Robust Randomized Model Predictive Control for Energy Balance in Smart Thermal Grids

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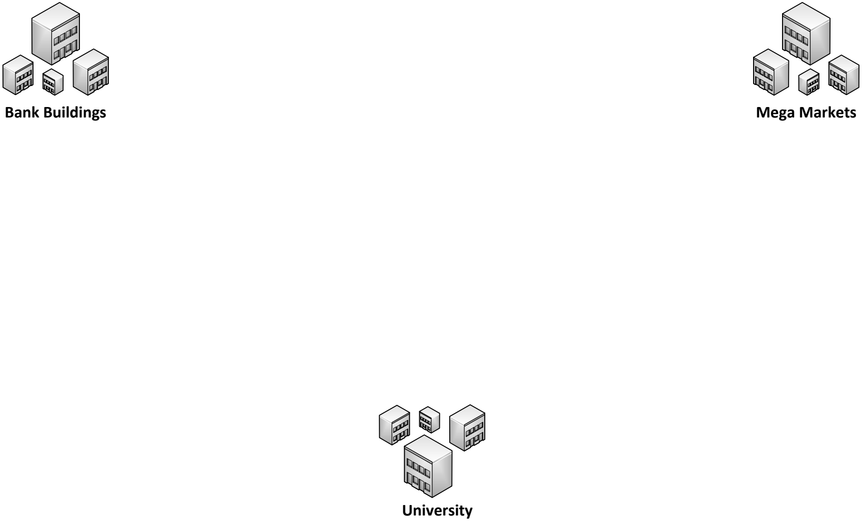
Smart Thermal Grids: Conceptual Representation



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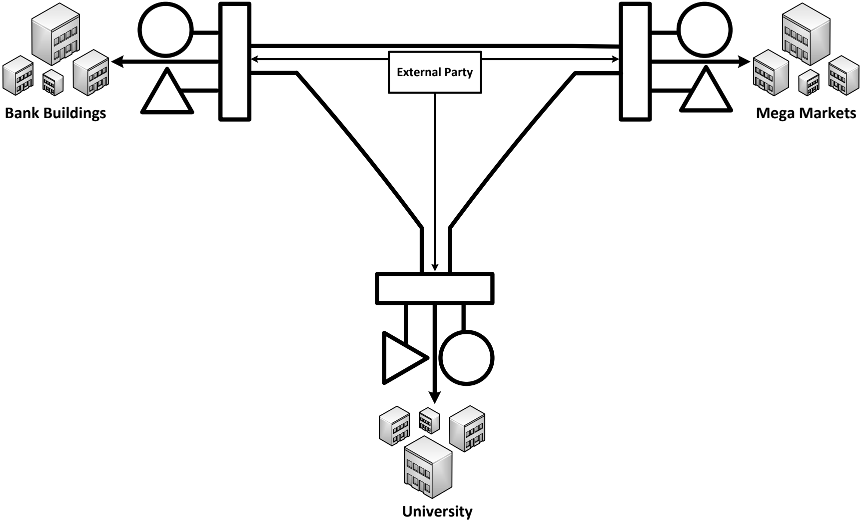

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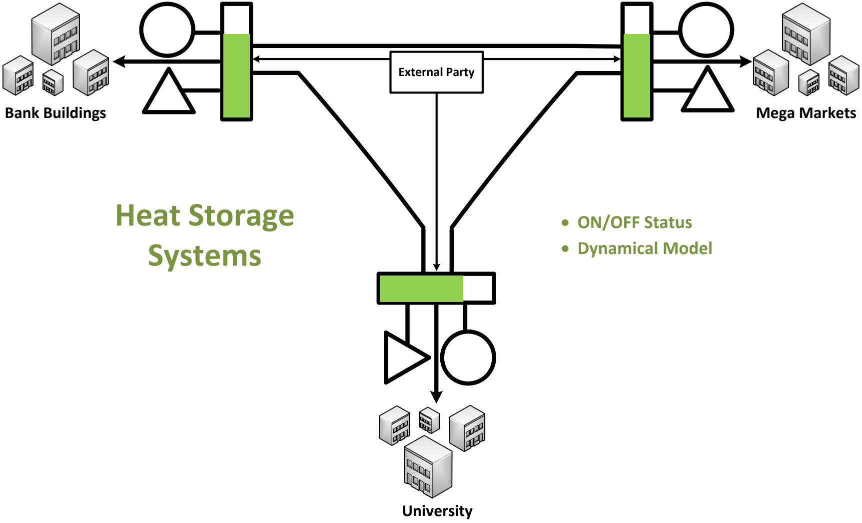
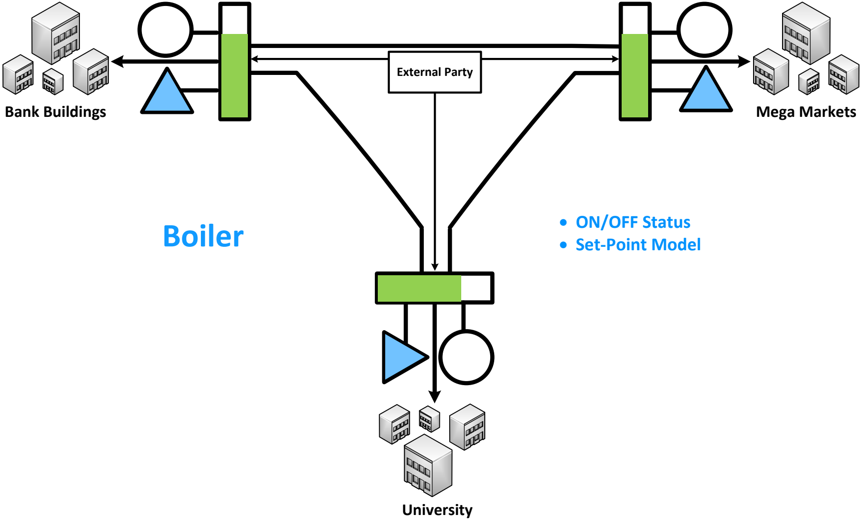


