Stochastic Constrained Control

For Large Scale Complex Systems

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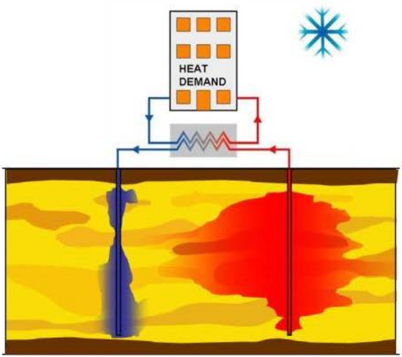
Aquifer Thermal Energy Storage (ATES)

- A large-scale natural subsurface storage for thermal energy
- An innovative method for thermal energy balance in smart grids

Cold season:

- The building requests thermal energy for the heating purpose
- Water is injected into cold well and is taken from warm well
- The stored water contains cold thermal energy for next season



Aquifer Thermal Energy Storage (ATES)

- A large-scale natural subsurface storage for thermal energy
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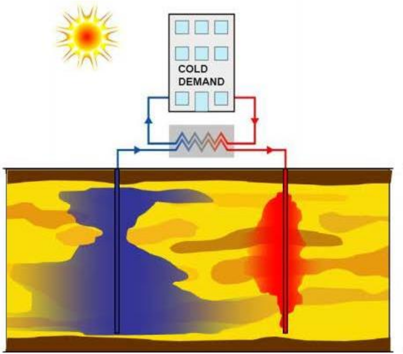
Warm season:

- The building requests thermal energy for the cooling purpose
- Water is injected into warm well and is taken from cold well
- The stored water contains warm thermal energy for next season



Smart Thermal Grids: ATES Systems

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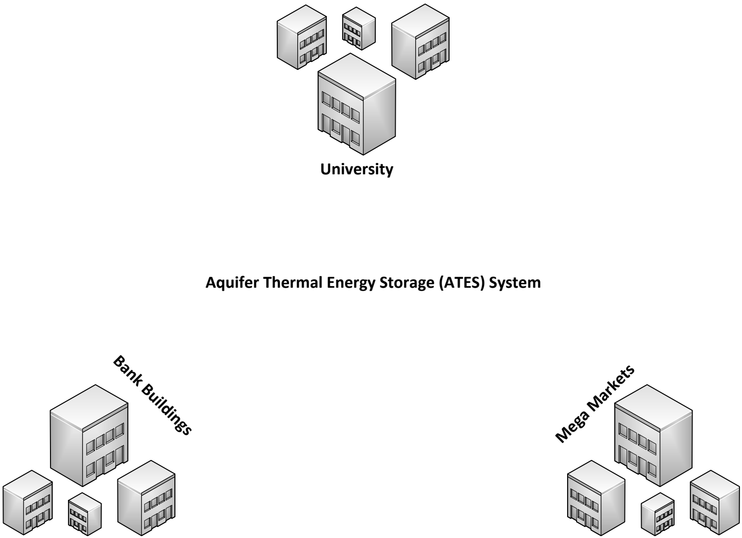

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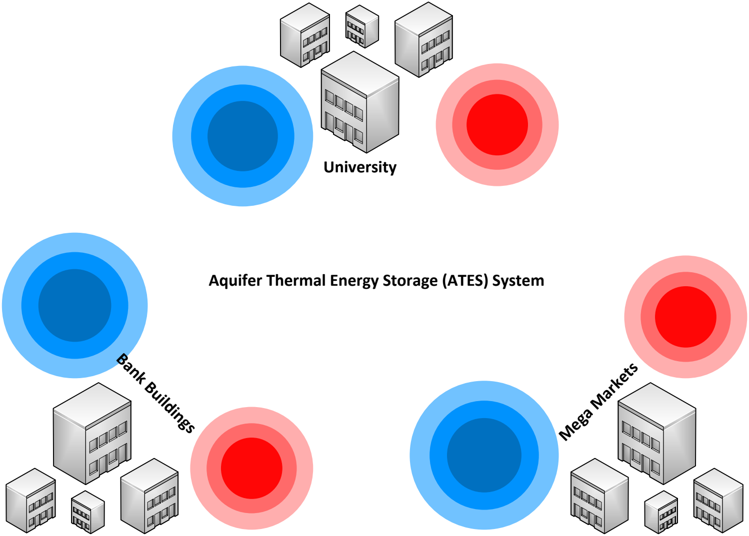

