A Control-Oriented Model for Combined Building Climate Comfort and ATES System

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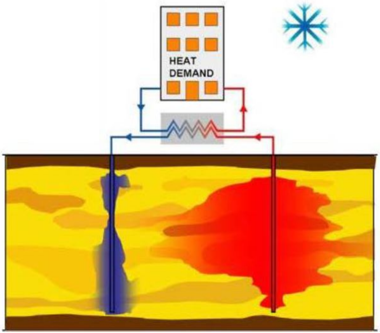
Aquifer Thermal Energy Storage (ATES)

- A large-scale natural subsurface storage for thermal energy
- An innovative method for thermal energy balance in smart grids

Cold season:

- The building requests thermal energy for the heating purpose
- Water is injected into cold well and is taken from warm well
- The stored water contains cold thermal energy for next season

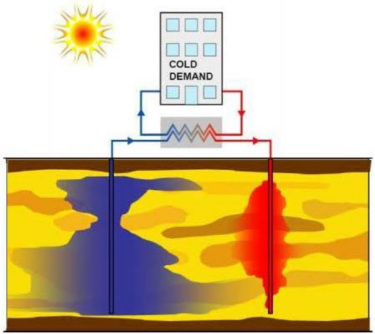


Aquifer Thermal Energy Storage (ATES)

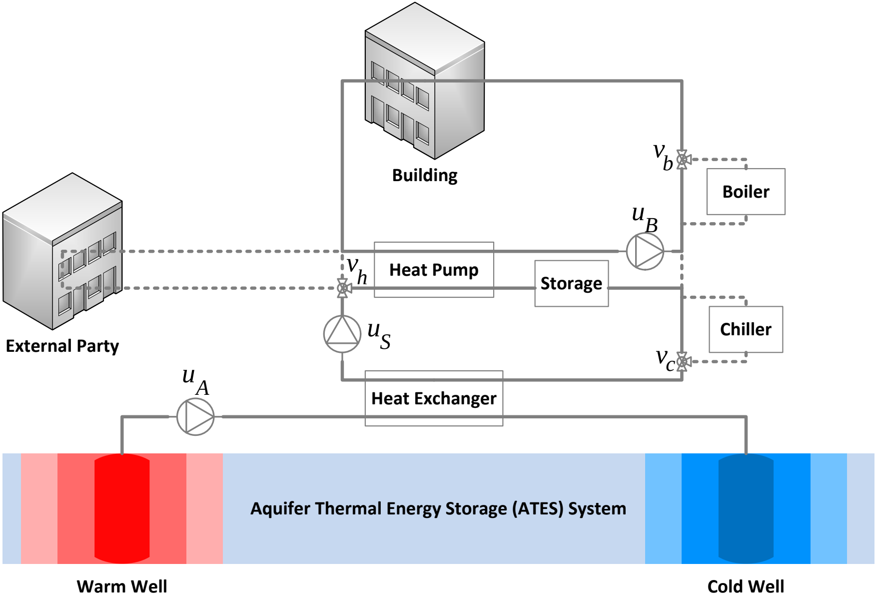
- A large-scale natural subsurface storage for thermal energy
- An innovative method for thermal energy balance in smart grids

Warm season:

- The building requests thermal energy for the cooling purpose
- Water is injected into warm well and is taken from cold well
- The stored water contains warm thermal energy for next season

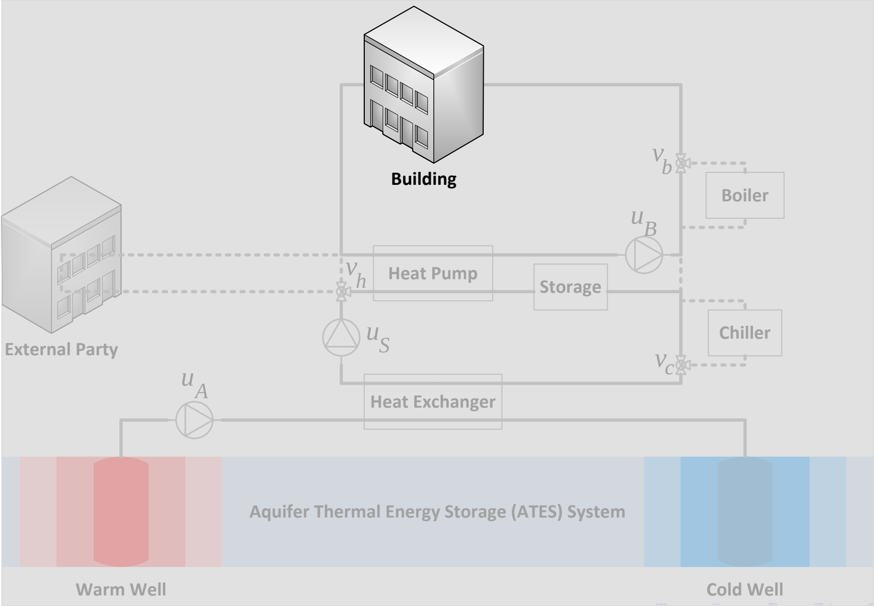


Building Climate Comfort with ATES System (Agent)

