Chance-Constrained Model Predictive Controller Synthesis for Stochastic Max-Plus Linear Systems

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Abstract

- Stochastic NMPC strategy for a class of discrete event systems, namely stochastic max-plus linear systems
- Using stochastic approach to interpret the constraints probabilistically, allowing for a sufficiently small violation level
- The proposed scheme does not require any assumption on the underlying probability distribution of the system parameters
- The developed framework is applicable to high dimensional problems, which makes it suitable for real industrial applications.

Max-Plus Algebra

$\mathcal{S}_{\text{mpns}}$ is the set of max-plus-nonnegative-scaling functions, i.e. functions f of the following form

$$f(z) = \max_{i\in\mathbb{N}_m}\{\alpha_{i,1}z_1 + \cdots + \alpha_{i,n}z_n + \beta_i\} \,, \tag{1}$$

where $z \in \mathbb{R}^n_\varepsilon$, $\alpha_{i,\cdot} \in \mathbb{R}^+$ and $\beta_i \in \mathbb{R}$.

Proposition 1. *Given $\mathcal{S}_{mpns}$ a max-plus-nonnegative-scaling function of z with each element $f(z) \in \mathcal{S}_{mpns}$. Then for any $\theta \in [0,1]$:*

1. the set $\mathcal{S}_{mpns}$ is a convex set, if

$$\forall g(z), h(z) \in \mathcal{S}_{mpns} \;\Rightarrow\; \theta g(z) + (1-\theta)h(z) \in \mathcal{S}_{mpns}.$$

2. $f(\cdot)$ is a convex function in the convex set $\mathcal{S}_{mpns}$, if for all $v, w \in \mathbb{R}^n_\varepsilon$

$$f\Big(\theta v + (1-\theta)w\Big) \le \theta f(v) + (1-\theta)f(w).$$

Stochastic Max-Plus Linear Systems

A stochastic MPL system is an extension of event-invariant MPL system where the system matrices are uncertain. This system is described as

$$\begin{cases} x_{k+1} &= A(\delta_k)\otimes x_k \oplus B(\delta_k)\otimes u_k \,, \\ y_k &= C(\delta_k)\otimes x_k \,, \end{cases} \tag{2}$$

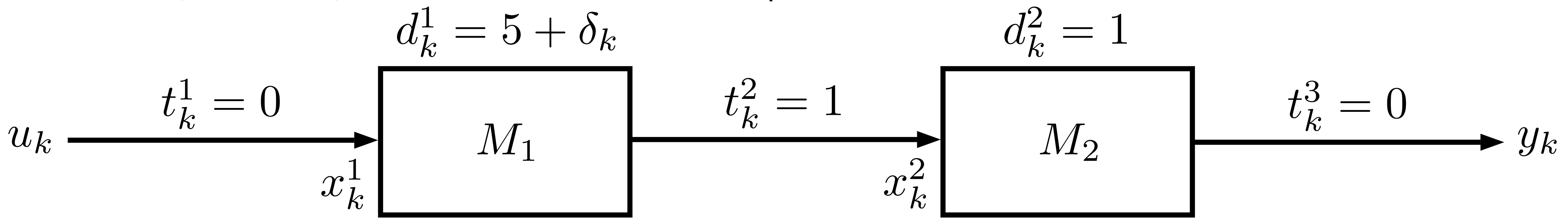
where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input vector, $y_k \in \mathbb{R}^\ell$ is the output vector, $\delta_k \in \Delta \subseteq \mathbb{R}^d$ is the random vector defined on a probability space $(\Delta, \mathbb{P})$ for $k \in \mathbb{N}\cup\{0\}$.

