Building Heating and Cooling System with ATES
Part One: Model

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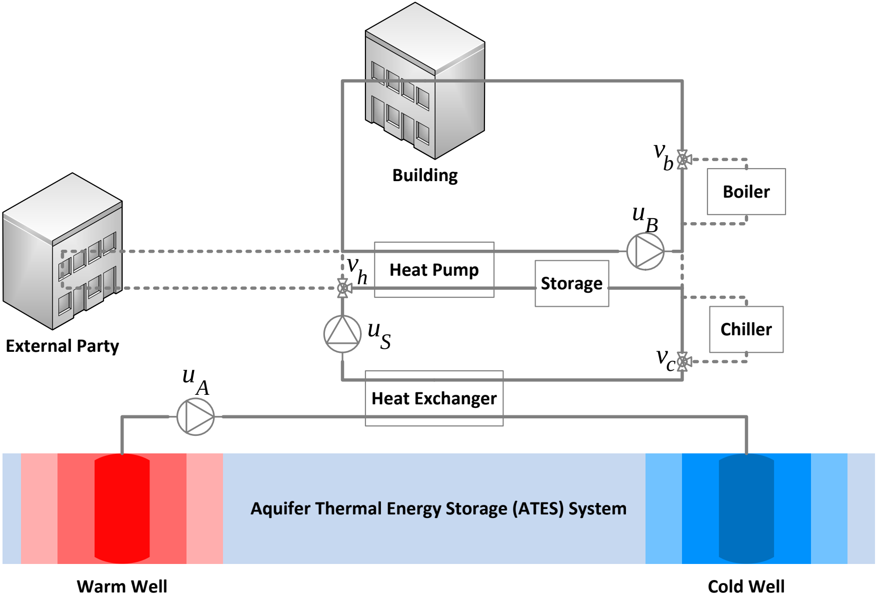

Outline

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Building Thermal Comfort Energy Demand

Main goal is to keep **building zone temperature at the desired level**.

- Building energy demand level is: $E_d = E_{\text{gain}} - E_{\text{loss}}$
- Endogenous source of losses: $E_{\text{loss}} = Q_{zo} + Q_{so} + Q_{\text{vent}}$
- Convection heat transfer from zone and solid to outside air: Q_{zo}, Q_{so}
- Ventilation thermal energy lost: Q_{vent}
- Endogenous source of energy: $E_{\text{gain}} = Q_{\text{rad}z} + Q_{\text{rad}s} + Q_p + Q_e$
- Radiation absorption by building zone and solid: $Q_{\text{rad}z}, Q_{\text{rad}s}$
- Occupancy and heat gain due to the electrical devices: Q_p, Q_e