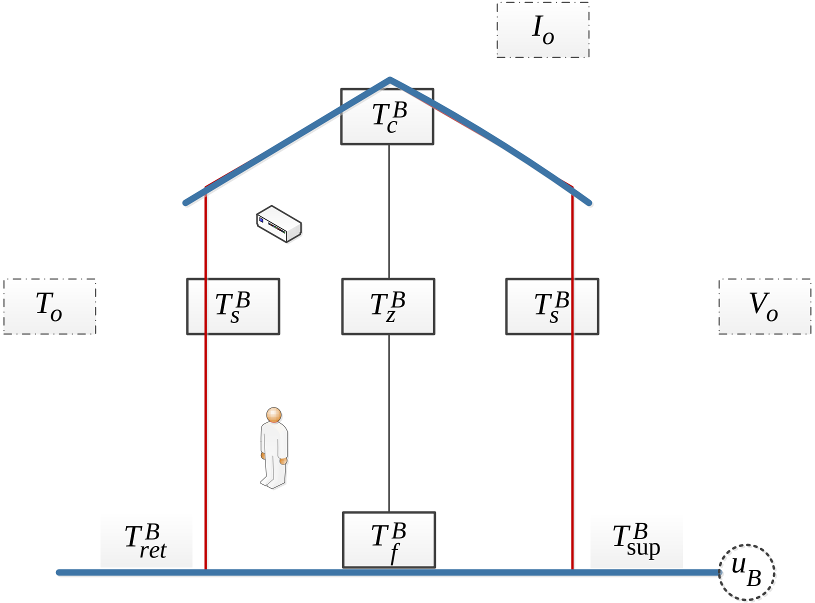


Outline

Single Agent System



Building Thermal Comfort Relations



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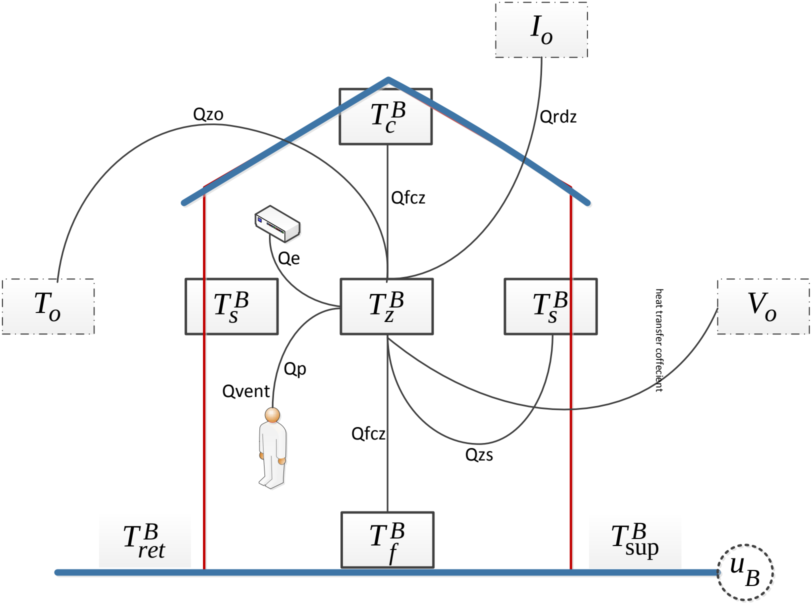
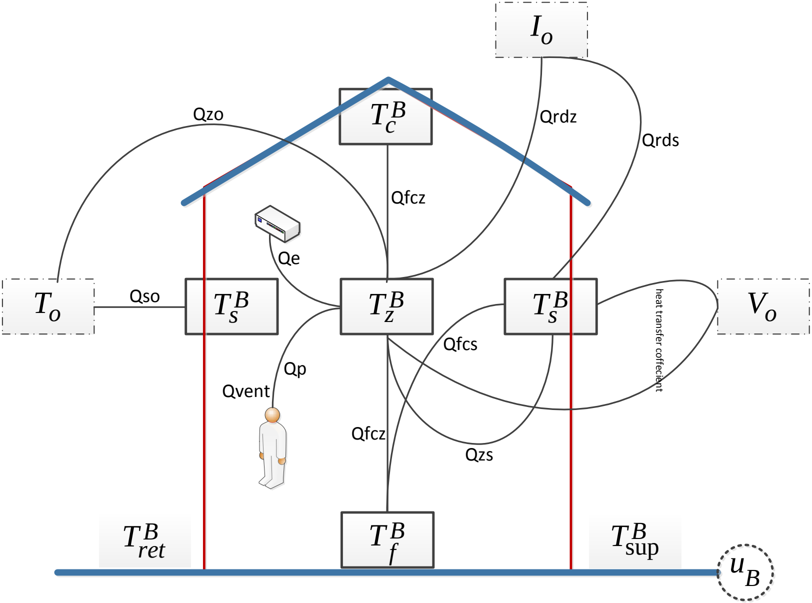
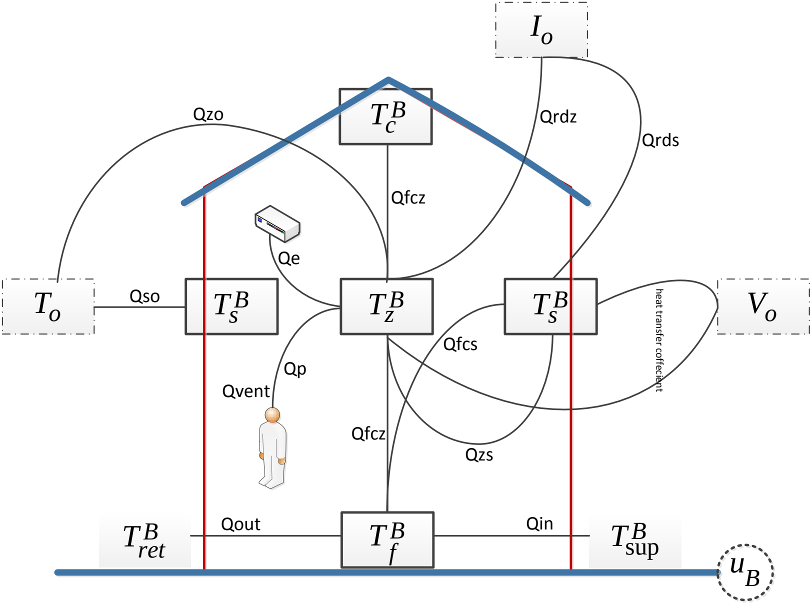


Building Thermal Comfort Relations



Building Thermal Comfort Relations



Building Thermal Comfort Model Formulation

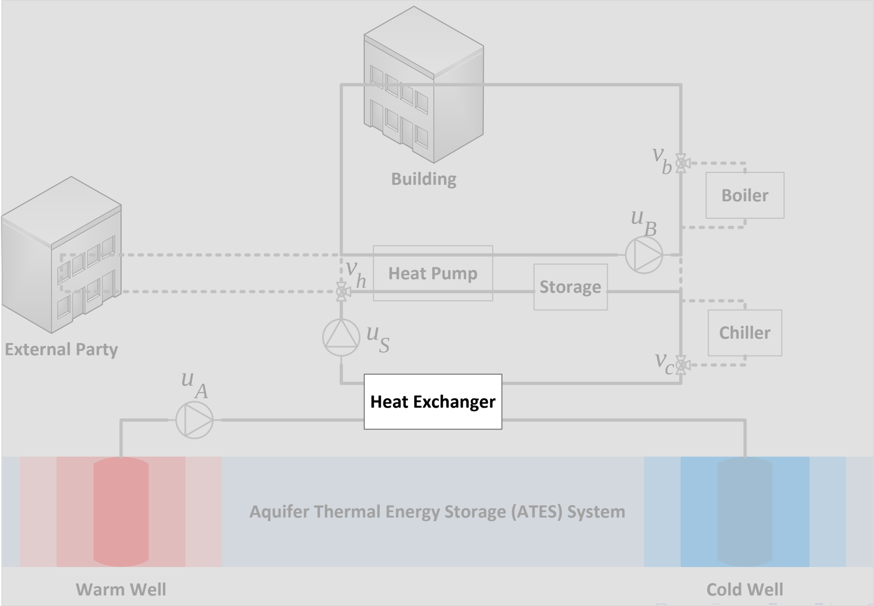
We define the following model:

Building Dynamical Model

$$x_{B,k+1} = x_{B,k} + f_B(x_{B,k}, u_{B,k}, \nu_{B,k}, \nu_{Bext,k})\tau$$
$$y_{B,k} = g_B(x_{B,k}, u_{B,k})$$

- Building inside variables (states): $x_{B,k} \in \mathbb{R}^3$
- Building outside variables (uncertain): $\nu_{Bext,k} \in \mathbb{R}^3$
- Pump flow rate variable (control): $u_{B,k}$
- Supplied water temperature: $\nu_{B,k}$
- Returned water temperature: $y_{B,k}$
- Sampling period: τ

Single Agent System

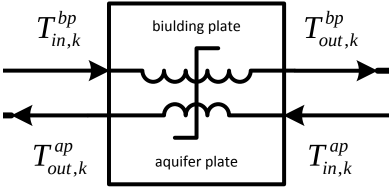


Heat Exchanger Model

A countercurrent heat exchanger is used and it presents via a static model.

