# A Symbolic Motion Planning Approach for the Reach-avoid Problem

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# Motivation

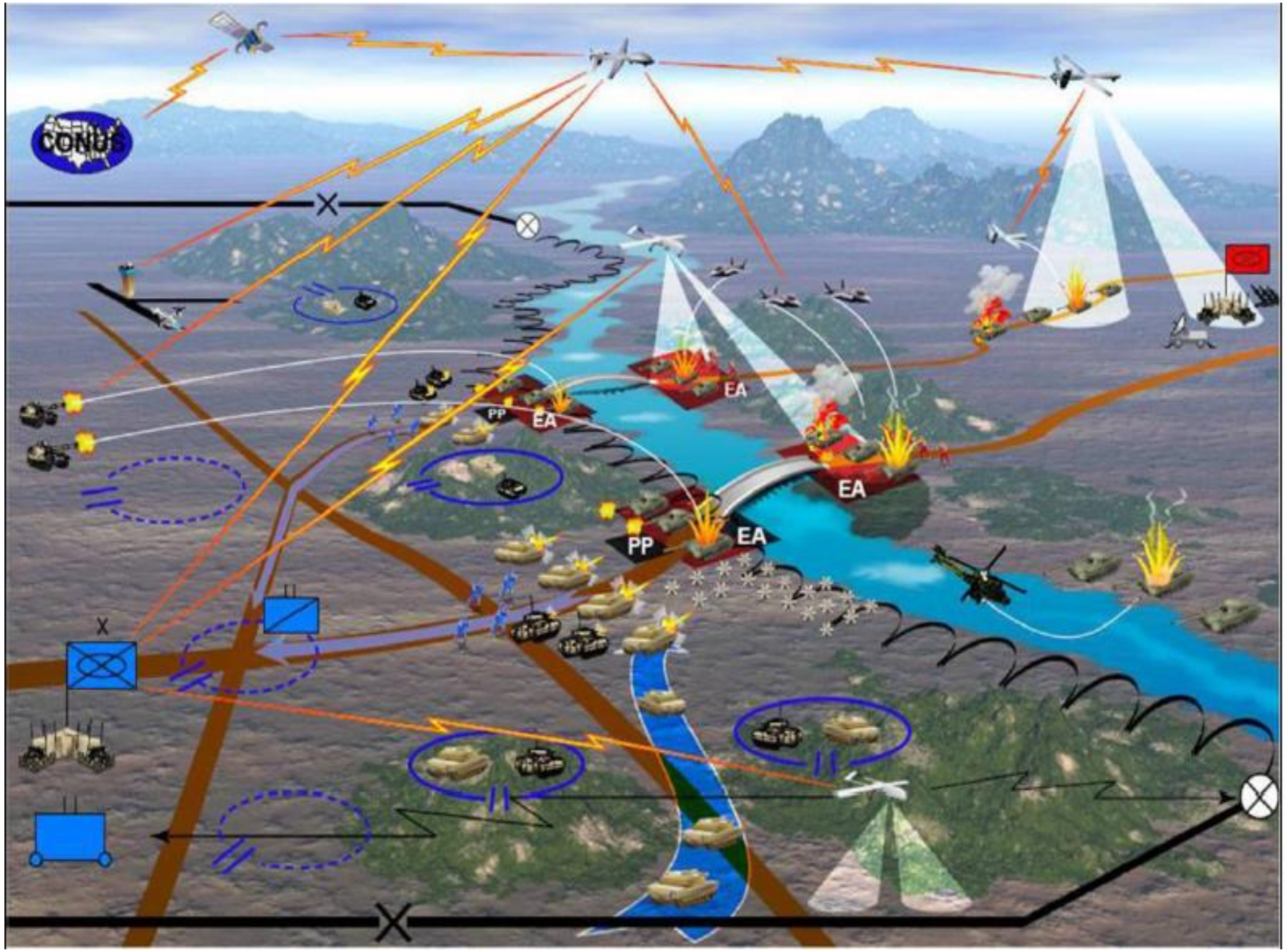
With advances in the capabilities of autonomous vehicles, it is becoming possible to autonomously control them to accomplish sophisticated tasks in dynamic environments such as a battlefield.

**Objective of this research:**

To develop a systematic coordination and planning method for autonomous vehicles in a dynamic adversarial environment.