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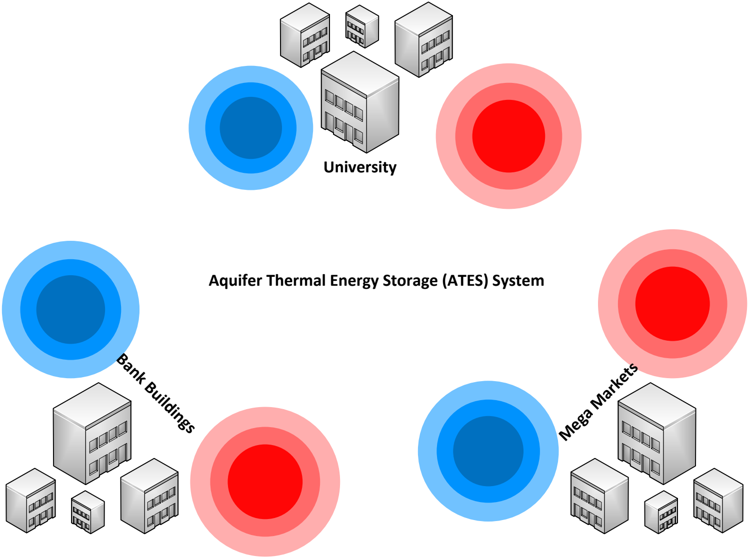

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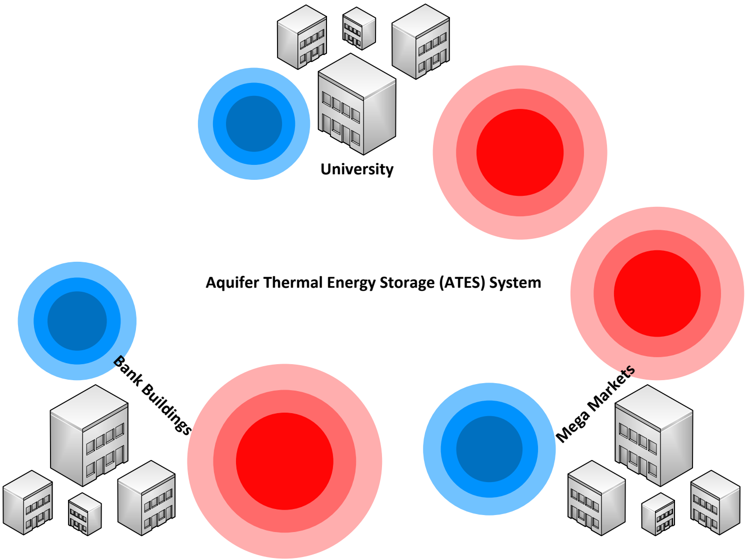

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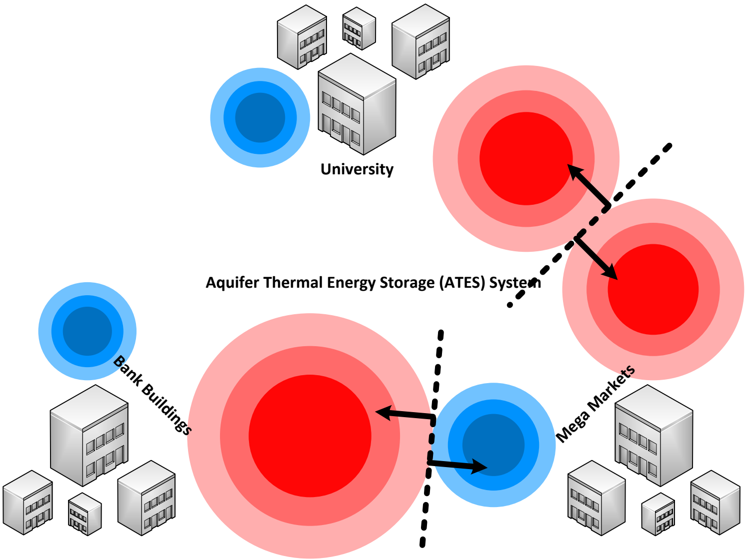
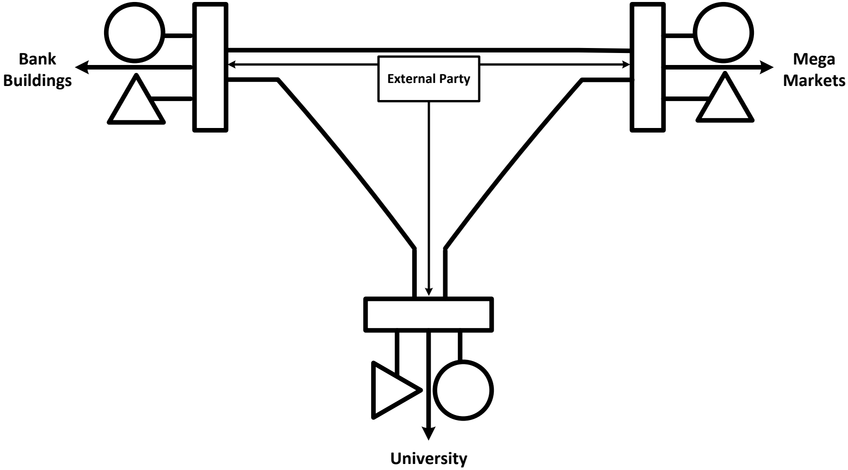