Proposition 2. *Given a stochastic MPL system in the form of (2), the future output events $y_{k+i|k}$ belong to the set of max-plus-nonnegative-scaling functions $\mathcal{S}_{mpns}$ for all $i \in \mathbb{N}_N$.*

Definition 1 (Probabilistically Feasible). *Given $\alpha \in (0,1)$ as an admissible constraint violation parameter, the state variables are called probabilistically feasible if*

$$\mathbb{P}_{\boldsymbol{\delta}}\left[\, x_{k+i|k} \in \mathbb{X} \,,\; i \in \mathbb{N}_N \,\right] \ge 1-\alpha \,. \tag{3}$$

Note that the index of $\mathbb{P}_{\boldsymbol{\delta}}$ denotes the dependency of $x_{k+i|k}$ on the string of random scenarios $\{\delta_0, \delta_1, \cdots, \delta_{N-1}\}$.



Stochastic Model Predictive Control

Define the objective function to be

$$\|\max\{y_{k+i|k} - r_{k+i}, 0\}\|_1 - \gamma\|u_{k+i|k}\|_1 := \mathcal{J}\,(x_k, \boldsymbol{u}, \boldsymbol{\delta}) \,,$$

where r_{k+i} is the deadline for the $(k+i)$-th occurrence of the output events and γ is a cost coefficient term for the input variables. Now we can formulate a chance-constrained finite-horizon optimal control problem for each event step k:

$$\min_{\boldsymbol{u}\in\mathbb{U}} \quad \mathbb{E}\left[\mathcal{J}(x_k, \boldsymbol{u}, \boldsymbol{\delta})\right] \tag{4a}$$

$$\text{s.t.} \quad u_{k+(i+1)|k} - u_{k+i|k} \ge 0 \,, \tag{4b}$$

$$\mathbb{P}_{\boldsymbol{\delta}}\left[\begin{cases} y_{k+i|k} \in \mathbb{Y} \\ \mathsf{E}\, u_{k+i|k} + \mathsf{F}\, y_{k+i|k} \le \mathsf{H} \\ i \in \{1, \cdots, N\} \end{cases}\right] \ge 1-\alpha \,, \tag{4c}$$

where $\mathbb{Y} \subseteq \mathbb{R}^\ell$ represents a desired convex bound on the predicted output events at each horizon step $i \in \mathbb{N}_N$.

Scenario Model Predictive Control

Theorem 1 (Calafiore, Campi, TAC, 2008). *Define the positive constant parameters $\alpha, \beta \in (0,1)$ to be a probability of constraint violations and a confidence level, respectively. If*

$$S_1 \ge g(\alpha, \beta, mN) := \frac{2}{\alpha}\ln\frac{1}{\beta} + 2mN + \frac{2mN}{\alpha}\ln\frac{2}{\alpha} \,,$$

then the optimal solution of the tractable formulation (5) is a feasible solution for the chance-constrained optimization problem (4) with confidence level of $(1-\beta)$.

Consider now the following tractable formulation of (4), called randomized MPC:

$$\min_{\boldsymbol{u}\in\mathbb{U}} \; \sum\nolimits_{\boldsymbol{\delta}^{(k)}\in\boldsymbol{W}_0} \mathcal{J}(x_k, \boldsymbol{u}, \boldsymbol{\delta}^{(k)}) \,, \tag{5a}$$