**Interested scenario:**

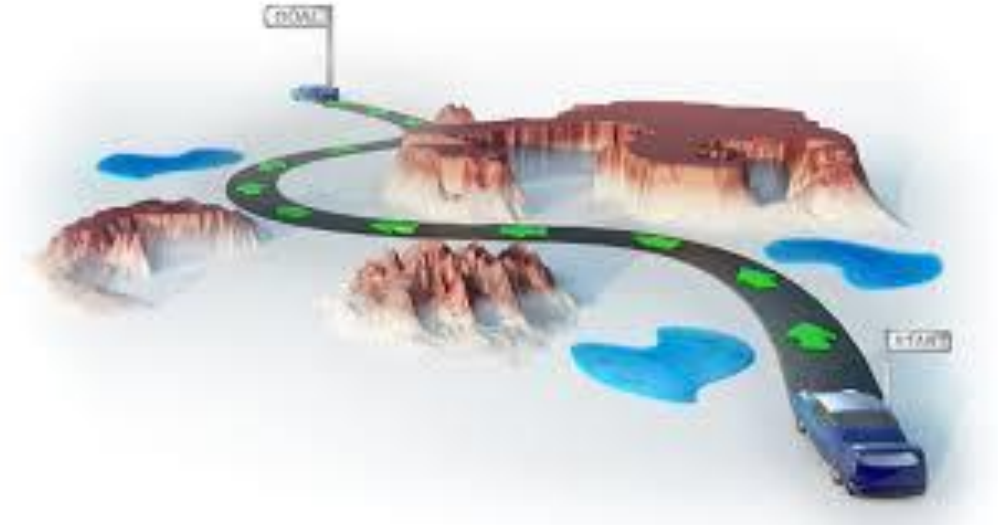
Planning and coordination for an attacker/defender involved in a reach-avoid scenario.



# Motion planning

**Conventional problem statement for motion planning:**

Optimally transit from an initial region to the desired location while avoiding (static) obstacles in an (unchanging environment).



**Challenge**:

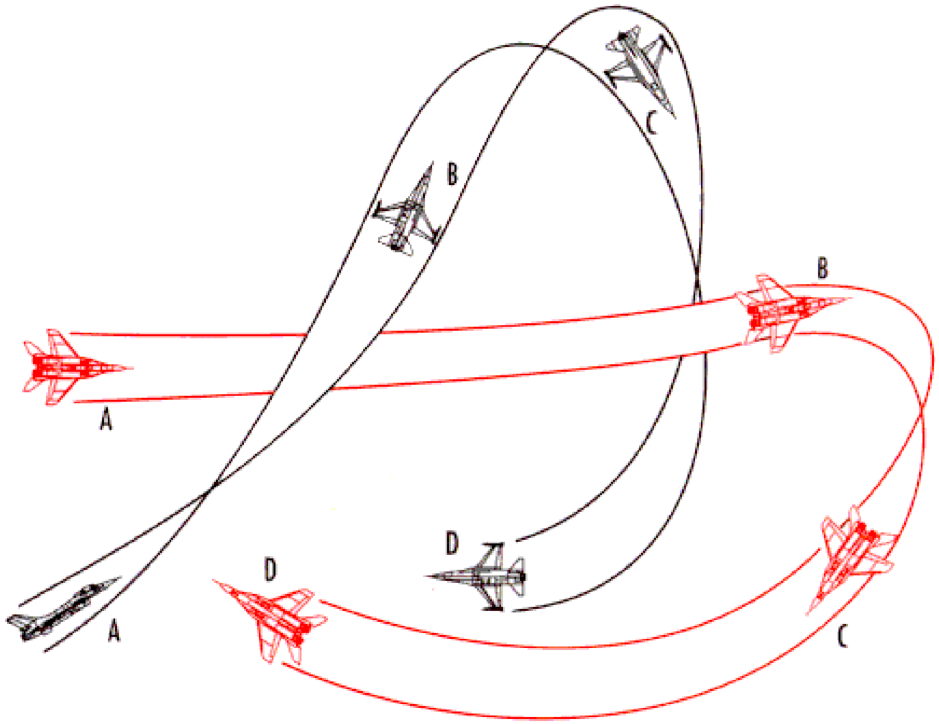
How to coordinate autonomous vehicles to achieve their goals in an adversarial environment?