Building Thermal Comfort Relations

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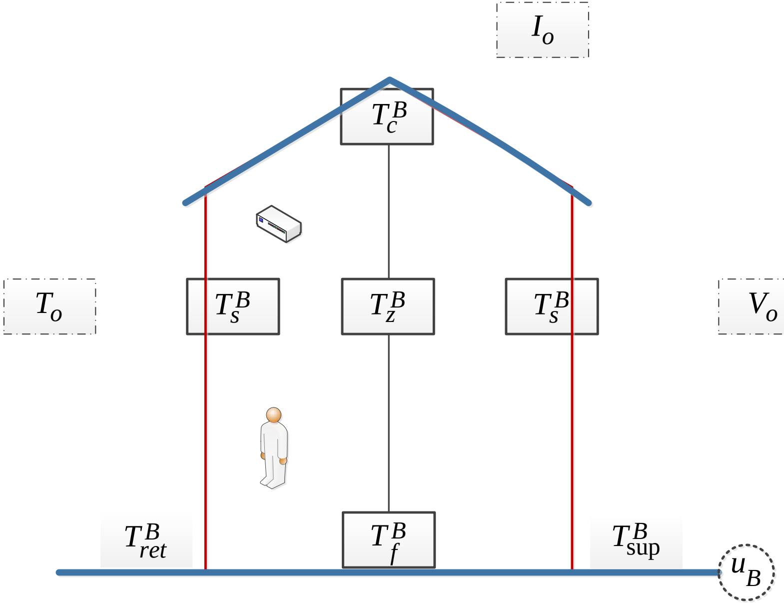
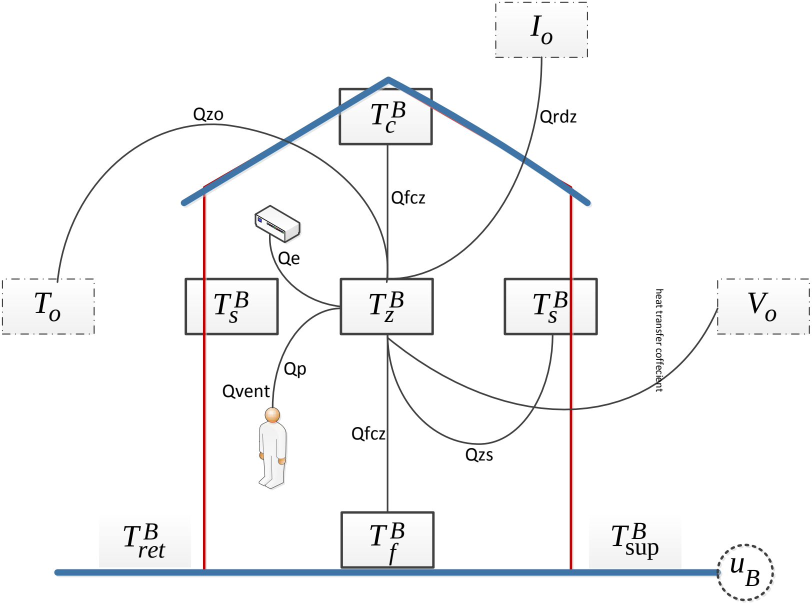
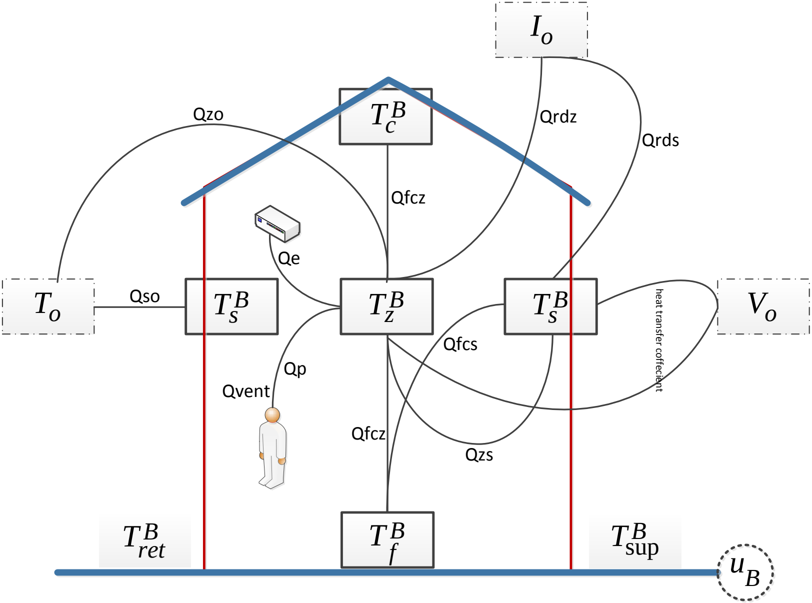