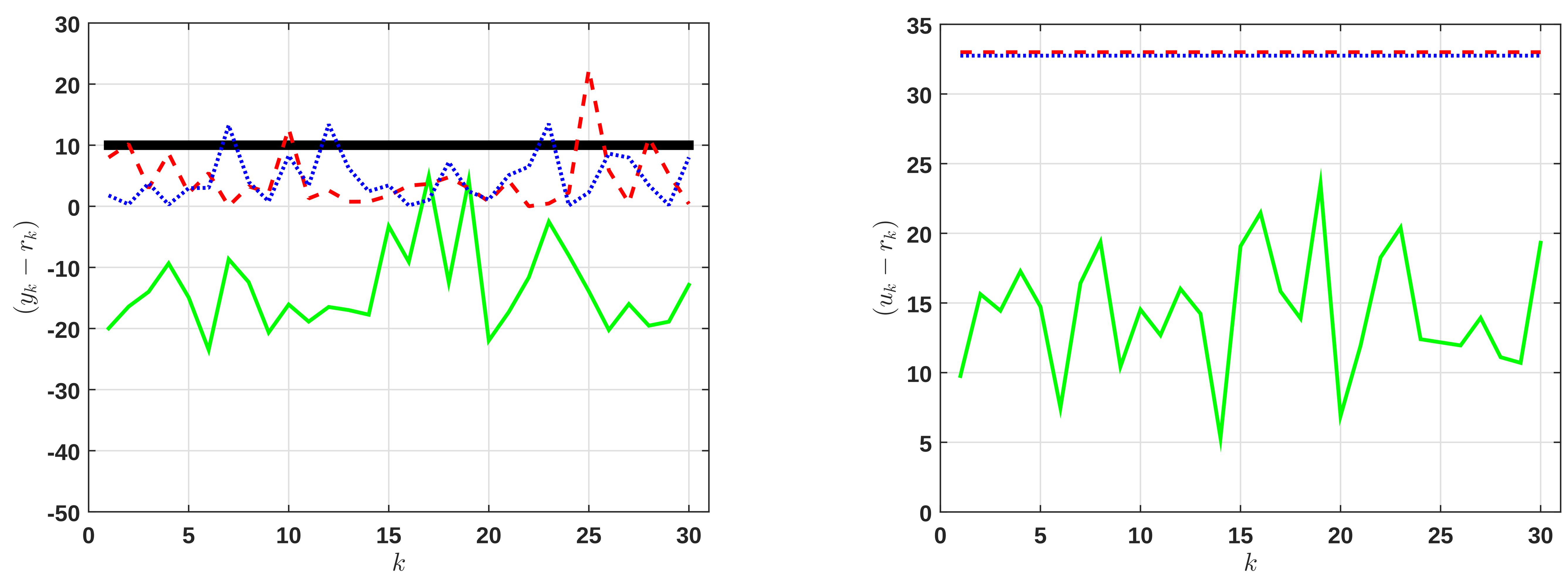
$$\text{s.t.} \quad u_{k+(i+1)|k} - u_{k+i|k} \ge 0 \,, \tag{5b}$$

$$\begin{cases} y_{k+i|k} = \varphi(x_k, \boldsymbol{u}, \boldsymbol{\delta}^{(l)}) \in \mathbb{Y} \\ \mathsf{E}\, u_{k+i|k} + \mathsf{F}\, y_{k+i|k} \le \mathsf{H} \end{cases}, \; \begin{cases} \forall\, i \in \mathbb{N}_N \\ \forall\, \boldsymbol{\delta}^{(l)} \in \boldsymbol{W}_1 \end{cases}, \tag{5c}$$

Remark 1. *The proposed framework provides a solution to the stochastic MPL system (2) with a probabilistic feasibility certificate and it does not necessarily lead to the optimal solution. This is due to the fact that a set of S_0 random scenarios is used as a tuning variable to empirically approximate the cost function $\mathcal{J}$.*

Results

- $\mathcal{S}_{\text{mpns}}$ is a set of convex function with respect to the control input
- Chance-Constrained MPC formulation for feasibility of system trajectories along the prediction horizon
- Randomized MPC framework to approximate the underlying problem with a-priori probabilistic performance guarantees for the feasibility of obtained solution with high confidence level.
- Simulation study for two different cases: production system case and Dutch railways to illustrate scalability of our framework



Production system case study. Left figure: the difference between the output signal and the due date signal. Right figure: the difference between the input signal and the due date signal. The 'green' solid line is related to the result of our developed framework. The 'red' dashed line corresponds to the nominal system model, whereas the 'blue' dotted line shows the results for the nominal system model with taking into account the mean value of the uncertain elements. The 'black' line is the boundary for $(y_k - r_k) \le 10$.