# Reach-avoid problem

Consider a defender agent and an attacker agent:

- The defender tries to capture the attacker before reaching the defending area.
- The attacker tries to reach the target while avoiding of capture.

Depending on our interest, the problem is then how to coordinate the defender or the attacker to be win this conflict.



# Challenges and Gaps

**Reach-avoidance**

- **Graph theory**
- **Discrete games**
- **Path planning with obstacle**
- **model-predictive control**
- **Differential game**

- The exiting methods mostly suffer from high computational cost.
- The exiting methods mostly assume the agents are driven in an unchanging environment.
- The integration of other requirements such as avoiding obstacles, unsafe or no-fly zones into the existing methods would not be straightforward.

# Proposed Approach

To develop a computationally effective suboptimal hybrid reactive symbolic motion planning approach for the reach-avoid problem.

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To reduce the computation cost

To capture the interactions with a dynamic environment

To uniformly capture the continuous motion planning and discrete logics

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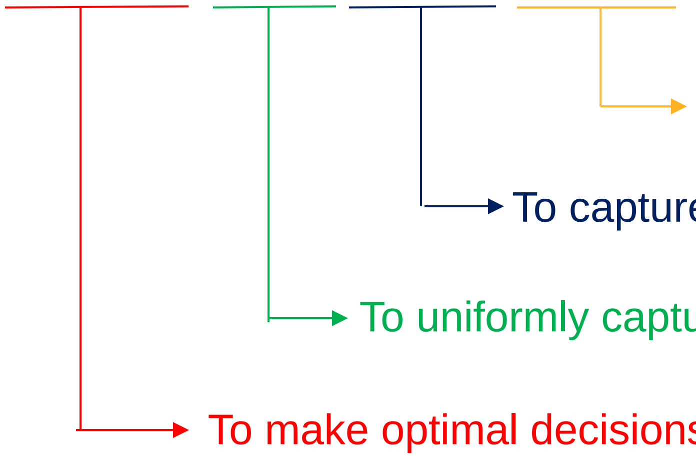

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To make optimal decisions over a bisimilarly-equivalent partitioned space

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# Contributions

- ✓ To the best of our knowledge, **this is the first work on symbolic motion planning for reach-avoid problem**.
- ✓ Our proposed symbolic motion planning framework can effectively handle reach-avoid problems within the context of hybrid supervisory control theory.
- ✓ We formulated the reach-avoid problem as a discrete game over a bisimilarly-equivalent partitioned environment. This allows to consider the decision making process for the players as a finite two-player zero-sum game over the partitioned environment. As a result, the players can make high-level decisions about their next immediate actions based on the current positions of their opponents the players, without the use of explicit prediction models or imposing any limit on the players actions.
- ✓ The desired objectives of the defender and the adversarial behavior of the attacker are captured by linear temporal logic (LTL).
- ✓ We integrated the Reactive synthesis techniques into the proposed algorithm to achieve winning strategies for the players, while capturing the interactions with the environment and satisfying the desired objectives.
- ✓ We developed a hybrid controller that can generate continues control signals for driving the players over the partitioned game space