Static Model Variables:

- Input water temperatures: $\boldsymbol{\nu}_{\text{he},k} \in \mathbb{R}^2$
- Pump flow rates (control variables): $\boldsymbol{u}_{\text{A},k}, \boldsymbol{u}_{\text{S},k}$
- Output water temperatures: $\boldsymbol{y}_{\text{he},k} \in \mathbb{R}^2$

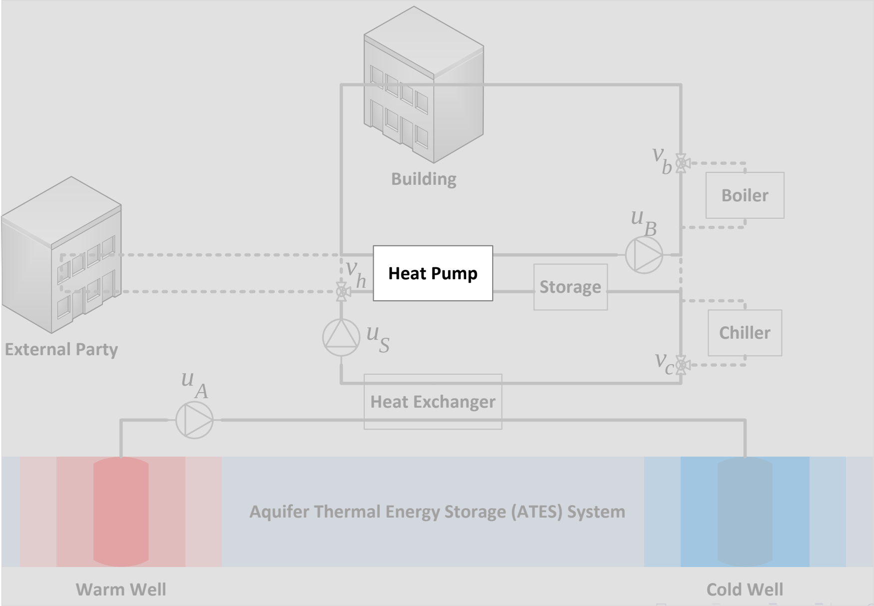


Heat Exchanger

Heat Exchanger Static Model

$$\boldsymbol{y}_{\text{he},k} = \text{H}(\boldsymbol{\nu}_{\text{he},k}, \boldsymbol{u}_{\text{A},k}, \boldsymbol{u}_{\text{S},k})$$

Single Agent System

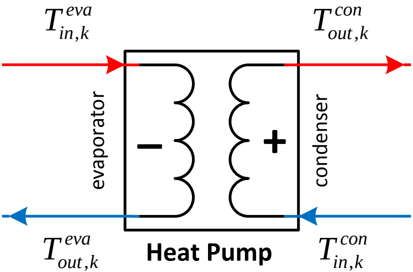


Heat Pump Model

An electrical water to water heat pump is used with static model.

Static Model Variables:

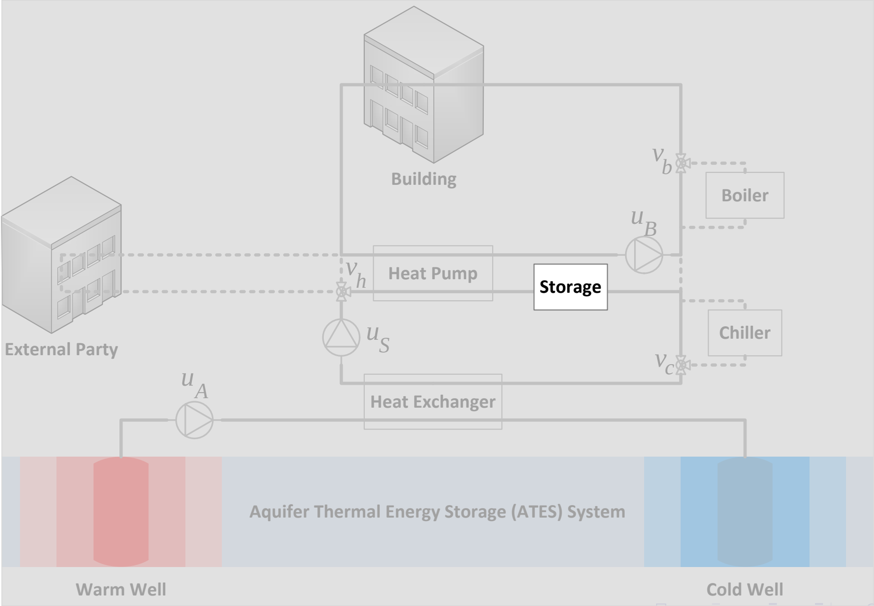
- Input water temperatures: $\boldsymbol{\nu}_{\mathrm{hp},k} \in \mathbb{R}^2$
- Pump flow rates (control variables): $\boldsymbol{u}_{\mathrm{B},k}, \boldsymbol{u}_{\mathrm{S},k}$
- Output water temperatures: $\boldsymbol{y}_{\mathrm{hp},k} \in \mathbb{R}^2$



Heat Pump Static Model

$$\boldsymbol{y}_{\mathrm{hp},k} = \mathrm{P}(\boldsymbol{\nu}_{\mathrm{hp},k}, \boldsymbol{u}_{\mathrm{B},k}, \boldsymbol{u}_{\mathrm{S},k})$$

Single Agent System



Storage Tank Model

We define an storage tank model with the following first order difference equations:

$$\mathrm{V}_{s,k+1} = \mathrm{V}_{s,k} + \mathrm{V}_{in,k} - \mathrm{V}_{out,k}$$

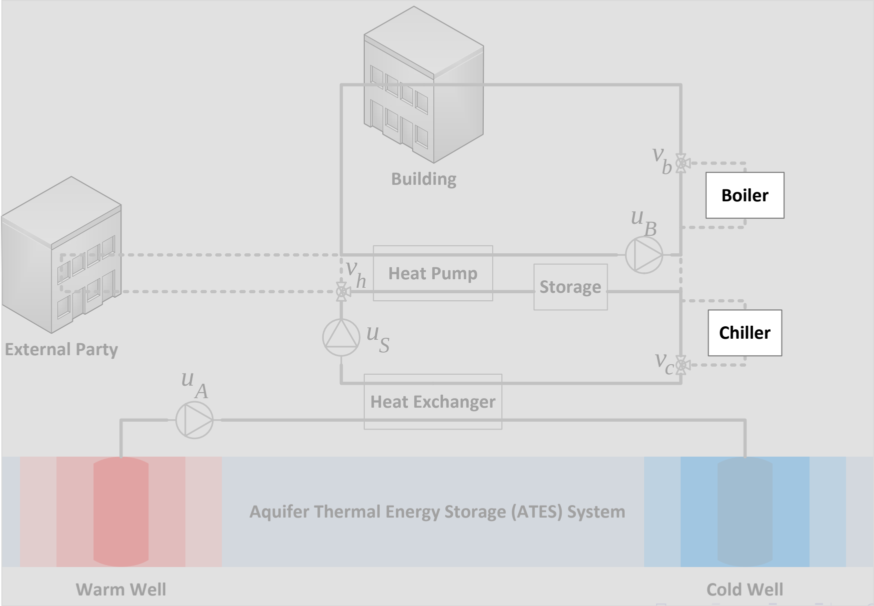
$$\mathrm{T}_{s,k+1} = \frac{\mathrm{V}_{s,k}}{\mathrm{V}_{s,k} + \mathrm{V}_{in,k}}\mathrm{T}_{s,k} + \frac{\mathrm{V}_{in,k}}{\mathrm{V}_{s,k} + \mathrm{V}_{in,k}}\mathrm{T}_{in,k}$$

Storage Dynamical Model

$$\begin{aligned} x_{\mathrm{S},k+1} &= f_{\mathrm{S}}(x_{\mathrm{S},k}, u_{\mathrm{S},k}, \nu_{\mathrm{S},k}) \\ y_{\mathrm{S},k} &= g_{\mathrm{S}}(x_{\mathrm{S},k}) \end{aligned}$$

- Tank temperature and volume variables (state): $x_{\mathrm{S},k} \in \mathbb{R}^2$
- Pump flow rate variable (control): $u_{\mathrm{S},k}$
- Input water temperature: $\nu_{\mathrm{S},k}$
- Output water temperature: $y_{\mathrm{S},k}$

