FD Threshold Design for Uncertain Nonlinear Systems

With Probabilistic Performance Guarantees

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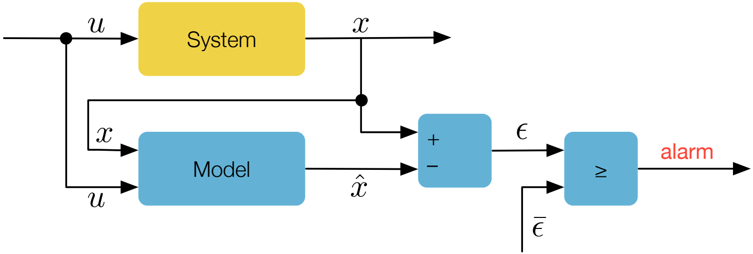
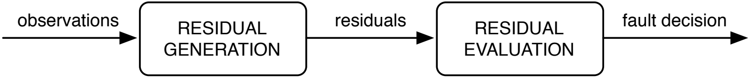
Model Based Fault Detection Concepts

- The basis of fault detection concept is:



- A basic point is to have observer that generate residual:



Outline

1. Uncertain Nonlinear Systems Fault Detection

2. Fault Threshold Design Frameworks

3. Simulation Study

4. Conclusions

Observers for Model Based Fault Detection

- Uncertain nonlinear system under fault: additive terms of system **nominal behavior, uncertainties, and fault functions**

$$x_{k+1} = \underbrace{g(x_k, u_k)}_{\text{nominal}} + \underbrace{\eta(x_k, u_k, w_k)}_{\text{uncertainty}} + \underbrace{\phi(x_k, u_k, f_k)}_{\text{fault function}}$$

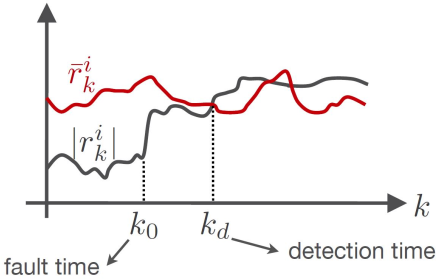
$$y_k = \underbrace{Hx_k}_{\text{nominal}} + \underbrace{v_k}_{\text{uncertainty}}$$

- w_k , v_k are independent and identically distributed random variables
- Estimator can be built such as:

$$\hat{x}_{k+1} = g(y_k, u_k) + \Lambda\,(\hat{x}_k - y_k)$$

Observers for Model Based Fault Detection

Define the residual signal to be as: $r_k = y_k - \hat{x}_k$



- $\bar{r}_k$: corresponding threshold
- faults can be detected by comparing $|r_k|$ with $\bar{r}_k$
- the threshold is needed to be robust w.r.t. uncertainties

Detectability Concept

A fault will be detected if the absolute value of at least one component of the residual crosses the threshold.

Observers for Model Based Fault Detection

- The residual dynamics obey the following equation:

$$r_{k+1} = y_{k+1} - \hat{x}_{k+1} = \boxed{\Lambda} r_k + \boxed{\delta_k} + \boxed{\phi(x_k, u_k, f_k)}$$

$\Lambda := diag(\lambda^i)$ — total uncertainty — fault function

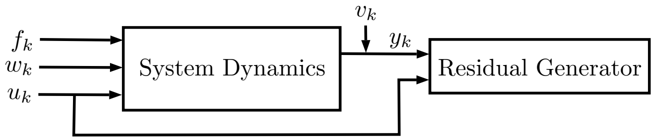
- δ_k represents all the sources of uncertainty (model and output):

$$\delta_{k+1} = \boxed{g(x_k, u_k) - g(y_k, u_k)} + \boxed{\eta(x_k, u_k, w_k)} + \boxed{v_{k+1}}$$

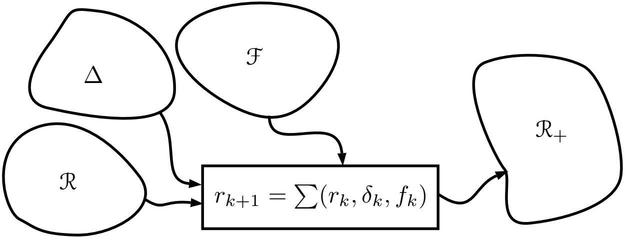
effect of output uncertainty on evaluating nominal dynamics — model uncertainty — output uncertainty

Residual Generator System Dynamics

- General configuration of FD with dynamical residual:



- Residual image set with uncertainty δ_k and faulty signal f_k:



Outline

Robust Threshold Design of Fault Detection

- Robust threshold problem[1] assumes a known uncertainties bound:

$$|\delta_k^i| \leq \bar{\delta}_k^i$$

- Robust threshold problem leads to a deterministic solution to avoid false alarms in the absence of faults:

$$\bar{r}_{k+1}^i \;=\; \lambda^i \; \bar{r}_k^i + \bar{\delta}_k^i$$

- This is a very strong assumption on the uncertainties:

$$\mathbb{P}\left[|\delta_k^i| > \bar{\delta}_k^i\right] = 0 \;,\; \forall k$$

[1] B. Svetozarevic, et al. ACC 2013

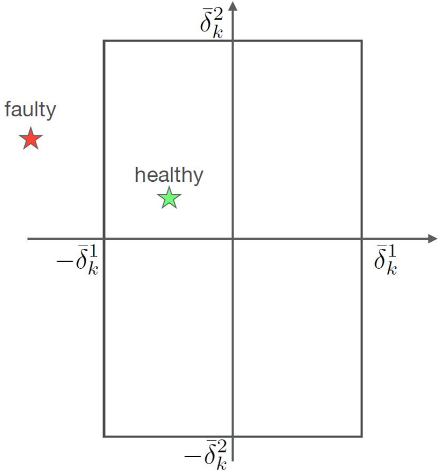
Robust Threshold Design of Fault Detection

- Robust approach guarantees: zero false alarms probability, in practice **excessively conservative**
- In healthy conditions the bounded uncertainty set:

$$r^i_{k+1} - \lambda^i \; r^i_k = \delta^i_k \in \Delta$$

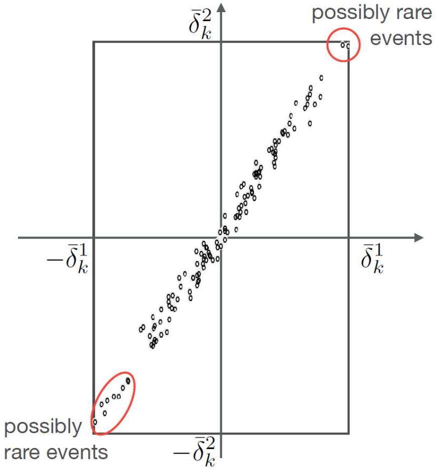
- A hyper-rectangular healthy set:

$$\Delta = \Pi_i[-\bar{\delta}^i_k, \bar{\delta}^i_k] \;, \; \forall k$$



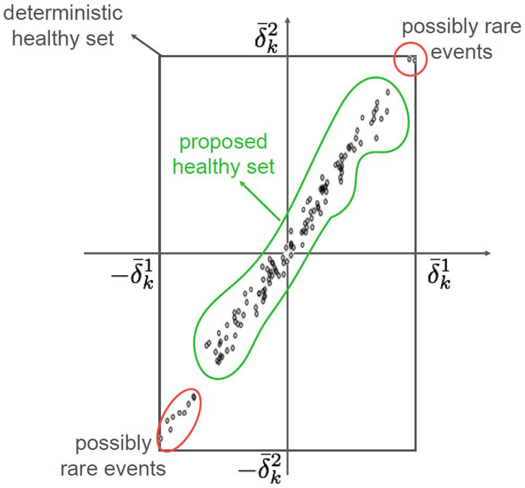
Robust Threshold Design of Fault Detection

- A cloud of uncertainty samples will in general **not be rectangular**!
- A worst-case uncertainty even **very rare values** are considered!
- This leads to the worst fault **detectability issues**!



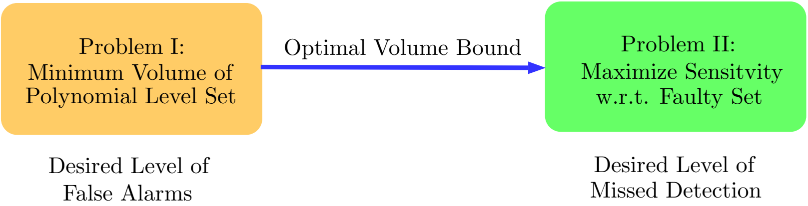
Probabilistic Threshold Design of Fault Detection[2]

- Relax zero false alarm probability improves detectability
- No longer a robust guarantee on the false alarms' absence
- Design healthy sets with arbitrary shape, rather than very conservative hyper-rectangles



[2]V. Rostampour, et al. 2016

Proposed Novel Framework[3]



Probabilistic Performance Guarantees

Given decision dimension (d), fix the false alarm (ε) and missed detection (α) ratios with the desired confidence level (β). If $N_s(d, \varepsilon, \beta)$ samples is considered, then the optimal solution will be probabilistically feasible with high confidence level $(1 - \beta)$.