# General structure of the proposed symbolic planning framework

*Human Operator*

*Players dynamics*
*and the environment*

# General structure of the proposed symbolic planning framework

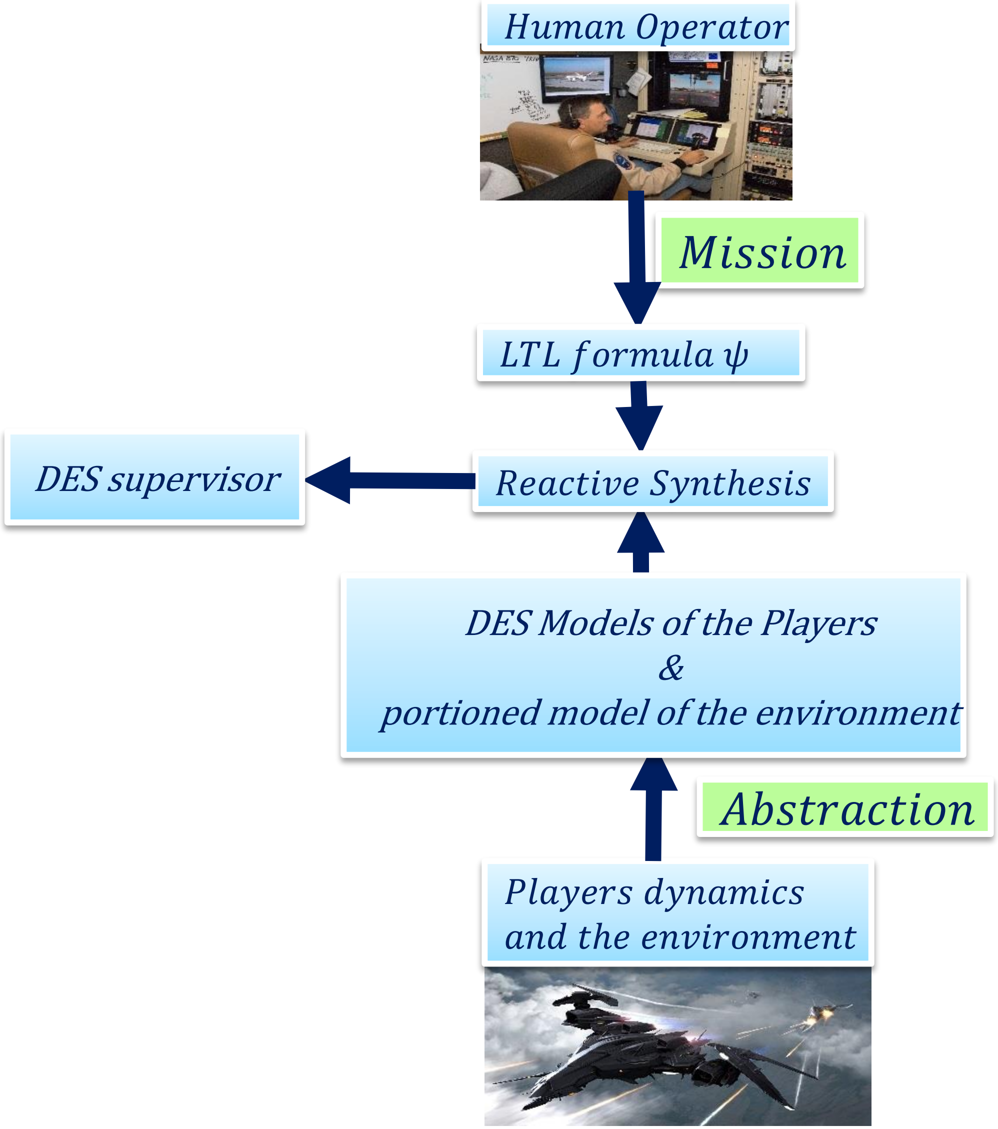
Human Operator

Mission

LTL formula $\psi$

Players dynamics
and the environment

# General structure of the proposed symbolic planning framework



*Human Operator*

*Mission*

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*Reactive Synthesis*

*DES supervisor*

*DES Models of the Players*
*&*
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*Abstraction*