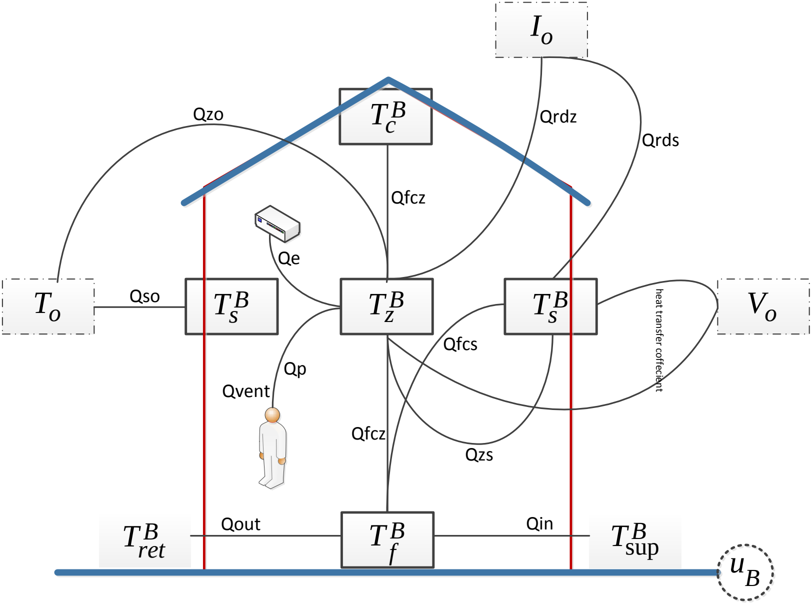
Building Thermal Comfort Relations



Building Thermal Comfort Relations



Building Thermal Comfort Relations



Building Thermal Comfort Model Formulation

We define the following model:

Building Dynamical Model

$$\boldsymbol{\Sigma}_{\mathrm{B}} := \begin{cases} \boldsymbol{x}_{\mathrm{B},k+1} = \boldsymbol{x}_{\mathrm{B},k} + \boldsymbol{f}_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B},k}, \boldsymbol{u}_{\mathrm{B},k}, \boldsymbol{v}_{\mathrm{B},k}, \boldsymbol{v}_{\mathrm{B}ext,k})\tau \\ \boldsymbol{y}_{\mathrm{B},k} = \boldsymbol{g}_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B},k}, \boldsymbol{u}_{\mathrm{B},k}) \end{cases}$$

- State variables: $\boldsymbol{x}_{\mathrm{B},k} := [\mathrm{T}^{\mathrm{B}}_{z,k}, \mathrm{T}^{\mathrm{B}}_{fc,k}, \mathrm{T}^{\mathrm{B}}_{s,k}] \in \mathbb{R}^3$
- External variable: $\boldsymbol{v}_{\mathrm{B}ext,k} := [\mathrm{T}_{o,k}, \mathrm{I}_{o,k}, \mathrm{V}_{o,k}] \in \mathbb{R}^3$
- Control variable: pump flow rate $\boldsymbol{u}_{\mathrm{B},k}$
- Input variable: $\boldsymbol{v}_{\mathrm{B},k} := \mathrm{T}^{\mathrm{B}}_{\mathrm{sup},k}$
- Output variable: $\boldsymbol{y}_{\mathrm{B},k} := \mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}$
- Sampling period: τ

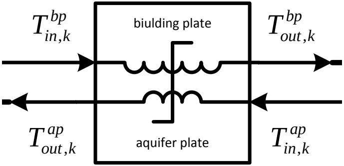
Outline

Heat Exchanger Model

A countercurrent heat exchanger is used and it presents via a static model.

Static Model Variables:

- Input water temperatures:
  $\boldsymbol{v}_{\mathrm{he},k} := [\mathrm{T}^{\mathrm{ap}}_{in,k}, \mathrm{T}^{\mathrm{bp}}_{in,k}] \in \mathbb{R}^2$
- Control variables:
  pump flow rates: $\boldsymbol{u}_{\mathrm{A},k}, \boldsymbol{u}_{\mathrm{S},k}$
- Output water temperatures:
  $\boldsymbol{y}_{\mathrm{he},k} := [\mathrm{T}^{\mathrm{ap}}_{out,k}, \mathrm{T}^{\mathrm{bp}}_{out,k}] \in \mathbb{R}^2$



Heat Exchanger

How to determine output from input water temperatures?

Heat Exchanger Model

Having the following relations:

- Aquifer plate thermal energy: $Q_{he,k} = \rho_w c_{p,w} u_{A,k} (T^{ap}_{out,k} - T^{ap}_{in,k})$
- Building plate thermal energy: $Q_{he,k} = \rho_w c_{p,w} u_{S,k} (T^{bp}_{in,k} - T^{bp}_{out,k})$
- Using the internal thermal energy conditions: $Q_{he,k} = k_{he} A_{he} \Delta T^{he}_{m}$
- ΔT^{he}_{m} is the mean temperature difference for the heat transfer.