Heat Storage Systems

- **ON/OFF Status**
- **Dynamical Model**

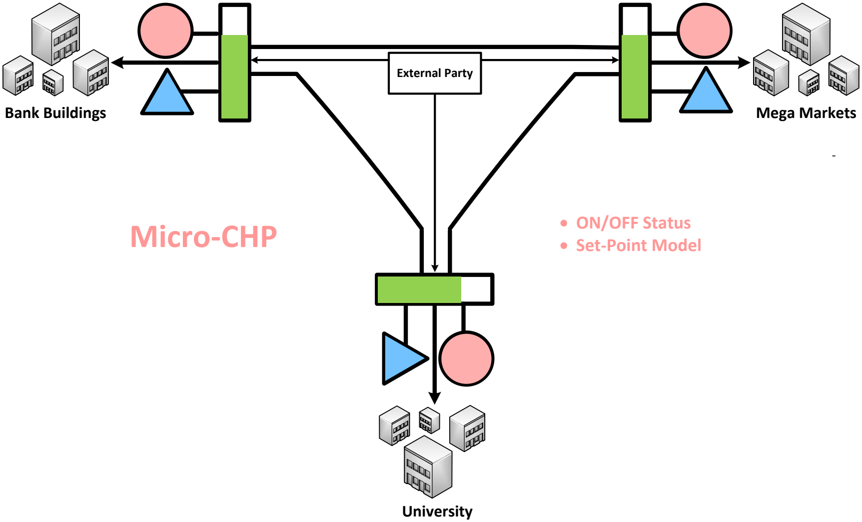
Smart Thermal Grids: Conceptual Representation



Boiler

- **ON/OFF Status**
- **Set-Point Model**

Smart Thermal Grids: Conceptual Representation



Micro-CHP

- ON/OFF Status
- Set-Point Model

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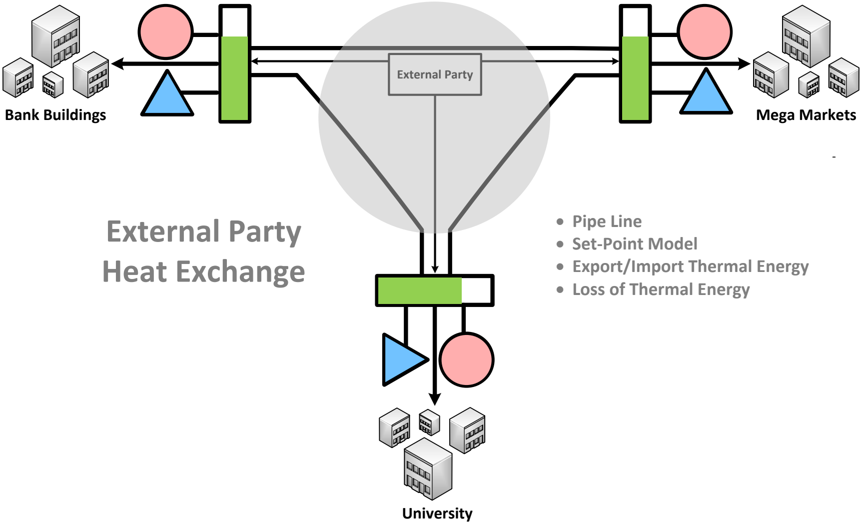
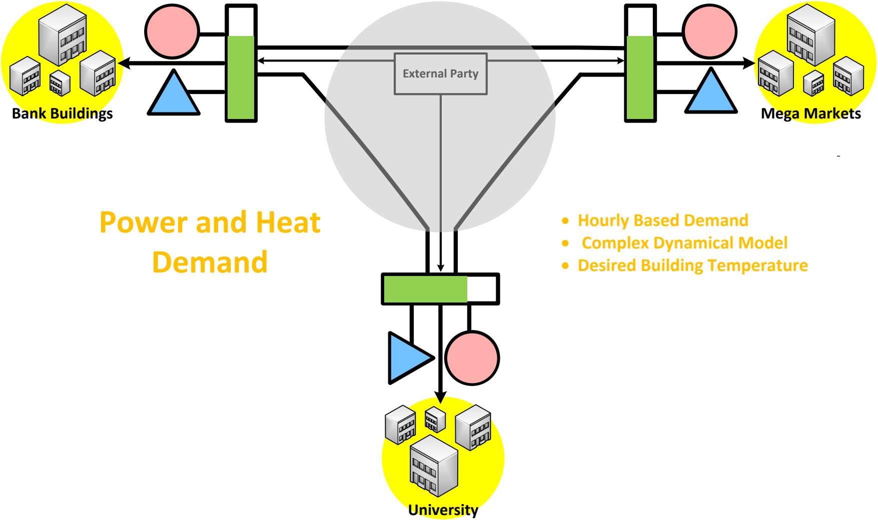