Single Agent System



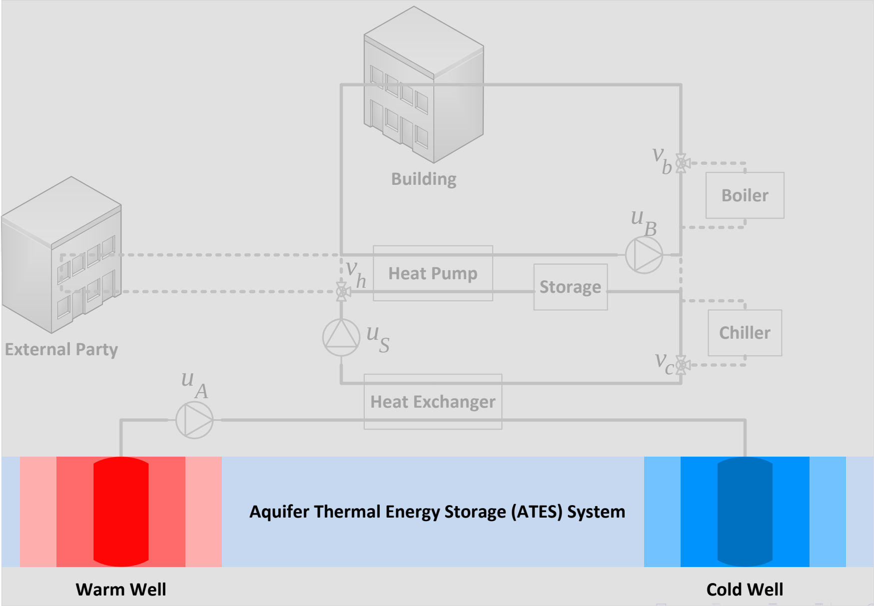
Boiler and Chiller Model

We define the boiler and chiller water temperatures with the following relations:

$$\text{Boiler: } \begin{cases} T^{boi}_{out,k} = 90^{\circ}C \\ T^{boi}_{in,k} = T_{bypass,k} \\ u_{b,k} = v_{b,k} u_{B,k} \end{cases} \qquad \text{Chiller: } \begin{cases} T^{chi}_{out,k} = 5^{\circ}C \\ T^{chi}_{in,k} = T_{bypass,k} \\ u_{c,k} = v_{c,k} u_{S,k} \end{cases}$$

- Boiler valve position (control): $v_{b,k} \in [0, 1]$
- Chiller valve position (control): $v_{c,k} \in [0, 1]$

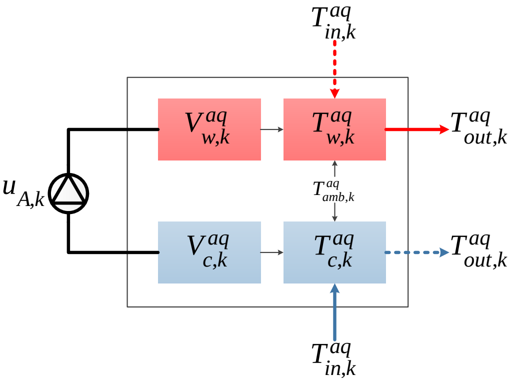
Single Agent System



Aquifer Thermal Energy Storage System Principle

Similar modeling as the storage model by introducing different modes:

- Water is taken from one of the wells and is injected into the counterpart well.
- Taken water has constant temperature until the aquifer water temperature dominates.
- Injected water has gained thermal energy and it is stored for the next upcoming season.



Aquifer Thermal Energy Storage System Model

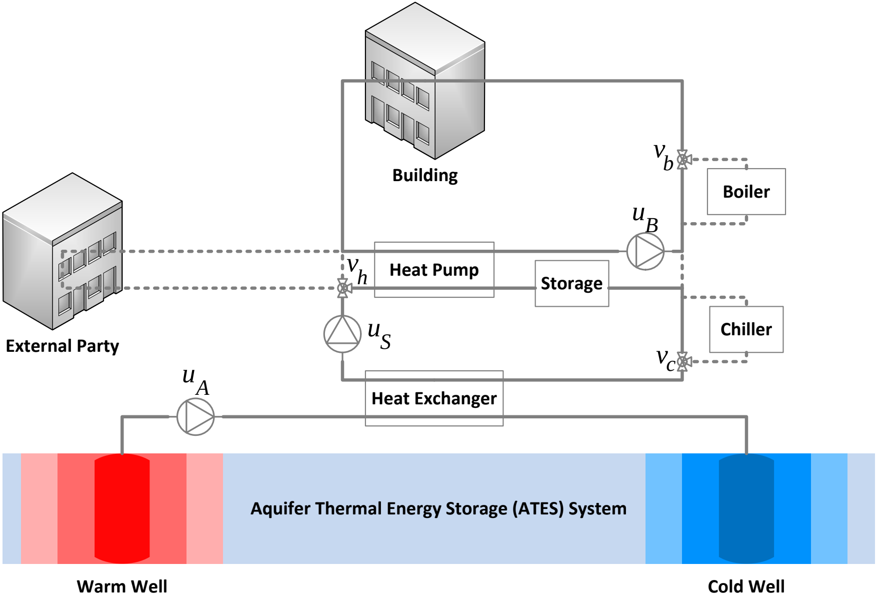
We define the following Model:

ATES system Dynamical Model

$$\begin{aligned} \boldsymbol{x}_{\mathrm{A},k+1} &= \boldsymbol{f}_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A},k}, \boldsymbol{u}_{\mathrm{A},k}, \boldsymbol{\nu}_{\mathrm{A},k}, \boldsymbol{s}_{w,k}, \boldsymbol{s}_{c,k}) \\ \boldsymbol{y}_{\mathrm{A},k} &= \boldsymbol{g}_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A},k}, \boldsymbol{s}_{w,k}, \boldsymbol{s}_{c,k}) \end{aligned}$$

- Wells temperature and volume variables (state): $\boldsymbol{x}_{\mathrm{A},k} \in \mathbb{R}^4$
- Pump flow rate variable (control): $\boldsymbol{u}_{\mathrm{A},k}$
- Output water temperature: $\boldsymbol{y}_{\mathrm{A},k}$
- Input water temperature: $\boldsymbol{\nu}_{\mathrm{A},k}$

Interconnections Between Each Subsystem



Interconnections Between Each Subsystem

1. **ATES system:** $\nu_{A,k} := T^{aq}_{in,k}, \quad y_{A,k} := T^{aq}_{out,k}$
   - $T^{aq}_{in,k} = T^{ap}_{out,k}$
2. **Heat exchanger:** $\nu_{he,k} := [T^{ap}_{in,k}, T^{bp}_{in,k}], \quad y_{he,k} := [T^{ap}_{out,k}, T^{bp}_{out,k}]$
   - $T^{ap}_{in,k} = T^{aq}_{out,k}$ and $T^{bp}_{in,k} = (1 - v_{c,k})T_{s,k} + v_{c,k}T^{chi}_{out,k}$
3. **Heat pump:** $\nu_{hp,k} := [T^{con}_{in,k}, T^{eva}_{in,k}], \quad y_{hp,k} := [T^{con}_{out,k}, T^{eva}_{out,k}]$
   - $T^{con}_{in,k} = s_{n,k}(s_{w,k}T^{bp}_{out,k} + s_{c,k}T^{B}_{ret,k}) + (1 - s_{n,k})(s_{w,k}T^{ext}_{out,k} + s_{c,k}T^{B}_{ret,k})$
   - $T^{eva}_{in,k} = s_{n,k}(s_{c,k}T^{bp}_{out,k} + s_{w,k}T^{B}_{ret,k}) + (1 - s_{n,k})(s_{w,k}T^{ext}_{out,k} + s_{c,k}T^{B}_{ret,k})$
4. **Storage model:** $\nu_{S,k} := T_{in,k}, \quad y_{S,k} := T^{s}_{out,k}$
   - $T^{s}_{in,k} = v_{h,k}(s_{w,k}T^{con}_{out,k} + s_{c,k}T^{eva}_{out,k}) + (1 - v_{h,k})T^{B}_{ret,k}$
5. **Building model:** $\nu_{B,k} := T^{B}_{sup,k}, \quad y_{B,k} := T^{B}_{ret,k}$

$$T^{B}_{sup,k} = v_{h,k}(s_{w,k}T^{eva}_{out,k} + s_{c,k}((1 - v_{b,k})T^{con}_{out,k} + v_{b,k}T^{boi}_{out,k})) + (1 - v_{h,k})T^{bp}_{out,k}$$