Heat Exchanger Static Model

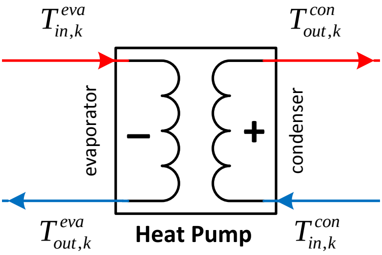
$$\Pi_{he} := \begin{cases} y_{he,k} = H(v_{he,k}, u_{A,k}, u_{S,k}) \\ \forall k \in \{0, 1, 2, \cdots\} \end{cases}$$

Heat Pump Model

An Electrical water to water heat pump is used with static model.

Static Model Variables:

- Input water temperatures: $\boldsymbol{v}_{\mathrm{hp},\boldsymbol{k}} := [\mathrm{T}^{\mathrm{con}}_{\boldsymbol{in},\boldsymbol{k}}, \mathrm{T}^{\mathrm{eva}}_{\boldsymbol{in},\boldsymbol{k}}] \in \mathbb{R}^2$
- Control variables: pump flow rates: $\boldsymbol{u}_{\mathrm{B},\boldsymbol{k}}, \boldsymbol{u}_{\mathrm{S},\boldsymbol{k}}$
- Output water temperatures: $\boldsymbol{y}_{\mathrm{hp},\boldsymbol{k}} := [\mathrm{T}^{\mathrm{con}}_{\boldsymbol{out},\boldsymbol{k}}, \mathrm{T}^{\mathrm{eva}}_{\boldsymbol{out},\boldsymbol{k}}] \in \mathbb{R}^2$



How to determine output from input water temperatures?

Heat Pump Model

Having the following relations:

- The thermal energy of condenser $Q_{h,k}$ and evaporator $Q_{c,k}$ sides:
  $Q_{h,k} = \rho_w c_{p,w} u_{B,k}(T^{con}_{out,k} - T^{con}_{in,k})$
  $Q_{c,k} = \rho_w c_{p,w} u_{S,k}(T^{eva}_{in,k} - T^{eva}_{out,k})$
- Using the internal thermal energies conditions:
  $Q_{h,k} = k_{hp} A_{hp} \Delta T^{hp}_{m,h}$ and $Q_{c,k} = k_{hp} A_{hp} \Delta T^{hp}_{m,c}$
- The coefficient of performance: $COP = Q_{h,k} \left(Q_{h,k} - Q_{c,k}\right)^{-1}$
- Using Carnot cycle: $COP = \eta_{hp} T_{hs} \left(T_{hs} - T_{cs}\right)^{-1}$

Heat Pump Static Model

$$\Pi_{hp} := \begin{cases} y_{hp,k} = P(v_{hp,k}, u_{B,k}, u_{S,k}) \\ \forall k \in \{0, 1, 2, \cdots\} \end{cases}$$

Storage Model

We define an storage model with the following first order difference equations:

$$V_{s,k+1} = V_{s,k} + V_{in,k} - V_{out,k}$$

$$T_{s,k+1} = \frac{V_{s,k}}{V_{s,k} + V_{in,k}} T_{s,k} + \frac{V_{in,k}}{V_{s,k} + V_{in,k}} T_{in,k}$$

Storage Dynamical Model

$$\Sigma_S := \begin{cases} x_{S,k+1} = f_S(x_{S,k}, u_{S,k}, v_{S,k}) \\ y_{S,k} = g_S(x_{S,k}) \end{cases}$$

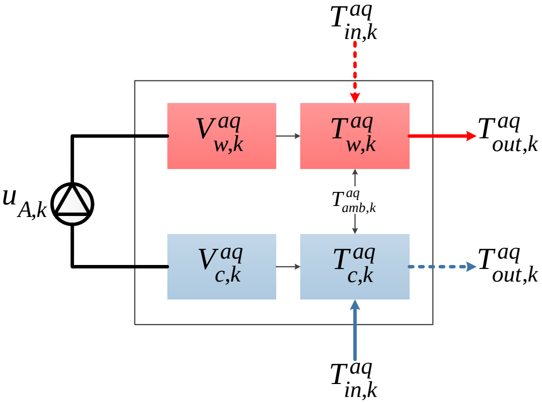
- State variables: $x_{S,k} := [V_{s,k}, T_{s,k}] \in \mathbb{R}^2$
- Control variable: pump flow rate $u_{S,k}$
- Output variable: $y_{S,k} := T_{s,k}$
- Input variable: $v_{S,k} := T_{in,k}$

Outline

Aquifer Thermal Energy Storage System Principle

Similar modeling as the storage model by introducing different modes:

- **Cold season:** water is injected into cold well and is taken from warm well.
- **Warm season:** water is injected into warm well and is taken from cold well.