External Party
Heat Exchange

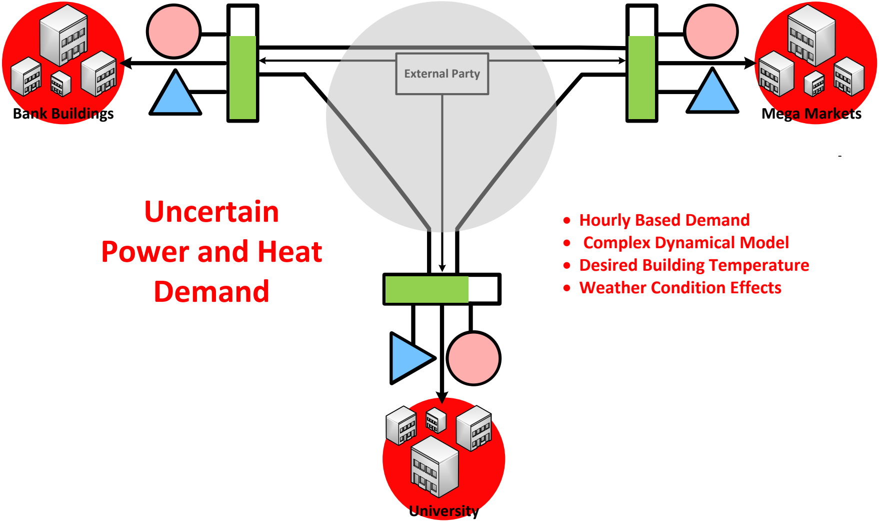
- Pipe Line
- Set-Point Model
- Export/Import Thermal Energy
- Loss of Thermal Energy

Smart Thermal Grids: Conceptual Representation



- Hourly Based Demand
- Complex Dynamical Model
- Desired Building Temperature

Smart Thermal Grids: Conceptual Representation



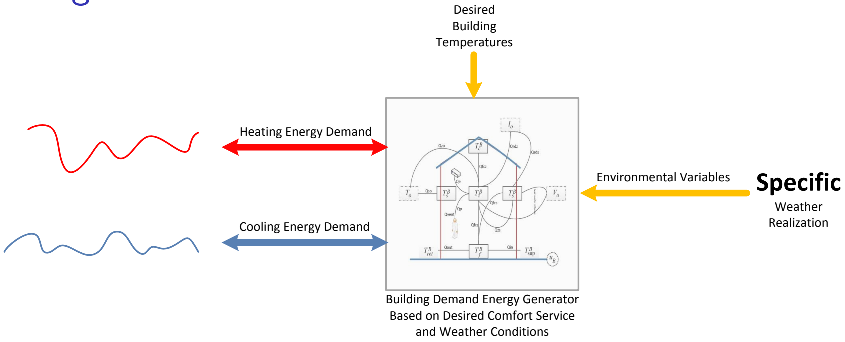
Uncertain Power and Heat Demand

- **Hourly Based Demand**
- **Complex Dynamical Model**
- **Desired Building Temperature**
- **Weather Condition Effects**