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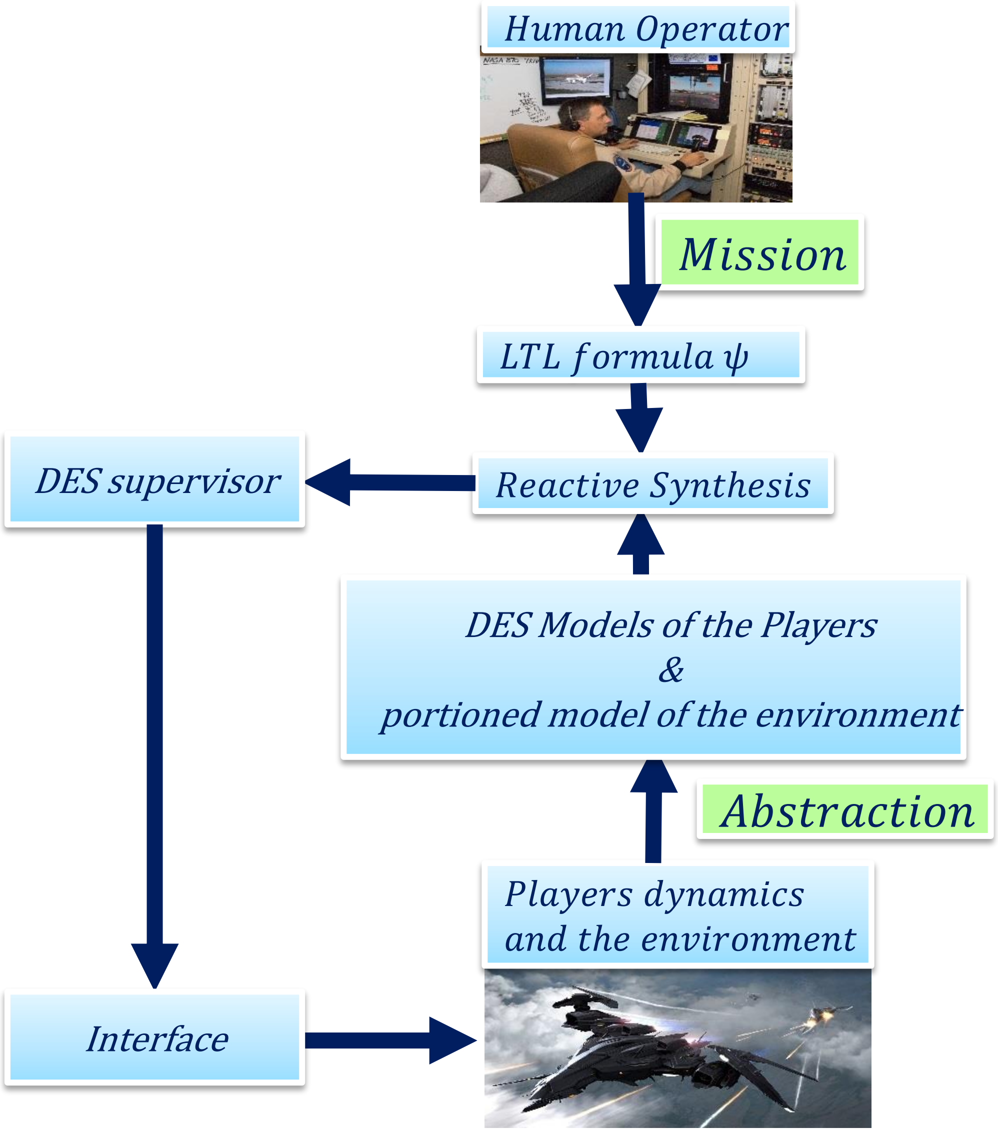

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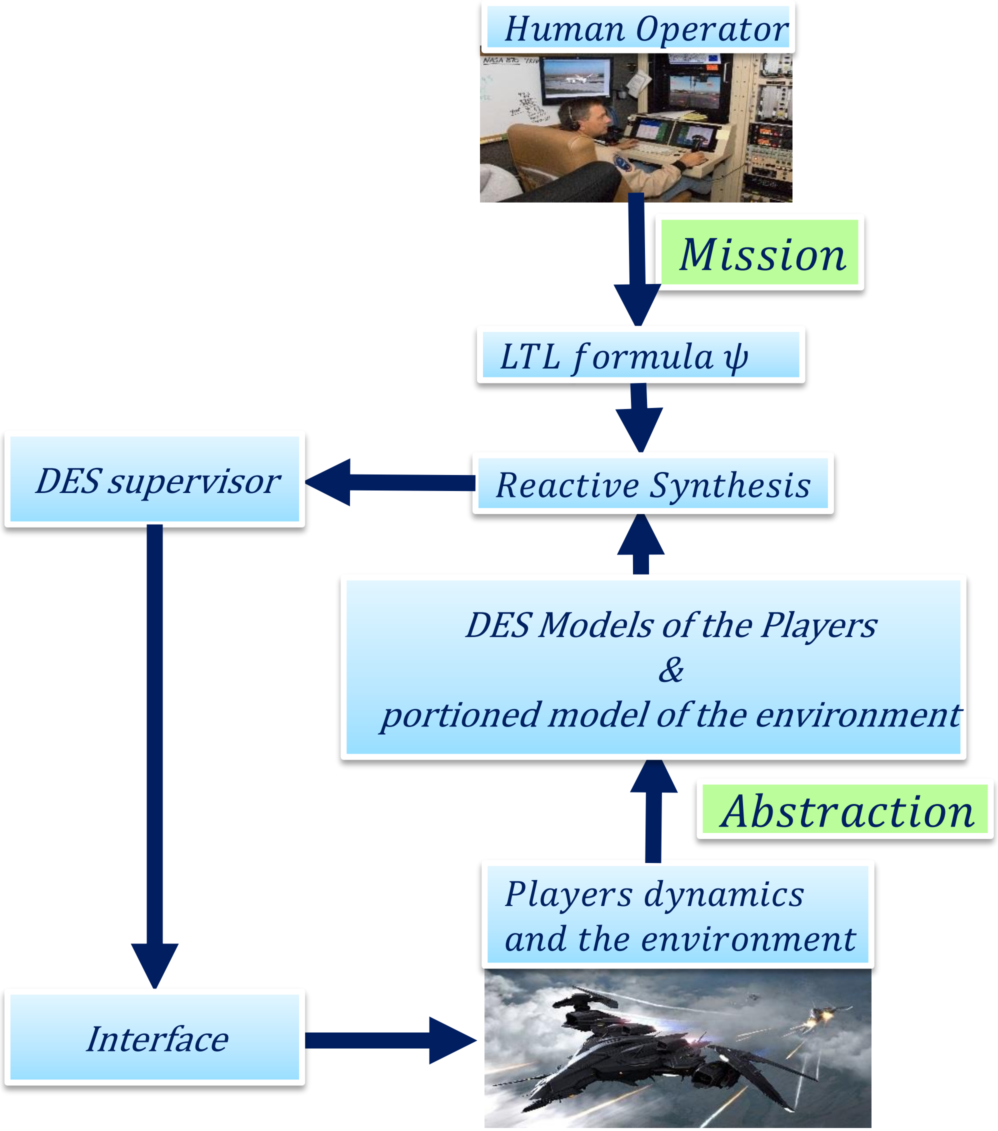