Smart Thermal Grids: Conceptual Representation



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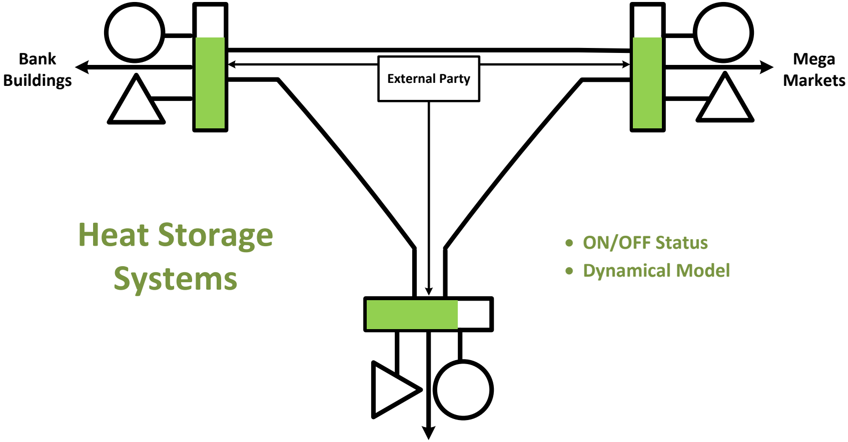

Heat Storage Systems

- **ON/OFF Status**
- **Dynamical Model**

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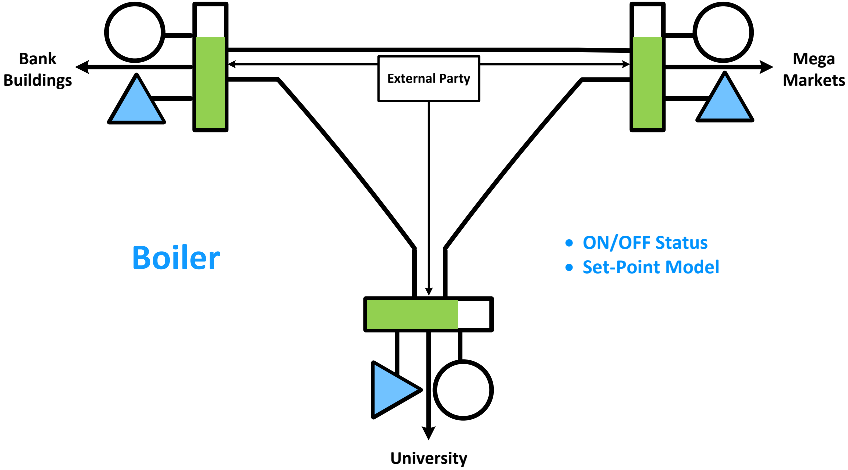

- ON/OFF Status
- Set-Point Model

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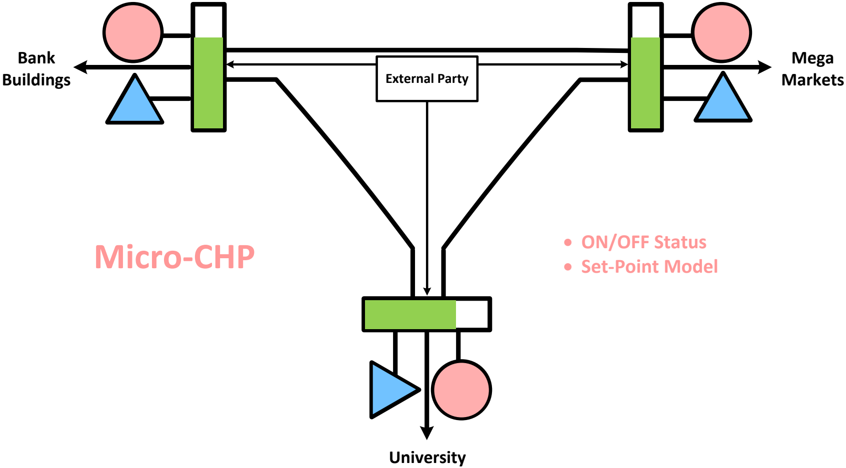
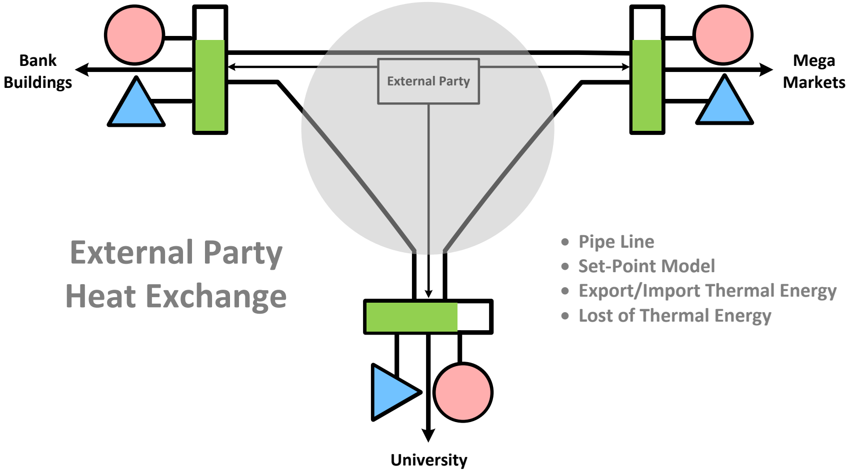