Outline

1. Mathematical Model
2. Stochastic Control
3. Simulation Study
4. Conclusions

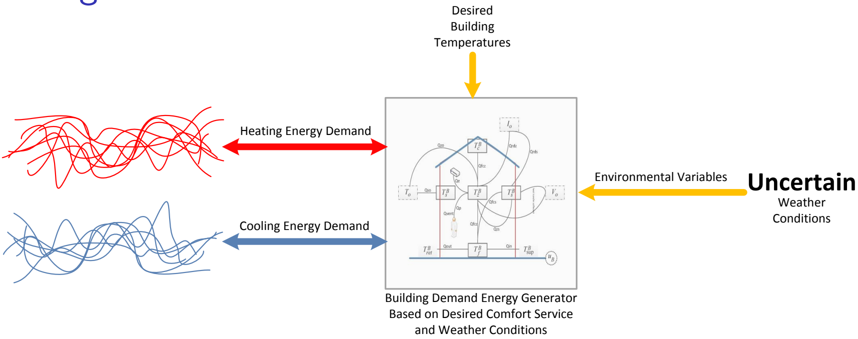
Building Demand Generator



Demand Profile Generator:

- Complete and detailed building dynamical model
- Desired building temperature (local controller unit)
- In a specific weather realization, deterministic demand profiles are generated

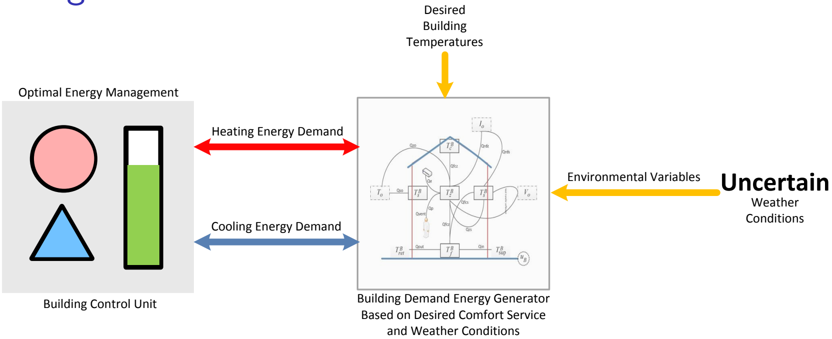
Building Demand Generator



Demand Profile Generator:

- Complete and detailed building dynamical model
- Desired building temperature (local controller unit)
- In uncertain weather conditions, uncertain demand profiles are generated

Building Comfort Service

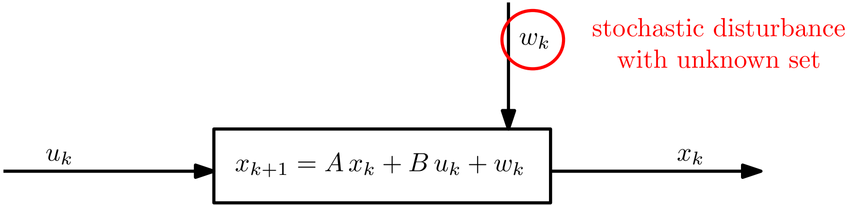


Building Control Unit:

- Main components: Boiler, HP, HE, micro-CHP, Buffer Storage
- **ON/OFF status** together with **production schedule** as decisions
- Thermal **energy balance** for dynamical systems

Mathematical Model

Define x_k to be **imbalance error** between production and building energy demand



Our objective: design a state feedback control policy that aims at:

- Keeping room temperatures between comfortable limits
- Minimizing building operational cost and energy consumption
- Taking into account **uncertain weather conditions**, operational constraints

Constrained Control Problem

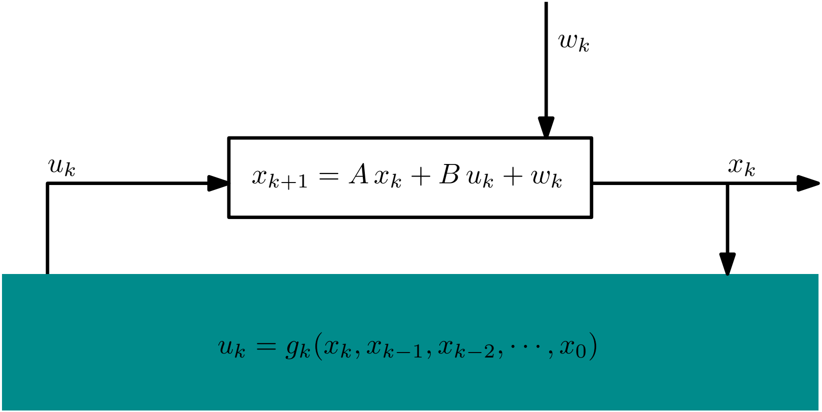
Finite horizon open loop control problem for each agent:

$$
\begin{aligned}
\underset{(u_k, y_k)_{k=1}^{M}}{\text{minimize}} \quad & J(x_k, u_k) := \mathbb{E}\left[\sum_{k=0}^{M} x_k^\top Q x_k + \sum_{k=0}^{M-1} u_k^\top R u_k\right], \; Q \succeq 0\,,\; R \succ 0 \\
\text{subject to} \quad & f_k(x_k, u_k, y_k) \le 0\,,\; y_k \in \{0, 1\} \\
& x_k \in \mathcal{X}\,,\; k = 0, 1, \cdots, M \qquad \Rightarrow \textbf{hard constraints}
\end{aligned}
$$