Single Agent Representation

Consider compact formulation of dynamical agent system:

Single Agent Model

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{v}_k, \boldsymbol{s}_k, \boldsymbol{w}_k)$$

- State variables: $\boldsymbol{x}_k := [x_{\mathrm{B},k}, x_{\mathrm{S},k}, x_{\mathrm{A},k}] \in \mathbb{R}^9$
- Pump flow rate variables: $\boldsymbol{u}_k := [u_{\mathrm{B},k}, u_{\mathrm{S},k}, u_{\mathrm{A},k}] \in \mathbb{R}^3$
- Valve position variables: $\boldsymbol{v}_k := [v_{b,k}, v_{c,k}, v_{h,k}] \in [0,1]^3$
- Operating mode variables: $\boldsymbol{s}_k := [s_{w,k}, s_{c,k}, s_{n,k}] \in \{0,1\}^3$
- Uncertain variables: $\boldsymbol{w}_k := [\mathrm{T}_{o,k}, \mathrm{I}_{o,k}, \mathrm{V}_{o,k}] \subseteq \boldsymbol{\Delta} \in \mathbb{R}^3$
- State variables are available at each sampling time $\boldsymbol{k}$.

Outline

Control Problem Formulation

We formulate an optimization problem as follows:

$$\min_{\{u_k, v_k\}_{k=1}^{N}} \quad \text{Objective Function: Reference Tracking}$$

subject to:
- Nonlinear System Dynamics
- State and Control Bounds
- Valves, Modes and Uncertainty Sets
- Heat Exchanger Capacity Constraints
- Heat Pump Capacity Constraints

Proposed Formulation

Stochastic Mixed-Integer Nonlinear Optimization Problem

Control Problem Formulation

We formulate an optimization problem as follows:

$$\min_{\{u_k, v_k\}_{k=1}^{N}} \quad \mathbb{E}\left[\sum_{k=1}^{N} \gamma(T_{z,k}^{B} - T_{set})^2\right]$$

subject to:

- Nonlinear System Dynamics
- State and Control Bounds
- Valves, Modes and Uncertainty Sets
- Heat Exchanger Capacity Constraints
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Proposed Formulation

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$$\min_{\{u_k, v_k\}_{k=1}^{N}} \quad \mathbb{E}\left[\sum_{k=1}^{N} \gamma(\mathrm{T}^{\mathrm{B}}_{z,k} - \mathrm{T}_{\mathrm{set}})^2\right]$$

subject to: $\quad x_{k+1} = f(x_k, u_k, v_k, s_k, w_k)$

State and Control Bounds

Valves, Modes and Uncertainty Sets

Heat Exchanger Capacity Constraints

Heat Pump Capacity Constraints

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subject to:

$$x_{k+1} = f\left(x_k, u_k, v_k, s_k, w_k\right)$$

$$x_{\min} \leq x_k \leq x_{\max} \;,\; u_{\min} \leq u_k \leq u_{\max}$$

Valves, Modes and Uncertainty Sets

Heat Exchanger Capacity Constraints

Heat Pump Capacity Constraints

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$$x_{\min} \leq x_k \leq x_{\max}\ ,\ u_{\min} \leq u_k \leq u_{\max}$$

$$0 \leq v_k \leq 1\ ,\ s_k \in \{0, 1\}\ ,\ w_k \in \Delta$$

Heat Exchanger Capacity Constraints

Heat Pump Capacity Constraints

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subject to:

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$$0 \leq v_k \leq 1\ ,\ s_k \in \{0,1\}\ ,\ w_k \in \Delta$$

$$\nu_{\mathrm{he}}^{\min} \leq \nu_{\mathrm{he},k} \leq \nu_{\mathrm{he}}^{\max}\ ,\ y_{\mathrm{he}}^{\min} \leq y_{\mathrm{he},k} \leq y_{\mathrm{he}}^{\max}$$

Heat Pump Capacity Constraints

Proposed Formulation

Stochastic Mixed-Integer Nonlinear Optimization Problem

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We formulate an optimization problem as follows:

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subject to:

$$x_{k+1} = f(x_k, u_k, v_k, s_k, w_k)$$

$$x_{\min} \leq x_k \leq x_{\max}\ ,\ u_{\min} \leq u_k \leq u_{\max}$$

$$0 \leq v_k \leq 1\ ,\ s_k \in \{0, 1\}\ ,\ w_k \in \Delta$$

$$\nu_{\mathrm{he}}^{\min} \leq \nu_{\mathrm{he},k} \leq \nu_{\mathrm{he}}^{\max}\ ,\ y_{\mathrm{he}}^{\min} \leq y_{\mathrm{he},k} \leq y_{\mathrm{he}}^{\max}$$

$$\nu_{\mathrm{hp}}^{\min} \leq \nu_{\mathrm{hp},k} \leq \nu_{\mathrm{hp}}^{\max}\ ,\ y_{\mathrm{hp}}^{\min} \leq y_{\mathrm{hp},k} \leq y_{\mathrm{hp}}^{\max}$$

Proposed Formulation

Stochastic Mixed-Integer Nonlinear Optimization Problem

Control Problem Formulation

We formulate an optimization problem as follows:

$$\min_{\{u_k, v_k\}_{k=1}^{N}} \quad \mathbb{E}\left[\sum_{k=1}^{N} \gamma(\mathrm{T}^{\mathrm{B}}_{z,k} - \mathrm{T}_{\mathrm{set}})^2\right]$$

subject to:

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$$x_{\min} \leq x_k \leq x_{\max}\ ,\ u_{\min} \leq u_k \leq u_{\max}$$

$$0 \leq v_k \leq 1\ ,\ s_k \in \{0, 1\}\ ,\ w_k \in \Delta$$

$$\nu_{\mathrm{he}}^{\min} \leq \nu_{\mathrm{he},k} \leq \nu_{\mathrm{he}}^{\max}\ ,\ y_{\mathrm{he}}^{\min} \leq y_{\mathrm{he},k} \leq y_{\mathrm{he}}^{\max}$$

$$\nu_{\mathrm{hp}}^{\min} \leq \nu_{\mathrm{hp},k} \leq \nu_{\mathrm{hp}}^{\max}\ ,\ y_{\mathrm{hp}}^{\min} \leq y_{\mathrm{hp},k} \leq y_{\mathrm{hp}}^{\max}$$

Proposed Formulation

Stochastic Mixed-Integer Nonlinear Optimization Problem

Outline

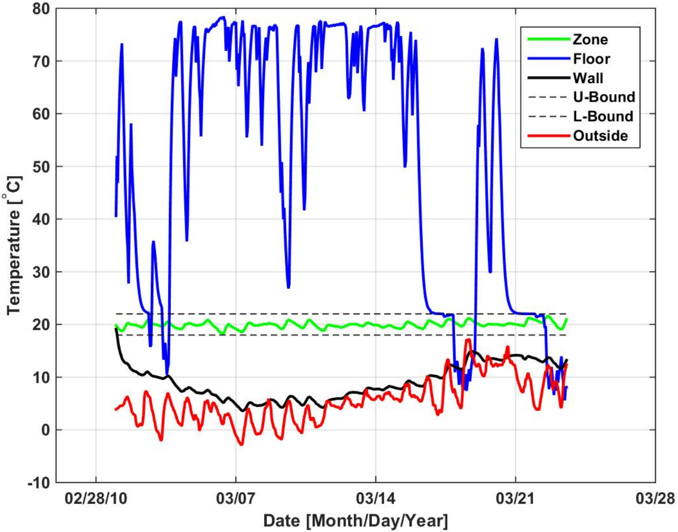
Simulation Study

A single agent model control problem formulation:

- Sampling period: **1**h
- Prediction horizon: **24**h
- No integer variables (fixed)
- No stochastic terms (deterministic controller)
- Linear approximation of HP & HE complex subsystems
- Remove complex constraints of HP & HE:

$$\boldsymbol{\nu}_{\text{he}}^{\mathbf{min}} \leq \boldsymbol{\nu}_{\text{he},\boldsymbol{k}} \leq \boldsymbol{\nu}_{\text{he}}^{\mathbf{max}} \;,\; \boldsymbol{y}_{\text{he}}^{\mathbf{min}} \leq \boldsymbol{y}_{\text{he},\boldsymbol{k}} \leq \boldsymbol{y}_{\text{he}}^{\mathbf{max}}$$
$$\boldsymbol{\nu}_{\text{hp}}^{\mathbf{min}} \leq \boldsymbol{\nu}_{\text{hp},\boldsymbol{k}} \leq \boldsymbol{\nu}_{\text{hp}}^{\mathbf{max}} \;,\; \boldsymbol{y}_{\text{hp}}^{\mathbf{min}} \leq \boldsymbol{y}_{\text{hp},\boldsymbol{k}} \leq \boldsymbol{y}_{\text{hp}}^{\mathbf{max}}$$

Simulation Results



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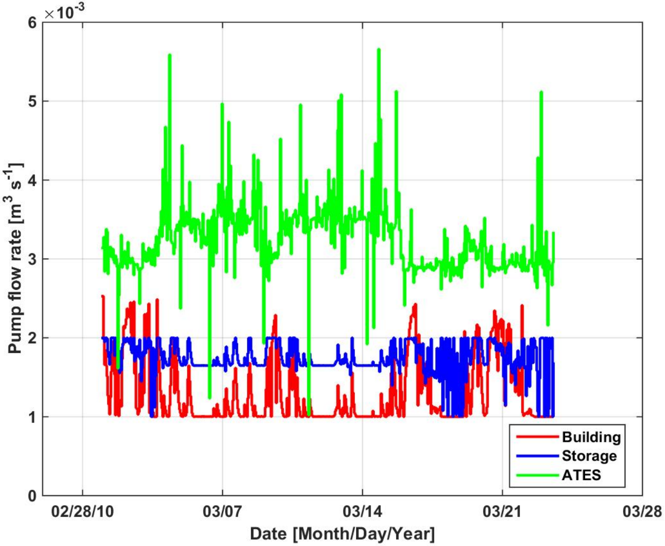
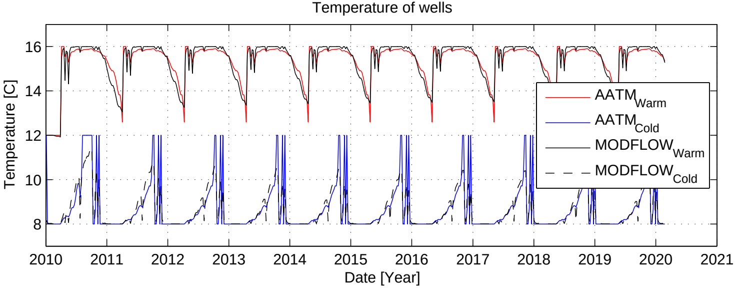
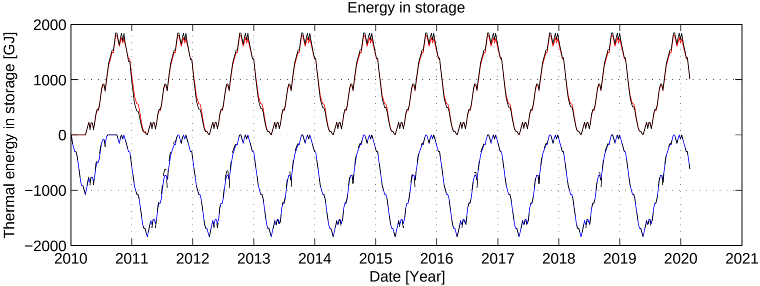


Parameter Estimation (loss-term)