Aquifer Thermal Energy Storage System Model

Consider the following mixed-integer first-order difference equations:

$$
\begin{aligned}
V^{aq}_{w,k+1} &= V^{aq}_{w,k} + (s_{w,k} - s_{c,k})V^{aq}_{in,k} \\
V^{aq}_{c,k+1} &= V^{aq}_{c,k} + (s_{c,k} - s_{w,k})V^{aq}_{in,k} \\
T^{aq}_{w,k+1} &= \frac{V^{aq}_{w,k}}{V^{aq}_{w,k} + s_{w,k}V^{aq}_{in,k}} T^{aq}_{w,k} + \frac{s_{w,k}V^{aq}_{in,k}}{V^{aq}_{w,k} + s_{w,k}V^{aq}_{in,k}} T^{aq}_{in,k} - \frac{\alpha(T^{aq}_{w,k} - T^{aq}_{amb,k})}{V^{aq}_{w,k} + s_{w,k}V_{in,k}} \\
T^{aq}_{c,k+1} &= \frac{V^{aq}_{c,k}}{V^{aq}_{c,k} + s_{c,k}V^{aq}_{in,k}} T^{aq}_{c,k} + \frac{s_{c,k}V^{aq}_{in,k}}{V^{aq}_{c,k} + s_{c,k}V^{aq}_{in,k}} T^{aq}_{in,k} - \frac{\alpha(T^{aq}_{c,k} - T^{aq}_{amb,k})}{V^{aq}_{c,k} + s_{c,k}V_{in,k}}
\end{aligned}
$$

- Integer variables of warm and cold season: $s_{w,k}, s_{c,k} \in \{0, 1\}$
- Output water temperature is: $T^{aq}_{out,k} = s_{c,k}T^{aq}_{w,k} + s_{w,k}T^{aq}_{c,k}$

Aquifer Thermal Energy Storage System Model

We define the following Model:

ATES system Dynamical Model

$$\Sigma_{\mathrm{A}} := \begin{cases} x_{\mathrm{A},k+1} = f_{\mathrm{A}}(x_{\mathrm{A},k}, u_{\mathrm{A},k}, v_{\mathrm{A},k}, s_{w,k}, s_{c,k}) \\ y_{\mathrm{A},k} = g_{\mathrm{A}}(x_{\mathrm{A},k}, s_{w,k}, s_{c,k}) \end{cases}$$

- State variables: $x_{\mathrm{A},k} := [\mathrm{V}^{\mathrm{aq}}_{w,k}, \mathrm{T}^{\mathrm{aq}}_{w,k}, \mathrm{V}^{\mathrm{aq}}_{c,k}, \mathrm{T}^{\mathrm{aq}}_{c,k}] \in \mathbb{R}^4$
- Control variable: pump flow rate $u_{\mathrm{A},k}$
- Output variable: $y_{\mathrm{A},k} := \mathrm{T}^{\mathrm{aq}}_{out,k}$
- Input variable: $v_{\mathrm{A},k} := \mathrm{T}^{\mathrm{aq}}_{in,k}$