Challenges:

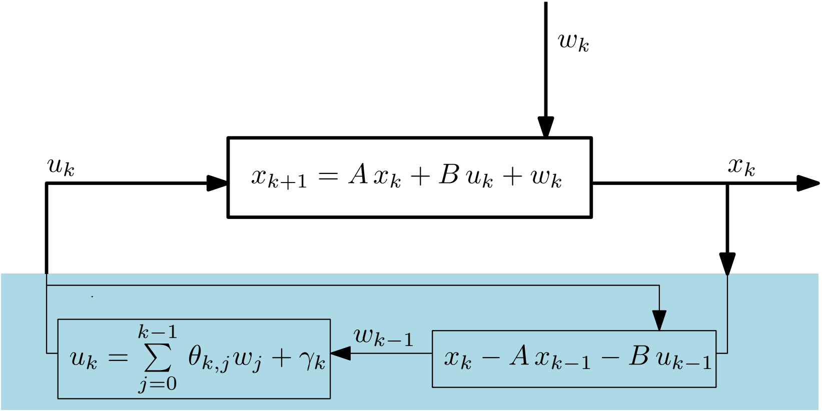
1. Control policy parametrization to obtain a less conservative formulation
2. Probabilistic interpretation of robustness feature of hard constraints
3. Handling mixed-integer optimization together with stochastic programming

Step 1: Control Policy Parametrization



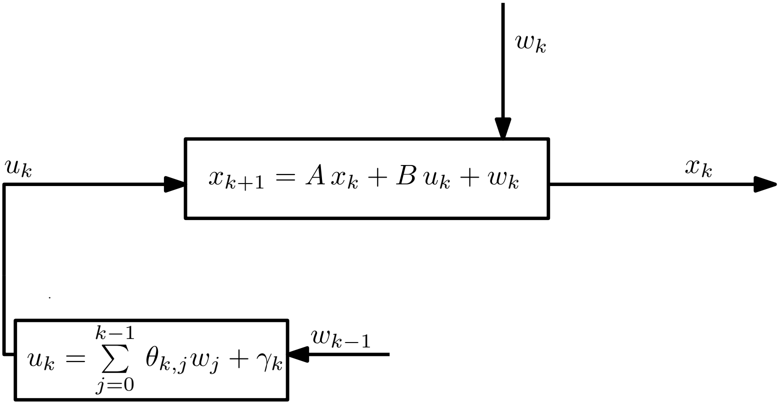
state feedback control policy

Step 1: Control Policy Parametrization



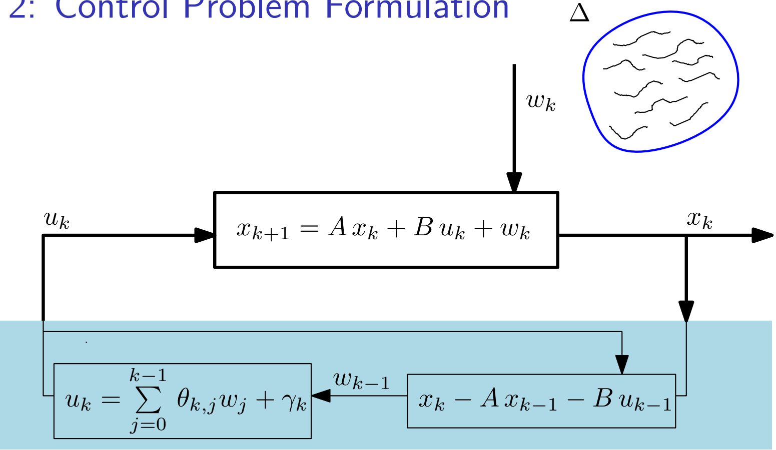
Affine feedback policy in the reconstructed (and possibly saturated) noise

Step 1: Control Policy Parametrization



Control input and state variables depend linearly on the parameters: $\theta_{k,j}, \gamma_k$