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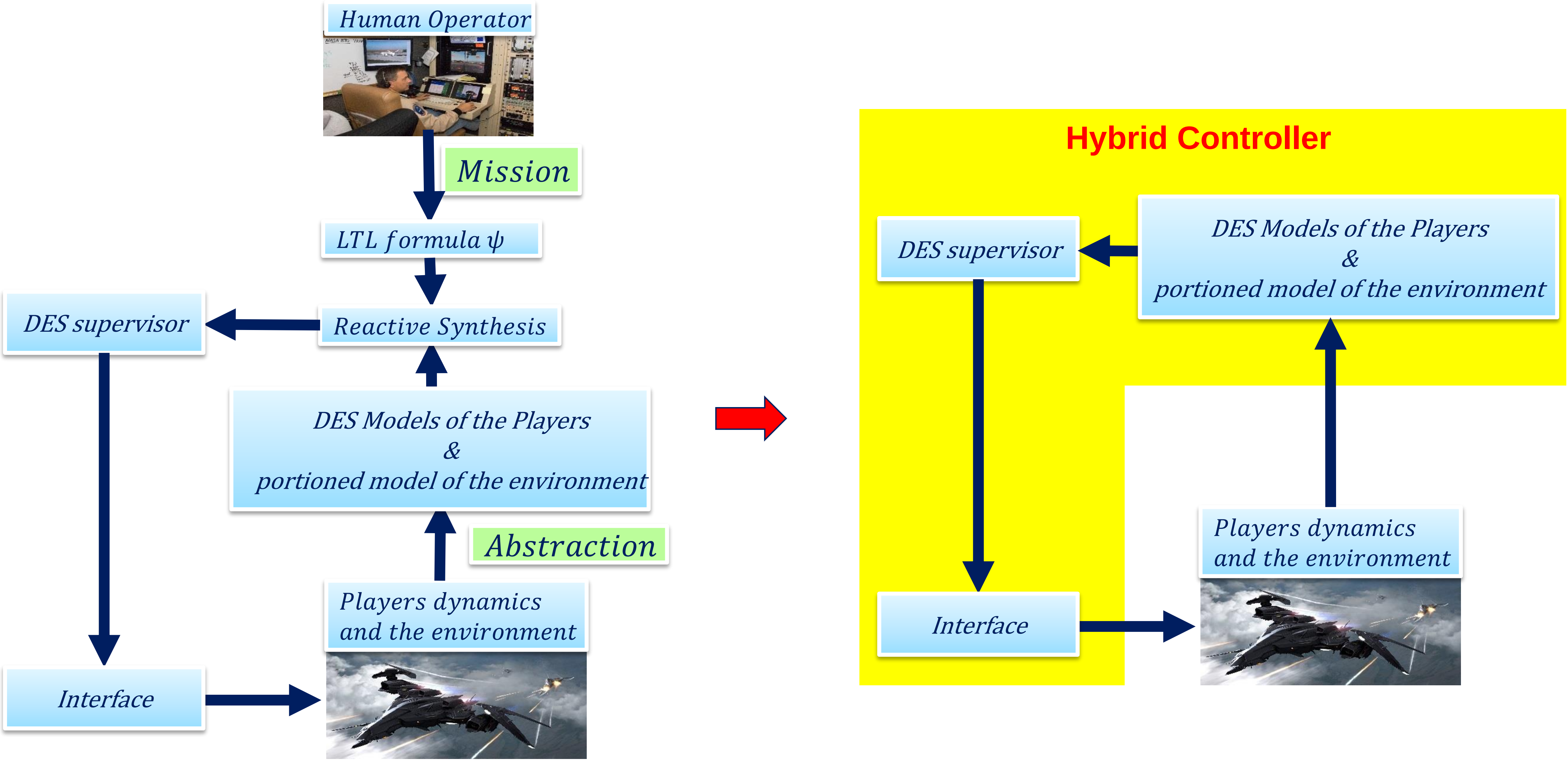