Micro-CHP

- **ON/OFF Status**
- **Set-Point Model**

Smart Thermal Grids: Conceptual Representation



External Party Heat Exchange

- Pipe Line
- Set-Point Model
- Export/Import Thermal Energy
- Lost of Thermal Energy

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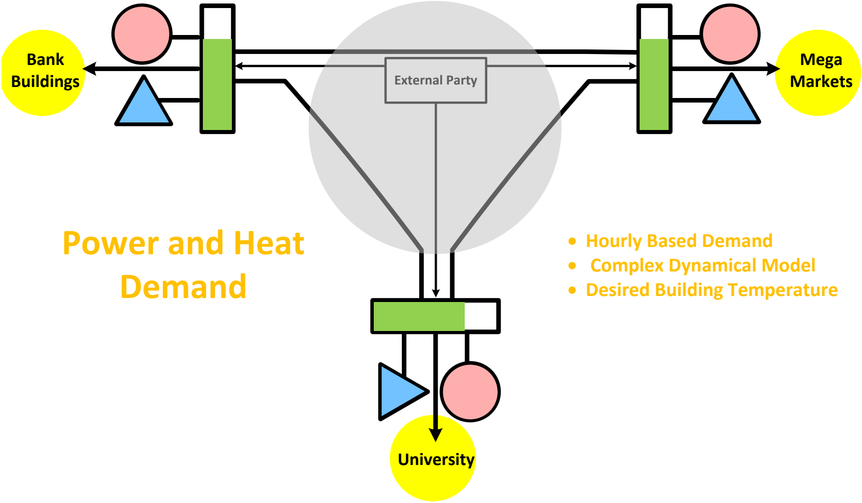

Power and Heat Demand

- Hourly Based Demand
- Complex Dynamical Model
- Desired Building Temperature

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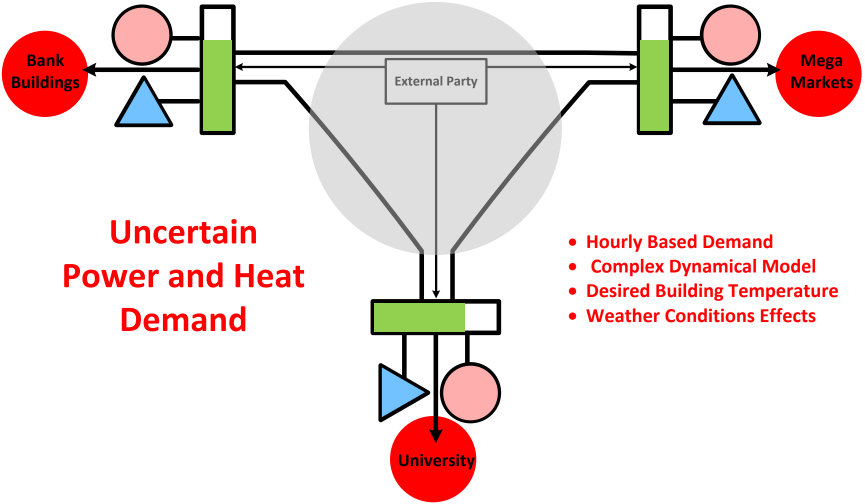