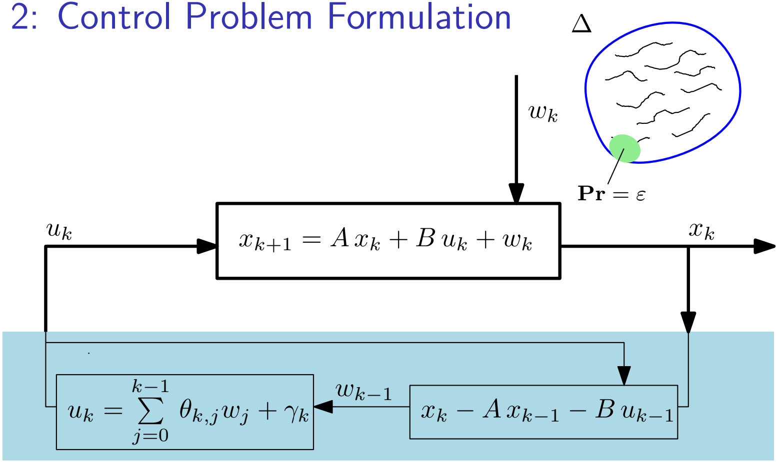
Step 2: Control Problem Formulation



Robust approach:

- Constraints must be satisfied for every disturbance realization in Δ
  Disturbance realizations are treated equally likely (hard constraints)
- Intractable problem formulation due to the unknown disturbance set Δ

Step 2: Control Problem Formulation



Chance constrained approach:

- Constraints must be satisfied for most disturbance realizations except for a set of probability $\leq \varepsilon$ (soft constraints)
- Nonconvex optimization problem and in general hard to solve

Step 2: Randomized Approximation

optimization variables
$\eta = \{u_k | k = 1, \cdots, M\}$

uncertainty parameters
$\delta = \{w_k | k = 1, \cdots, M\}$

$$\begin{aligned} \min_{\eta \in \mathbf{H}} \quad & J(\eta) \\ \text{s.t.} \quad & \mathbf{Pr}\left[h(\eta, \delta) \le 0\right] \ge 1 - \varepsilon \end{aligned}$$

The following **randomized approximation** that **only relies on data** can provide a (conservative) solution to the chance constrained problem:

$$\begin{aligned} \min_{\eta \in \mathbf{H}} \quad & J(\eta) \\ \text{s.t.} \quad & h(\eta, \delta^{(i)}) \le 0 \;, \quad i = 1, 2, \cdots, N \end{aligned}$$

N is the number of required disturbance realizations that one needs to generate. This approach provides a solution guaranteed to be probabilistically fulfilling the chance constraints[1].

[1] [Calafiore, Campi, TAC, 2005]

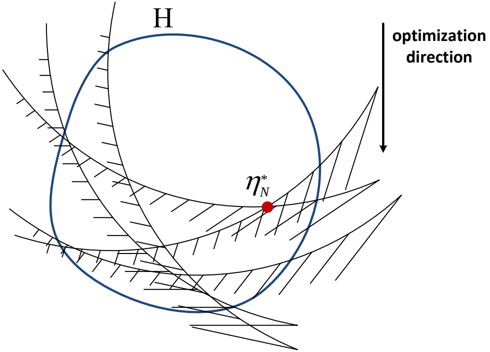
Step 2: Randomized Approximation & Constraint Removal

Advantages:

- Only relies on the data
- Reformulation is a convex optimization problem

Disadvantages:

- Convex reformulation is usually computationally demanding
- Still conservative performance with respect to the desired level of violation



One way, to improve performance of the solution, is by using **constraint removal** techniques such as greedy algorithm[2],etc.

[2] [Campi, Garatti, OTA, 2010]

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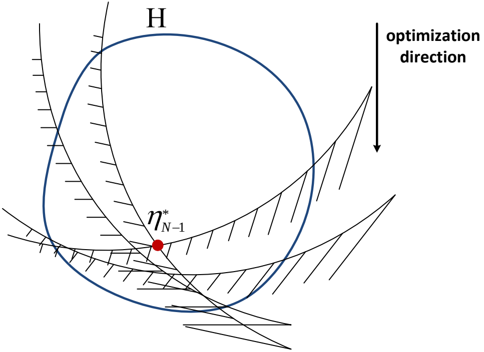

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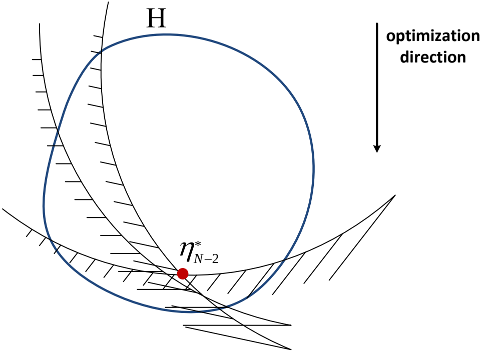

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Step 3: Nonconvex Randomized Approximation

- Mixed Integer Program

$$\begin{cases} \min\limits_{\eta \in \mathbf{H}, y} & J(\eta) \\ \text{s.t.} & \mathbf{Pr}\left[h(\eta, y, \delta) \leq 0\right] \geq 1 - \varepsilon \\ & y \in \{0,1\}^M \\ & y : \left(\begin{smallmatrix}\text{vector of integer variables} \\ \text{along the horizon length}\end{smallmatrix}\right) \end{cases}$$