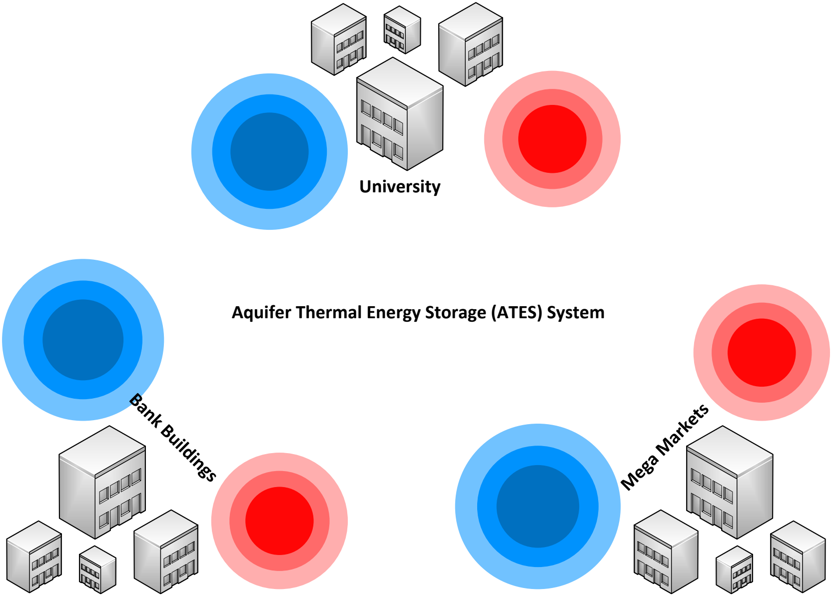




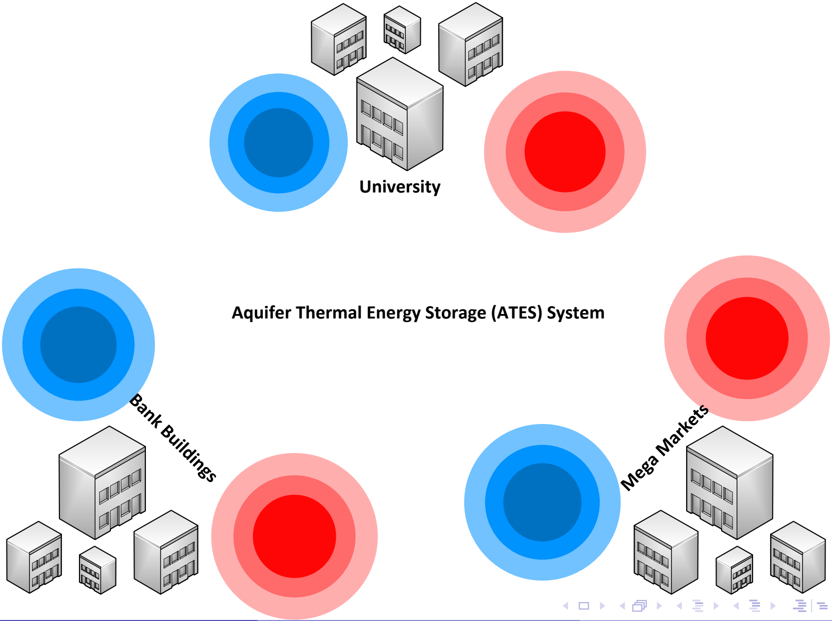
Outline

Large Scale Complex Systems: ATES Smart Grids

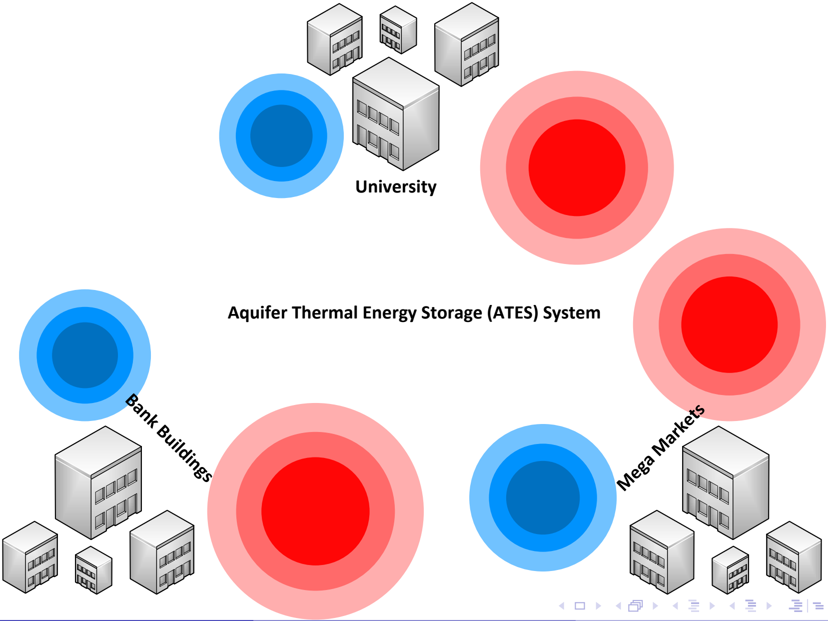
Ongoing Developments: Interactions Model



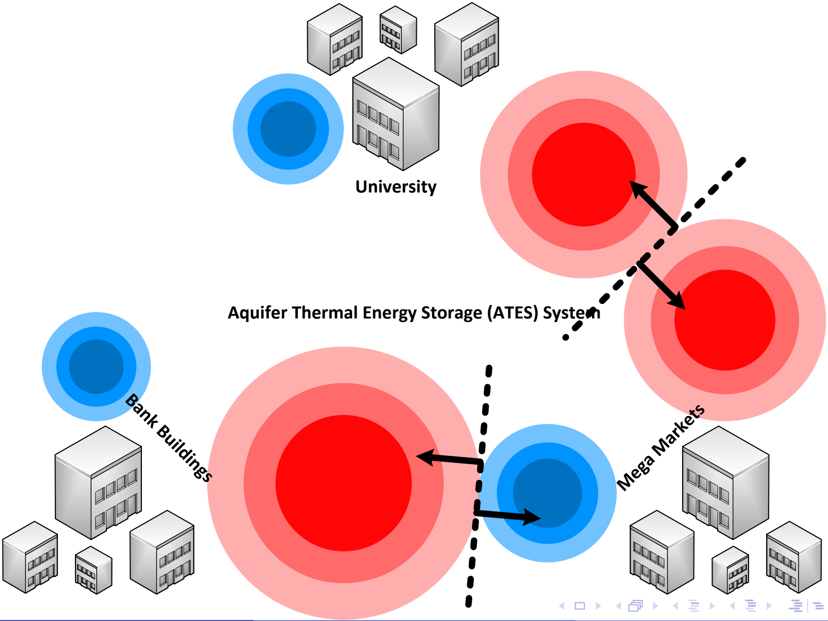
Ongoing Developments: Optimal Operation



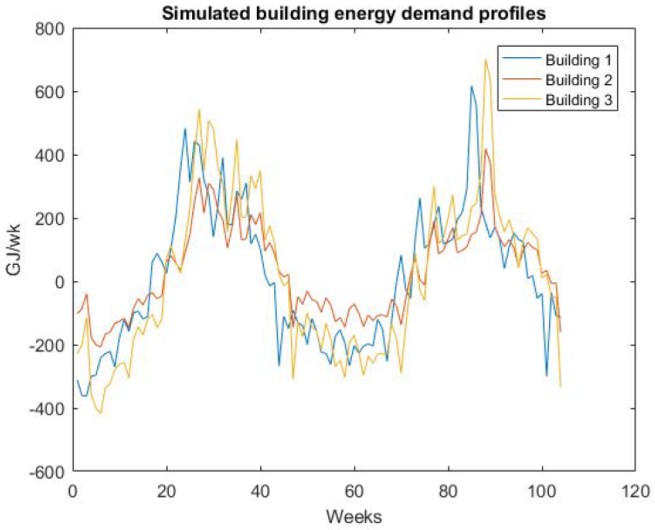
Ongoing Developments: Effective Operation



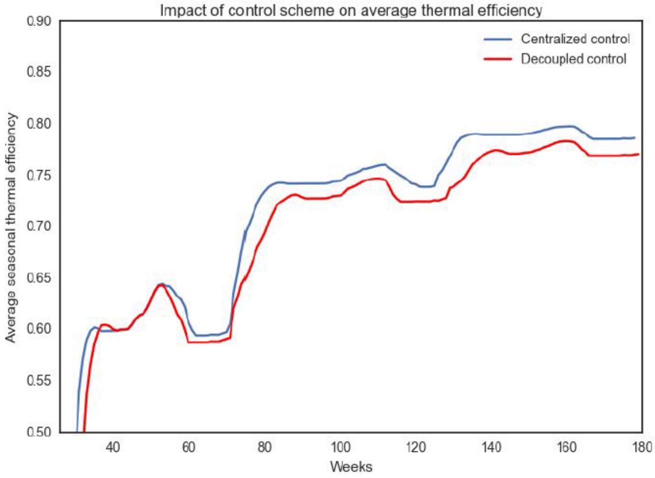
Ongoing Developments: Cooperative Approach



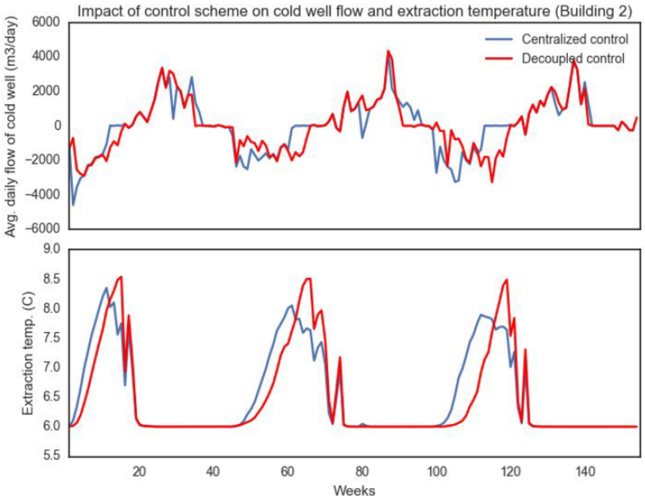
Results: Centralized vs. Decoupled



Results: Centralized vs. Decoupled



Results: Centralized vs. Decoupled



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Building Thermal Comfort Energy Demand

Main goal is to keep **building zone temperature at the desired level**.