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**Hybrid Controller**

*DES supervisor*

*DES Models of the Players*
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*Players dynamics*
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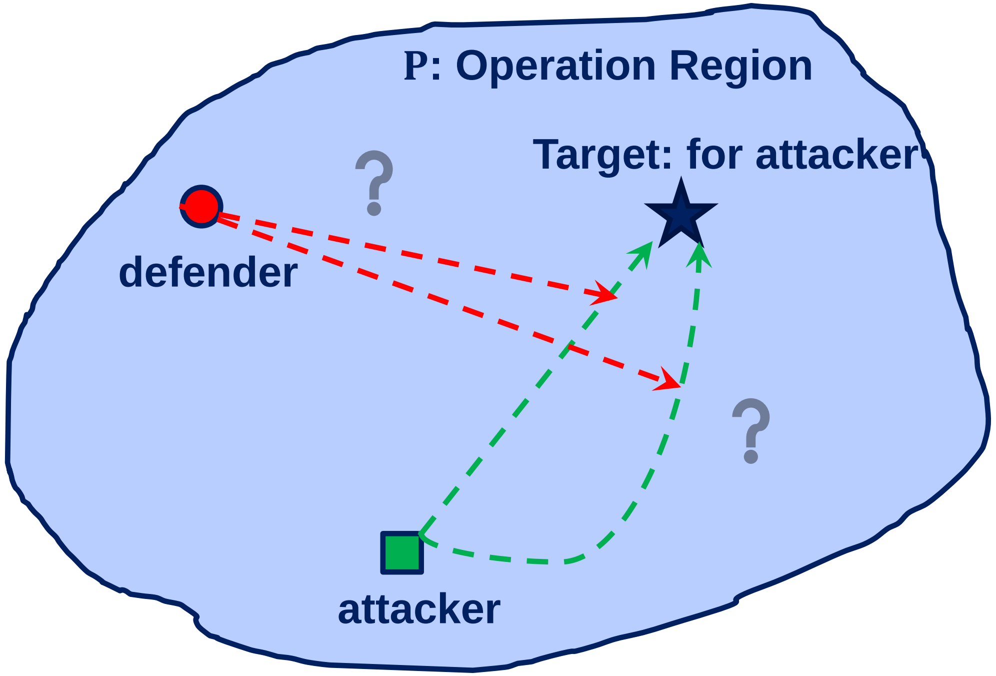
# Reach-avoid Problem Formulation