Can be reformulated via **robust (worst-case) programming** as follows:

- Worst Case Program
- $h_j(\eta, \delta) := h(\eta, y_j, \delta)$

$$\begin{cases} \min\limits_{\eta \in \mathbf{H}} & J(\eta) \\ \text{s.t.} & \max\limits_{j \in \{1, \cdots, 2^M\}} \mathbf{Pr}\left[h_j(\eta, \delta) \leq 0\right] \geq 1 - \varepsilon \end{cases}$$

Using randomized approximation, we need to generate at least $2^M\, N$ disturbance realizations to provide a solution guaranteed to be chance constrained feasible. This leads to **intractable optimization formulation**[3].

[3] [Esfahani, Sutter, et al., TAC, 2015]

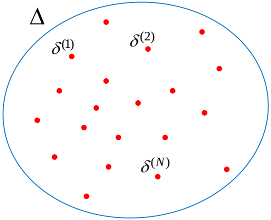
Step 3: Robust Randomized Optimization

Instead we provide two-step approach[4] in a receding horizon setting:

1. Determining a bounded set that contains $1-\varepsilon$ portion of Δ:

$$\begin{cases} \min\limits_{\gamma} & \sum\limits_{k=0}^{M-1} \overline{\gamma}_k - \underline{\gamma}_k \\ \text{s.t.} & \mathbf{Pr}\left[\delta_{i,k} \in [\underline{\gamma}_k, \overline{\gamma}_k], \forall k\right] \geq 1-\varepsilon \end{cases}$$

2. Solving the robust counterpart of problem w.r.t. the bounded set γ^*:



[4] [Margellos, Rostampour, et al., ECC, 2013]

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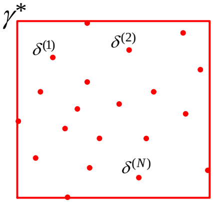
$$\begin{cases} \min\limits_{\gamma} & \sum\limits_{k=0}^{M-1} \overline{\gamma}_k - \underline{\gamma}_k \\ \text{s.t.} & \delta_{i,k}^{j} \in [\underline{\gamma}_k, \overline{\gamma}_k]\,, \begin{cases} \forall k \\ \forall j \end{cases} \end{cases}$$

2. Solving the robust counterpart of problem w.r.t. the bounded set γ^*:

$$\begin{cases} \min\limits_{\eta \in \mathbf{H}, y} & J(\eta) \\ \text{s.t.} & h(\eta, y, \gamma^o) + h(\eta, y, \gamma^{worst}) \leq 0 \\ & y \in \{0,1\}^M \end{cases}$$



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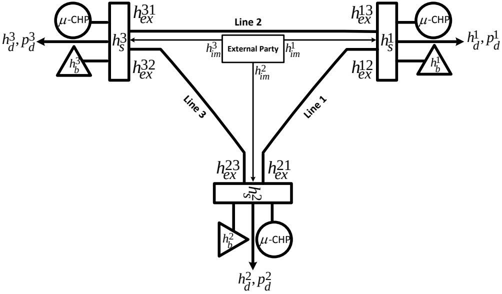
Three-Agent Case Study

households, greenhouses
smart thermal grid example

- Day-ahead control problem
- Economical cost function
- Operational constraints
- Uncertain energy demand
- Unit commitment problem
- Production scheduling problem



Mixed-Integer Chance-Constrained Linear Optimization Problem

Proposed Approach

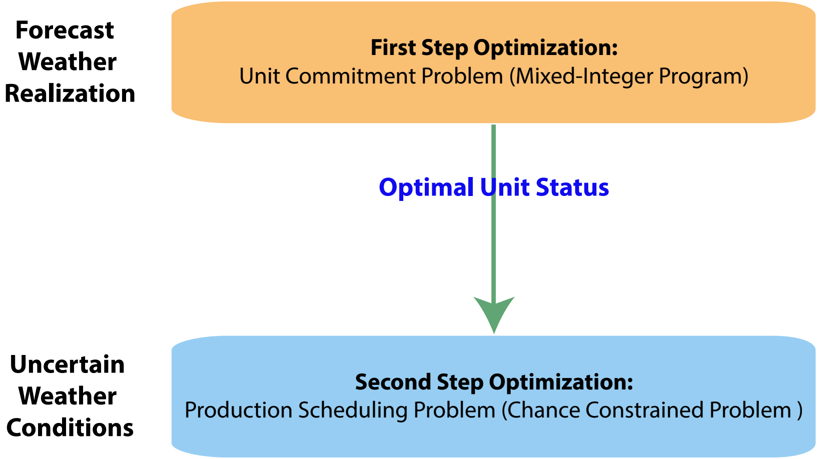
Resulting optimization problem for each agent:

$$\begin{aligned} \underset{(u_k, y_k)_{k=1}^{M}}{\text{minimize}} \quad & J(x_k, u_k) \\ \text{subject to} \quad & f_k(x_k, u_k, y_k) \leq 0\,,\ y_k \in \{0, 1\}\,,\ k = 0, 1, \cdots, M \\ & \mathbf{Pr}\{x_k \in \mathcal{X}\} \geq 1 - \varepsilon \quad \Rightarrow \textbf{chance constraints} \end{aligned}$$