Outline

Interconnections between each Subsystem

1. **ATES system:** $\boldsymbol{v}_{\mathrm{A},k} := \mathrm{T}^{\mathrm{aq}}_{in,k}, \quad \boldsymbol{y}_{\mathrm{A},k} := \mathrm{T}^{\mathrm{aq}}_{out,k}$
   - $\mathrm{T}^{\mathrm{aq}}_{in,k} = \mathrm{T}^{\mathrm{ap}}_{out,k}$
2. **Heat exchanger:** $\boldsymbol{v}_{\mathrm{he},k} := [\mathrm{T}^{\mathrm{ap}}_{in,k}, \mathrm{T}^{\mathrm{bp}}_{in,k}], \quad \boldsymbol{y}_{\mathrm{he},k} := [\mathrm{T}^{\mathrm{ap}}_{out,k}, \mathrm{T}^{\mathrm{bp}}_{out,k}]$
   - $\mathrm{T}^{\mathrm{ap}}_{in,k} = \mathrm{T}^{\mathrm{aq}}_{out,k}$ and $\mathrm{T}^{\mathrm{bp}}_{in,k} = (1 - v_{c,k})\mathrm{T}_{s,k} + v_{c,k}\mathrm{T}^{\mathrm{chi}}_{out,k}$
3. **Heat pump:** $\boldsymbol{v}_{\mathrm{hp},k} := [\mathrm{T}^{\mathrm{con}}_{in,k}, \mathrm{T}^{\mathrm{eva}}_{in,k}], \quad \boldsymbol{y}_{\mathrm{hp},k} := [\mathrm{T}^{\mathrm{con}}_{out,k}, \mathrm{T}^{\mathrm{eva}}_{out,k}]$
   - $\mathrm{T}^{\mathrm{con}}_{in,k} = s_{n,k}(s_{w,k}\mathrm{T}^{\mathrm{bp}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}) + (1 - s_{n,k})(s_{w,k}\mathrm{T}^{\mathrm{ext}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k})$
   - $\mathrm{T}^{\mathrm{eva}}_{in,k} = s_{n,k}(s_{c,k}\mathrm{T}^{\mathrm{bp}}_{out,k} + s_{w,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}) + (1 - s_{n,k})(s_{w,k}\mathrm{T}^{\mathrm{ext}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k})$
4. **Storage model:** $\boldsymbol{v}_{\mathrm{S},k} := \mathrm{T}_{in,k}, \quad \boldsymbol{y}_{\mathrm{S},k} := \mathrm{T}_{s,k}$
   - $\mathrm{T}_{in,k} = v_{h,k}(s_{w,k}\mathrm{T}^{\mathrm{con}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{eva}}_{out,k}) + (1 - v_{h,k})\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}$
5. **Building model:** $\boldsymbol{v}_{\mathrm{B},k} := \mathrm{T}^{\mathrm{B}}_{\mathrm{sup},k}, \quad \boldsymbol{y}_{\mathrm{B},k} := \mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}$

$$\mathrm{T}^{\mathrm{B}}_{\mathrm{sup},k} = v_{h,k}(s_{w,k}\mathrm{T}^{\mathrm{eva}}_{out,k} + s_{c,k}((1 - v_{b,k})\mathrm{T}^{\mathrm{con}}_{out,k} + v_{b,k}\mathrm{T}^{\mathrm{boi}}_{out,k})) + (1 - v_{h,k})\mathrm{T}^{\mathrm{bp}}_{out,k}$$

Single Agent Representation

Consider compact formulation of dynamical agent system:

Single Agent Model

$$\boldsymbol{x_{k+1} = f(x_k, u_k, v_k, s_k, \delta_k)}$$

- State variables vector: $\boldsymbol{x_k} := [x_{\mathrm{B},k}, x_{\mathrm{S},k}, x_{\mathrm{A},k}] \in \mathbb{R}^9$
- Pump flow rate variables vector: $\boldsymbol{u_k} := [u_{\mathrm{B},k}, u_{\mathrm{S},k}, u_{\mathrm{A},k}] \in \mathbb{R}^3$
- Valve position variables vector: $\boldsymbol{v_k} := [v_{b,k}, v_{c,k}, v_{h,k}] \in [0, 1]^3$
- Integer variables vector: $\boldsymbol{s_k} := [s_{w,k}, s_{c,k}, s_{n,k}] \in \{0, 1\}^3$
- Uncertain variables vector: $\boldsymbol{\delta_k} := [\mathrm{T}_{o,k}, \mathrm{I}_{o,k}, \mathrm{V}_{o,k}] \subseteq \boldsymbol{\Delta} \in \mathbb{R}^3$
- State variables are available at each sampling time $\boldsymbol{k}$.

Outline

Conclusion

Remarks:

- Detailed representation of each subsystem (components) in building heating and cooling system with ATES.
- Mathematical model of single dynamical agent system.

Next Steps:

- Formulating control optimal problem.
- Determining an objective function for each sampling time.
- Defining operational (state and control variables) constraints.

Thanks!
Questions?