- Building energy demand level is: $\boldsymbol{E_d = E_{\text{gain}} - E_{\text{loss}}}$
- Endogenous source of losses: $\boldsymbol{E_{\text{loss}} = Q_{zo} + Q_{so} + Q_{\text{vent}}}$
- Convection heat transfer from zone and solid to outside air: $\boldsymbol{Q_{zo}, Q_{so}}$
- Ventilation thermal energy lost: $\boldsymbol{Q_{\text{vent}}}$
- Endogenous source of energy: $\boldsymbol{E_{\text{gain}} = Q_{\text{rad}z} + Q_{\text{rad}s} + Q_p + Q_e}$
- Radiation absorption by building zone and solid: $\boldsymbol{Q_{\text{rad}z}, Q_{\text{rad}s}}$
- Occupancy and heat gain due to the electrical devices: $\boldsymbol{Q_p, Q_e}$

Heat Exchanger Model

Having the following relations:

- Aquifer plate thermal energy: $Q_{he,k} = \rho_w c_{p,w} u_{A,k}(T^{ap}_{out,k} - T^{ap}_{in,k})$
- Building plate thermal energy: $Q_{he,k} = \rho_w c_{p,w} u_{S,k}(T^{bp}_{in,k} - T^{bp}_{out,k})$
- Using the internal thermal energy conditions: $Q_{he,k} = k_{he} A_{he} \Delta T^{he}_{m}$
- ΔT^{he}_{m} is the mean temperature difference for the heat transfer.

Heat Exchanger Static Model

$$\Pi_{he} := \begin{cases} y_{he,k} = H(\nu_{he,k}, u_{A,k}, u_{S,k}) \\ \forall k \in \{0, 1, 2, \cdots\} \end{cases}$$

Heat Pump Model

Having the following relations:

- The thermal energy of condenser $Q_{h,k}$ and evaporator $Q_{c,k}$ sides:
  $Q_{h,k} = \rho_w c_{p,w} u_{B,k}(T^{con}_{out,k} - T^{con}_{in,k})$
  $Q_{c,k} = \rho_w c_{p,w} u_{S,k}(T^{eva}_{in,k} - T^{eva}_{out,k})$
- Using the internal thermal energies conditions:
  $Q_{h,k} = k_{hp} A_{hp} \Delta T^{hp}_{m,h}$ and $Q_{c,k} = k_{hp} A_{hp} \Delta T^{hp}_{m,c}$
- The coefficient of performance: $COP = Q_{h,k} \left(Q_{h,k} - Q_{c,k}\right)^{-1}$
- Using Carnot cycle: $COP = \eta_{hp} T_{hs} \left(T_{hs} - T_{cs}\right)^{-1}$

Heat Pump Static Model

$$\Pi_{hp} := \begin{cases} y_{hp,k} = P(\nu_{hp,k}, u_{B,k}, u_{S,k}) \\ \forall k \in \{0, 1, 2, \cdots\} \end{cases}$$

Aquifer Thermal Energy Storage System Model

Consider the following mixed-integer first-order difference equations:

$$
\begin{aligned}
V^{aq}_{w,k+1} &= V^{aq}_{w,k} + (s_{w,k} - s_{c,k})V^{aq}_{in,k} \\
V^{aq}_{c,k+1} &= V^{aq}_{c,k} + (s_{c,k} - s_{w,k})V^{aq}_{in,k} \\
T^{aq}_{w,k+1} &= \frac{V^{aq}_{w,k}}{V^{aq}_{w,k} + s_{w,k}V^{aq}_{in,k}} T^{aq}_{w,k} + \frac{s_{w,k}V^{aq}_{in,k}}{V^{aq}_{w,k} + s_{w,k}V^{aq}_{in,k}} T^{aq}_{in,k} - \frac{\alpha(T^{aq}_{w,k} - T^{aq}_{amb,k})}{V^{aq}_{w,k} + s_{w,k}V_{in,k}} \\
T^{aq}_{c,k+1} &= \frac{V^{aq}_{c,k}}{V^{aq}_{c,k} + s_{c,k}V^{aq}_{in,k}} T^{aq}_{c,k} + \frac{s_{c,k}V^{aq}_{in,k}}{V^{aq}_{c,k} + s_{c,k}V^{aq}_{in,k}} T^{aq}_{in,k} - \frac{\alpha(T^{aq}_{c,k} - T^{aq}_{amb,k})}{V^{aq}_{c,k} + s_{c,k}V_{in,k}}
\end{aligned}
$$

- Integer variables of warm and cold season: $s_{w,k}, s_{c,k} \in \{0, 1\}$
- Output water temperature is: $T^{aq}_{out,k} = s_{c,k}T^{aq}_{w,k} + s_{w,k}T^{aq}_{c,k}$

Interconnections Between Each Subsystem

1. **ATES system:** $\nu_{\mathrm{A},k} := \mathrm{T}^{\mathrm{aq}}_{in,k}, \quad y_{\mathrm{A},k} := \mathrm{T}^{\mathrm{aq}}_{out,k}$
   - $\mathrm{T}^{\mathrm{aq}}_{in,k} = \mathrm{T}^{\mathrm{ap}}_{out,k}$
2. **Heat exchanger:** $\nu_{\mathrm{he},k} := [\mathrm{T}^{\mathrm{ap}}_{in,k}, \mathrm{T}^{\mathrm{bp}}_{in,k}], \quad y_{\mathrm{he},k} := [\mathrm{T}^{\mathrm{ap}}_{out,k}, \mathrm{T}^{\mathrm{bp}}_{out,k}]$
   - $\mathrm{T}^{\mathrm{ap}}_{in,k} = \mathrm{T}^{\mathrm{aq}}_{out,k}$ and $\mathrm{T}^{\mathrm{bp}}_{in,k} = (1 - v_{c,k})\mathrm{T}_{s,k} + v_{c,k}\mathrm{T}^{\mathrm{chi}}_{out,k}$
3. **Heat pump:** $\nu_{\mathrm{hp},k} := [\mathrm{T}^{\mathrm{con}}_{in,k}, \mathrm{T}^{\mathrm{eva}}_{in,k}], \quad y_{\mathrm{hp},k} := [\mathrm{T}^{\mathrm{con}}_{out,k}, \mathrm{T}^{\mathrm{eva}}_{out,k}]$
   - $\mathrm{T}^{\mathrm{con}}_{in,k} = s_{n,k}(s_{w,k}\mathrm{T}^{\mathrm{bp}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}) + (1 - s_{n,k})(s_{w,k}\mathrm{T}^{\mathrm{ext}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k})$
   - $\mathrm{T}^{\mathrm{eva}}_{in,k} = s_{n,k}(s_{c,k}\mathrm{T}^{\mathrm{bp}}_{out,k} + s_{w,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}) + (1 - s_{n,k})(s_{w,k}\mathrm{T}^{\mathrm{ext}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k})$
4. **Storage model:** $\nu_{\mathrm{S},k} := \mathrm{T}_{in,k}, \quad y_{\mathrm{S},k} := \mathrm{T}^{s}_{out,k}$
   - $\mathrm{T}^{s}_{in,k} = v_{h,k}(s_{w,k}\mathrm{T}^{\mathrm{con}}_{out,k} + s_{c,k}\mathrm{T}^{\mathrm{eva}}_{out,k}) + (1 - v_{h,k})\mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}$
5. **Building model:** $\nu_{\mathrm{B},k} := \mathrm{T}^{\mathrm{B}}_{\mathrm{sup},k}, \quad y_{\mathrm{B},k} := \mathrm{T}^{\mathrm{B}}_{\mathrm{ret},k}$

$$\mathrm{T}^{\mathrm{B}}_{\mathrm{sup},k} = v_{h,k}(s_{w,k}\mathrm{T}^{\mathrm{eva}}_{out,k} + s_{c,k}((1 - v_{b,k})\mathrm{T}^{\mathrm{con}}_{out,k} + v_{b,k}\mathrm{T}^{\mathrm{boi}}_{out,k})) + (1 - v_{h,k})\mathrm{T}^{\mathrm{bp}}_{out,k}$$

Simulation Results