Uncertain Power and Heat Demand

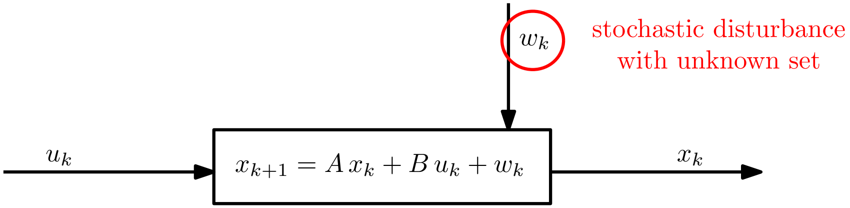
- **Hourly Based Demand**
- **Complex Dynamical Model**
- **Desired Building Temperature**
- **Weather Conditions Effects**

Outline

1. Mathematical Model
2. Stochastic Control
3. Simulation Study
4. Conclusions

Mathematical Model

Define x_k to be the imbalance error between demand and production level. This yields the following dynamical model for imbalance error:



Our objective: design a state feedback control policy that minimizes the energy consumption of buildings, while keeping room temperatures between comfortable limits, despite *uncertain weather conditions*, and subject to the operational constraints

Constrained Control Problem

Finite horizon open loop control problem:

$$\min_{(u_k,y_k)_{k=1}^{M}} \quad J(x_k,u_k) := \mathbb{E}\left[\sum_{k=0}^{M} x_k^\top Q x_k + \sum_{k=0}^{M-1} u_k^\top R u_k\right], Q \succeq 0\,,\; R \succ 0$$

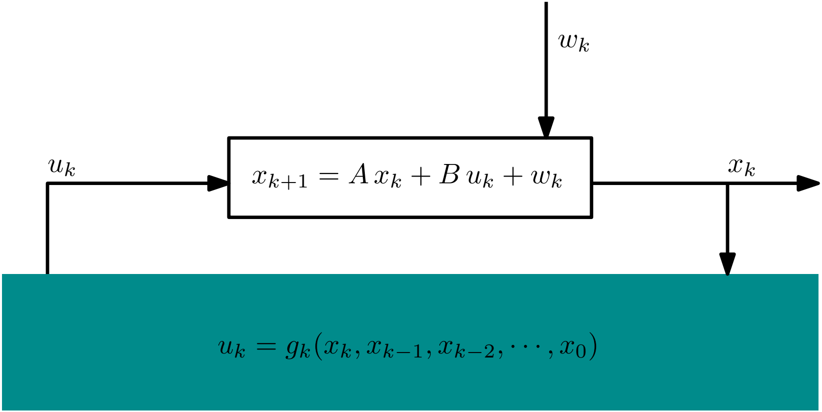
$$\text{subject to:} \quad f_k(x_k,u_k,y_k) \leq 0\,,\; y_k \in \{0,1\}\,,\; k = 0,1,\cdots,M$$

Comments:

- Easy to solve without constraints, e.g., LQG if noise is Gaussian
- Difficult in presence of constraints, binary variables (no closed-form solution)

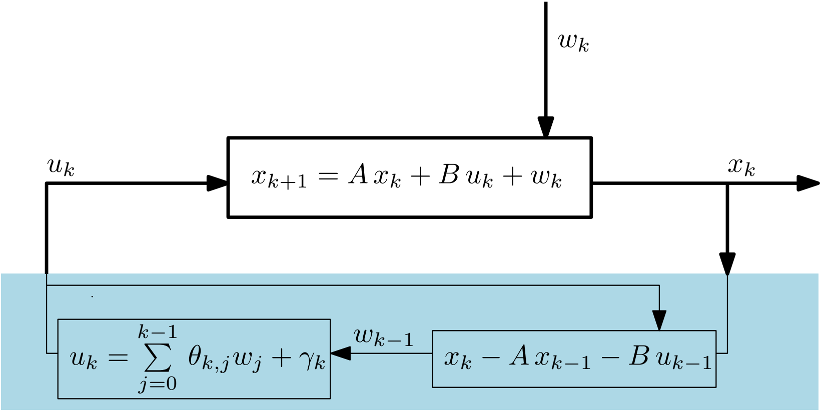
Parametrization of the state feedback control policy can be used to obtain a less conservative formulation for the price of **sub-optimal solution**

