmic Differentiation 2012
[2] Maria Parandini et al., A randomized approach to stochastic model predictive control, CDC 2012