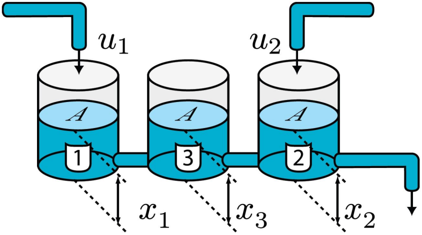
[3] V. Rostampour, et al. 2016

Outline

Benchmark Case Study: Three-Tank System[4]

- Uncertainties source is model mismatch: tanks and pipes' cross section and outflow coefficient
- Fault classes: the first or second pump shut down, leakage in the first tank
- A fault corresponding to a reduction in the inflow provided by the first pump is introduced

classical nonlinear system used as a **benchmark in FDI**



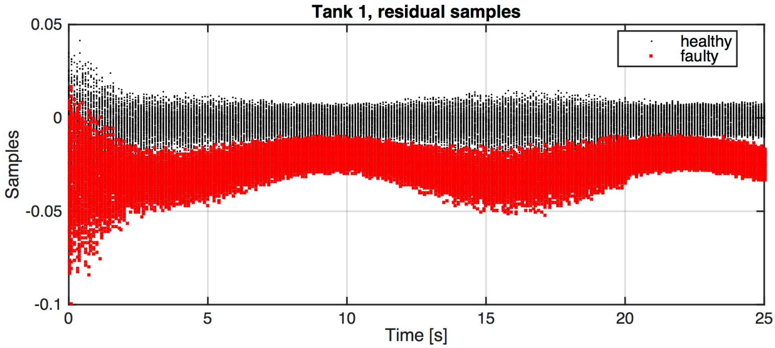
nominal dynamics can be easily written as a discrete-time nonlinear

[4] R. Ferrari, et al. ACC 2008

Healthy and Faulty Residuals

Analysis of healthy and faulty residuals:

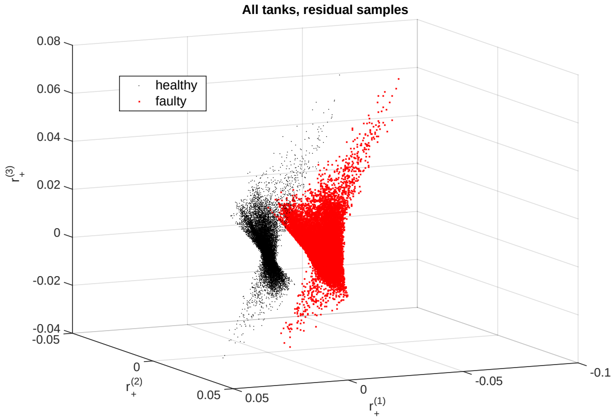
- the healthy and faulty residuals are very close



Healthy and Faulty Residuals

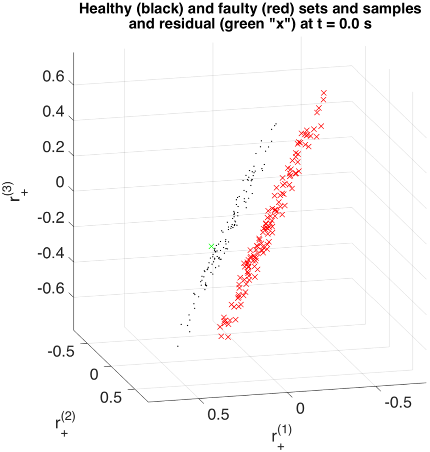
Analysis of healthy and faulty residuals:

- the healthy and faulty residuals for all residual samples



Solution of Proposed Framework

1. Healthy and faulty residual samples
2. Healthy polynomial set found in Problem I
3. Faulty polynomial set that will be used in Problem II
4. New healthy polynomial set found as solution of Problem II



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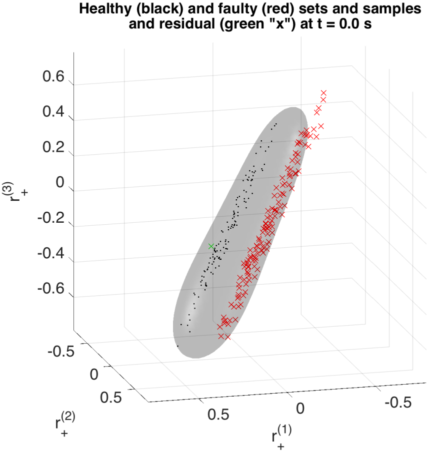