Control Policy Parametrization



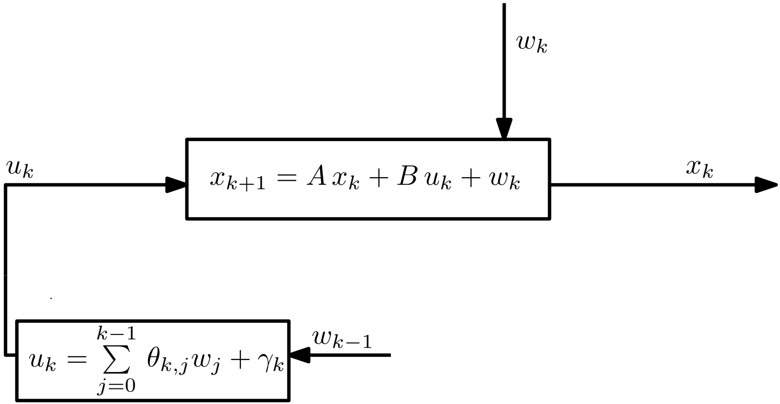
state feedback control policy

Control Policy Parametrization



Affine feedback policy in the reconstructed (and possibly saturated) noise

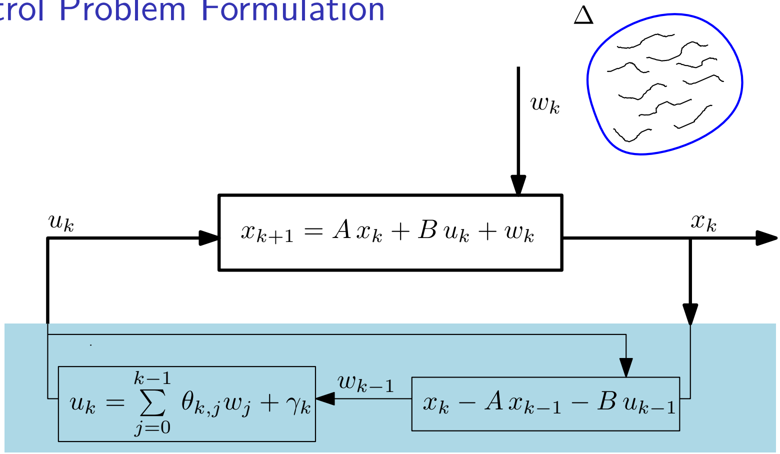
Control Policy Parametrization



Control input and state variables depend linearly on the parameters:

$\theta_{k,j}, \gamma_k$

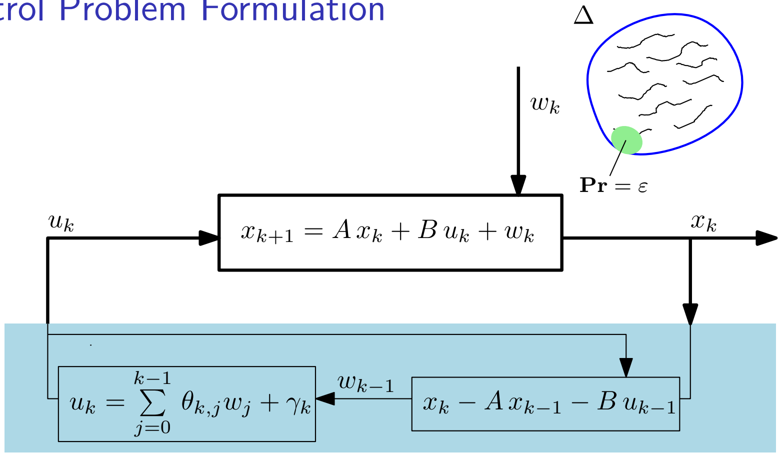
Control Problem Formulation



Robust approach:

- Constraints must be satisfied for every and each disturbance realization
  Disturbance realizations are treated equally likely (hard constraints)
- Intractable problem formulation due to the unknown disturbance set

Control Problem Formulation



Chance constrained approach:

- Constraints must be satisfied for most disturbance realizations except for a set of probability $\leq \varepsilon$ (soft constraints)
- Nonconvex optimization problem and in general hard to solve

Randomized Approximation

$$\min_{\eta \in \mathbf{H}} \quad J(\eta)$$
$$\text{s.t.} \quad \mathbf{Pr}\left[h(\eta, \delta) \leq 0\right] \geq 1 - \varepsilon$$

The following **randomized approximation** that **only relies on data** can provide (conservative) solution to the chance constrained problem:

$$\min_{\eta \in \mathbf{H}} \quad J(\eta)$$
$$\text{s.t.} \quad h(\eta, \delta^{(i)}) \leq 0 \ , \quad i = 1, 2, \cdots, N$$

N is the number of required disturbance realizations that one needs to generate. This approach provides a solution guaranteed to be probabilistically fulfilling the chance constraints

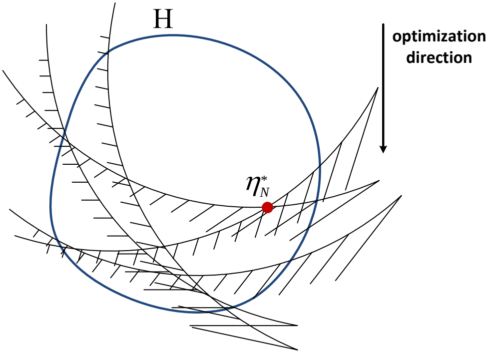
Randomized Approximation & Constraint Removal

Advantages:

- Only relies on the data
- Reformulation is a convex optimization problem

Disadvantages:

- Convex reformulation is usually computationally demanding
- Still conservative performance with respect to the desired level of violation



One way, to improve performance of the solution, is by using **constraint removal** techniques such as greedy algorithm, etc.