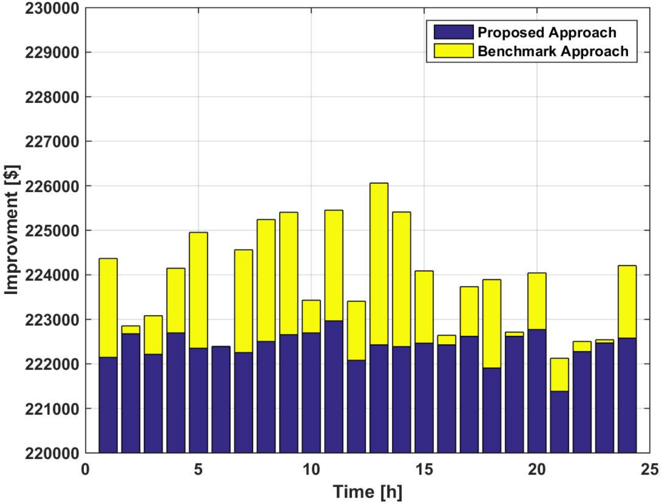
Theoretical features/contributions of proposed framework:

- Unified framework to solve mixed-integer stochastic optimization problems
- Robustness features of constraints in a relaxed probabilistic setting based on randomization of the constraints
- A-priori probabilistic guarantee on the feasibility of the optimal solution of the problem

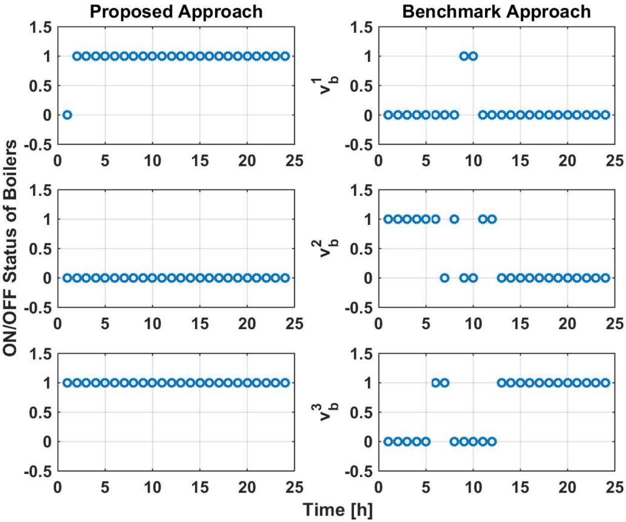
Comparison: Benchmark Approach



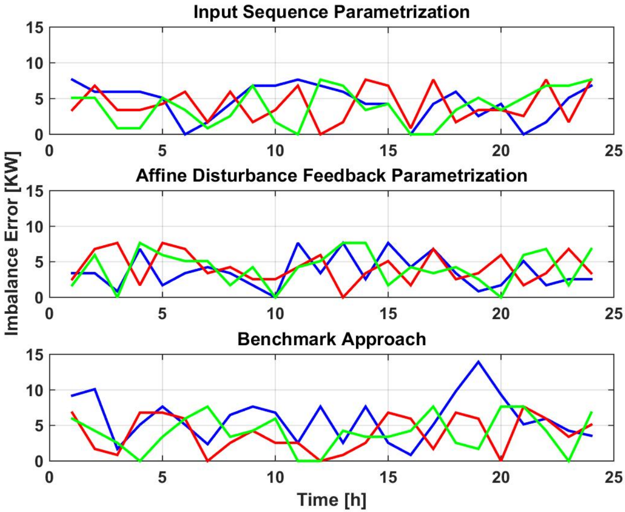
Simulation Results: Relative Cost Improvement



Simulation Results: ON/OFF Status of Boilers



Simulation Results: Imbalance Error Trajectories



Conclusions

Contributions:

- Centralized control problem formulation for a **SmartThermal Grid**
- **Affine Uncertainty Feedback Policy** with chance constraint formulation
- **Convex Reformulation** of the proposed stochastic constrained control
- **A-priori Probabilistic Feasibility Certificate** for a mixed-integer chance-constrained program in a receding horizon scheme

Next Steps:

- Developing a more realistic **Building Demand Profile Generator** by using a more detailed dynamical model
- Developing a new scheme to **Distribute Computations** in the developed framework among the agents

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