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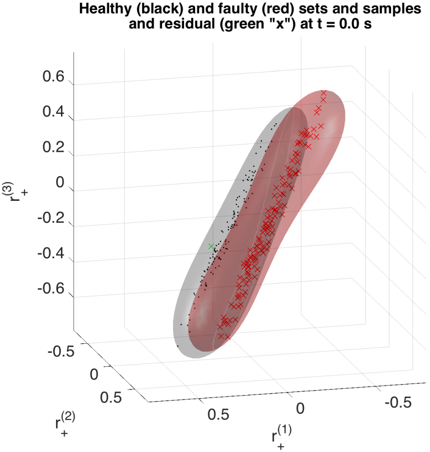

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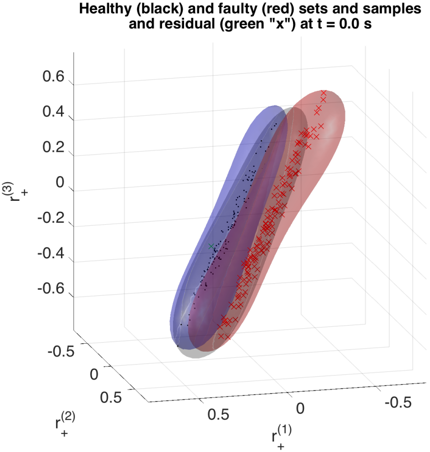


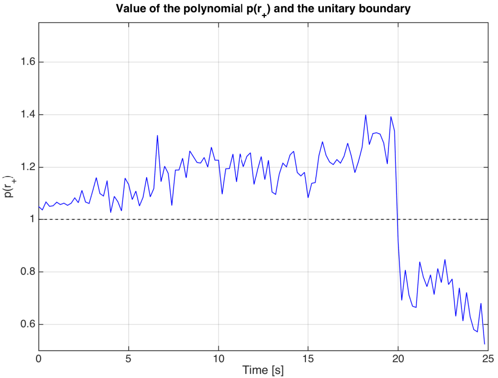
Fault Detection with Isolation Perspective

- Green $\times$ is the residual
- Black points are healthy residual samples
- Red points are faulty residual samples

1. Just before the fault
2. Just after the fault

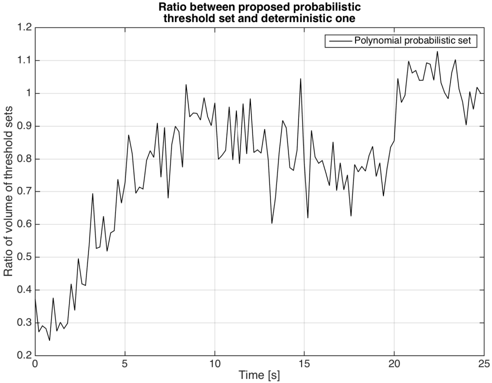
Dynamical Behavior of the Residual Signal

- Values of the polynomial threshold set computed at the residual
- Values greater than one means: **healthy**
- Values smaller than one means: **faulty**
- The fault is introduced at $T_f = 20s$ with sampling time $T_s = 0.1s$



Polynomial vs. Hyper-Rectangle Threshold Set

- Proposed probabilistic polynomial threshold set with guarantees vs. classic deterministic hyper rectangle
- Volume of proposed threshold set is smaller in healthy conditions
- Proposed framework leads to better detectability



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Concluding Remarks

- A novel approach to the design of fault detection thresholds for uncertain nonlinear systems
- A-priori probabilistic guarantees on the performance level in terms of false alarm and missed detection ratios as the theoretical results
- Validation of the advantages of the proposed framework using simulation results on the well known three-tank benchmark

Next steps:

- Extension to Fault Detectability, Isolability and Identifiability for the general faulty classes

Thank You!
Questions?

References:

- **V. Rostampour**, et al., Fault Detection Threshold Design for Uncertain Nonlinear Systems with Probabilistic Performance Guarantees, Submitted 2016
- B. Svetozarevic, et al., A Robust Fault Detection and Isolation Filter for a Horizontal Axis Variable Speed Wind Turbine, ACC 2013
- R. Ferrari, et al., A Robust Fault Detection and Isolation Scheme for a Class of Uncertain Input-output Discrete-time Nonlinear Systems,ACC 2008

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