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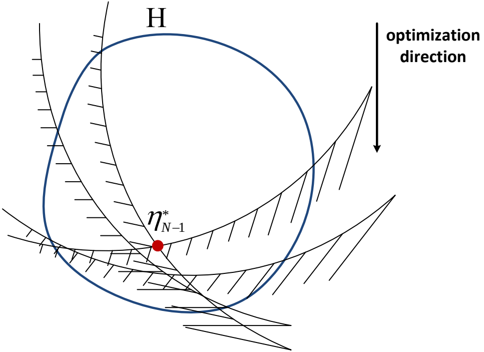

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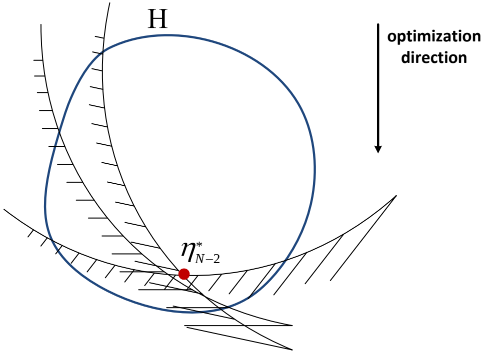

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Nonconvex Randomized Approximation

- Mixed Integer Program

$$\begin{cases} \min\limits_{\eta \in \mathbf{H}, y} & J(\eta) \\ \text{s.t.} & \mathbf{Pr}\left[h(\eta, y, \delta) \leq 0\right] \geq 1 - \varepsilon \\ & y \in \{0,1\}^M \\ & y : \binom{\text{vector of integer variables}}{\text{along the horizon length}} \end{cases}$$

Can be reformulated via **robust (worst-case) programing** as follows:

- Worst Case Program
- $h_j(\eta, \delta) := h(\eta, y_j, \delta)$

$$\begin{cases} \min\limits_{\eta \in \mathbf{H}} & J(\eta) \\ \text{s.t.} & \max\limits_{j \in \{1, \cdots, 2^M\}} \mathbf{Pr}\left[h_j(\eta, \delta) \leq 0\right] \geq 1 - \varepsilon \end{cases}$$

Using randomized approximation, we need to generate at least $2^M N$ disturbance realizations to provide a solution guaranteed to be chance constrained feasible. This leads to **intractable optimization formulation**.

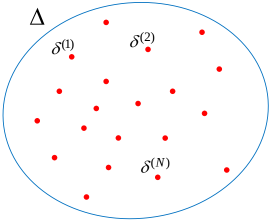
Robust Randomized Optimization

Instead we provide two-step approach:

1. Determining a bounded set that contains $1-\varepsilon$ portion of Δ:

$$\begin{cases} \min\limits_{\gamma} & \sum\limits_{k=0}^{M-1} \overline{\gamma}_k - \underline{\gamma}_k \\ \text{s.t.} & \mathbf{Pr}\left[\delta_{i,k} \in [\underline{\gamma}_k, \overline{\gamma}_k]\,,\, \forall k\right] \geq 1-\varepsilon \end{cases}$$

2. Solving the robust counterpart of problem w.r.t. the bounded set γ^*:



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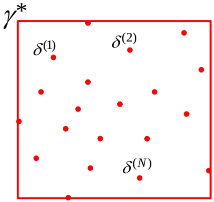
2. Solving the robust counterpart of problem w.r.t. the bounded set γ^*:

$$\begin{cases} \min\limits_{\eta \in \mathbf{H}, y} & J(\eta) \\ \text{s.t.} & h(\eta, y, \gamma^{o}) + h(\eta, y, \gamma^{worst}) \leq 0 \\ & y \in \{0,1\}^M \end{cases}$$



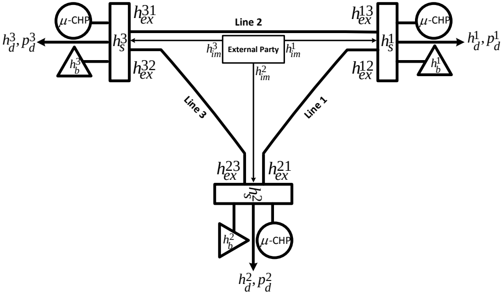
Simulation Results

A comparison against a **Benchmark approach** based on two-step solution

Simulation study settings:

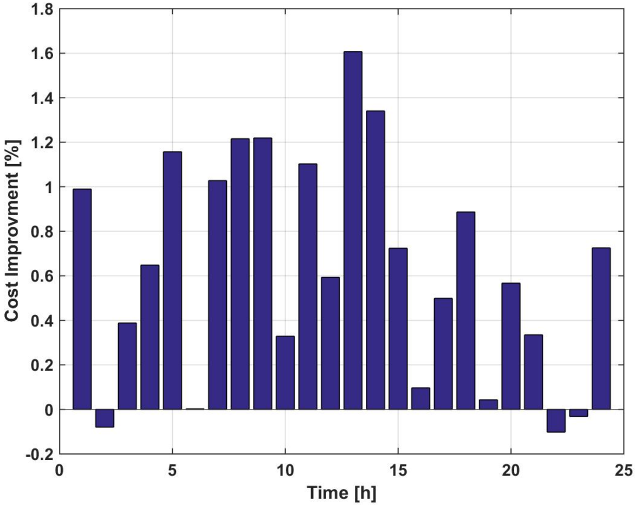
- Day-ahead control problem
- Economical cost function
- Operational constraints
- Uncertain thermal energy
- Unit commitment & scheduling problem

three-agent (households, greenhouses) smart thermal grid example

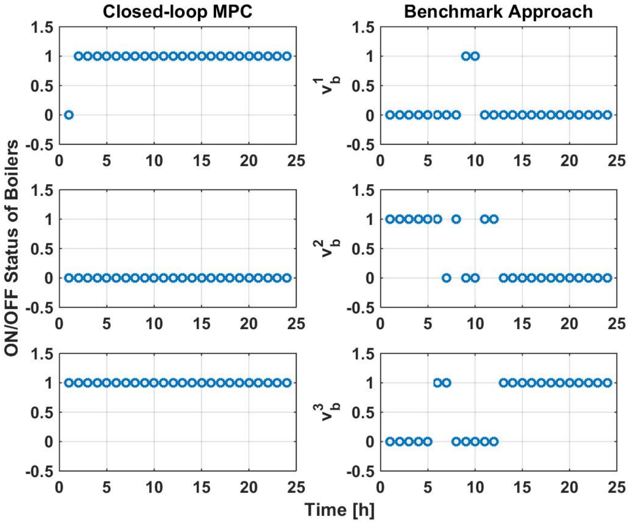


Mixed Integer Chance Constrained Linear Optimization Problem

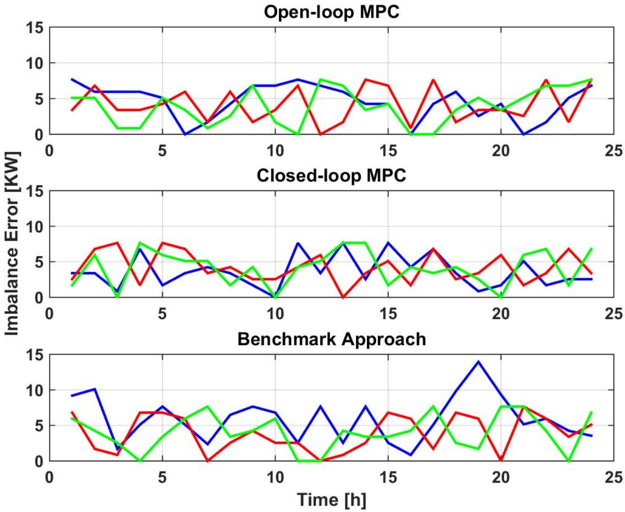
Simulation Results: Relative Cost Improvement



Simulation Results: ON/OFF Status of Boilers



Simulation Results: Imbalance Error Trajectories



Conclusions

Remarks:

- Centralized control problem formulation for a **SmartThermal Grid**
- **Affine Uncertainty Feedback Policy** with chance constraint formulation
- **Convex Reformulation** of the proposed stochastic constrained control

Next Steps:

- Developing a **Real Demand Profile Generator** by using a detailed building dynamical model
- Incorporating **Aquifer Thermal Energy Storage System (ATES)** in the developed framework

Future Directions

Decision making under uncertainty:

- **Traditional Approach**: perfect information, i.e. accurate system parameters and specific measures for the random variable
- **Modern Approach**: Big Data, i.e. historical data, data driven approaches
  - Randomization (Scenario) Based Optimization

Current works concentrate toward:

- Computational algorithm for **on-line data driven optimization**
- Distributed algorithm for **stochastic complex network** problem
- Incremental algorithm for **aggregated mathematical optimization**

Thank you! Questions?

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