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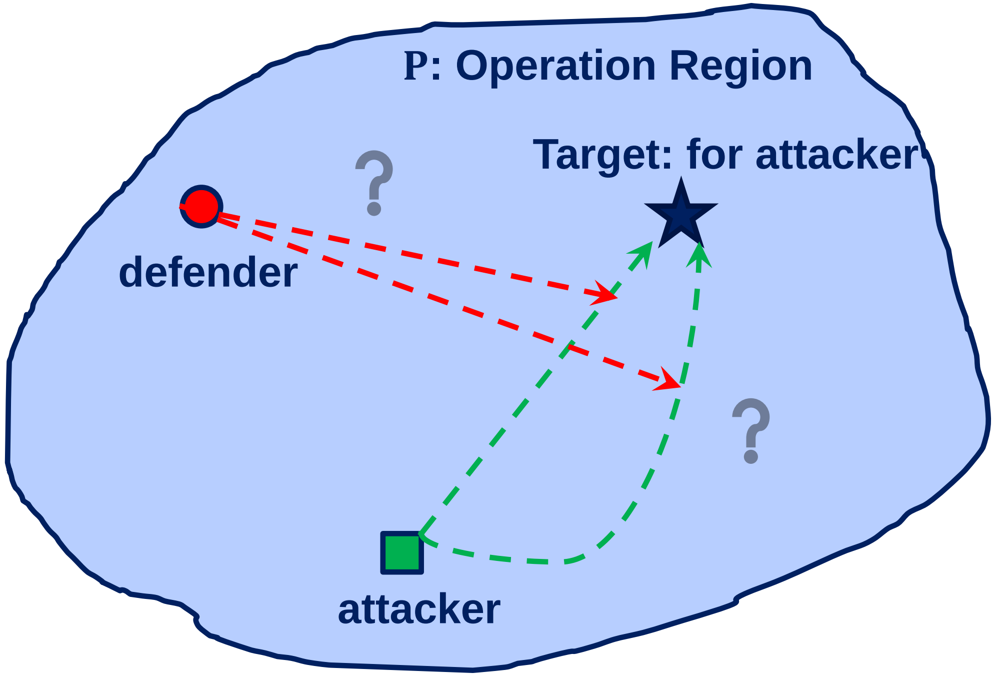

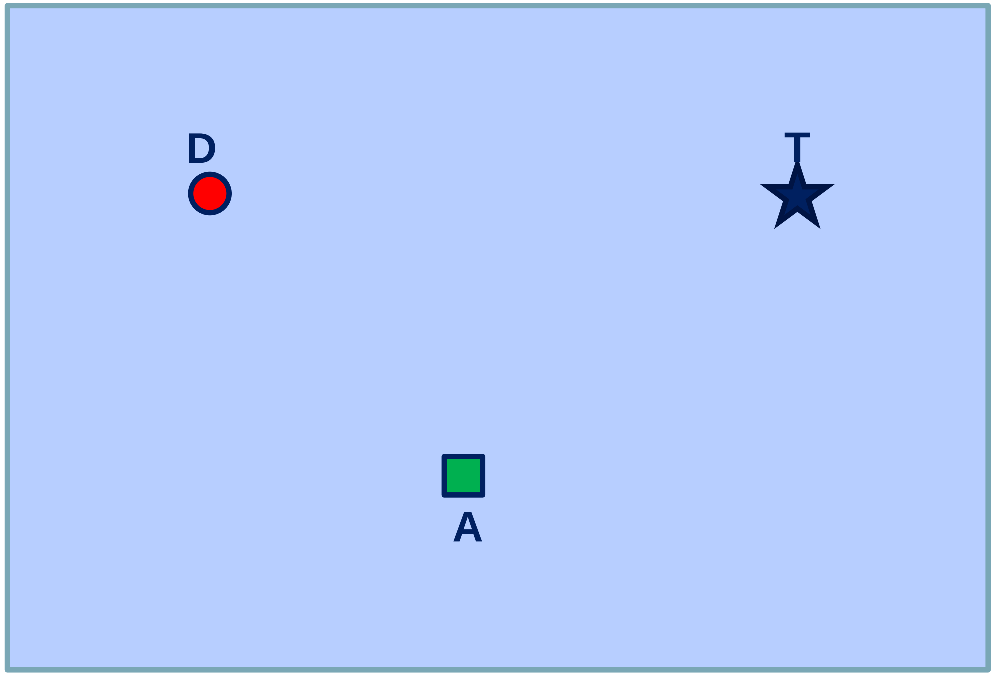

**Problem:**

For the above assumptions, design a controller to obtain trajectory $x(t) \in \mathrm{P} \in \mathbb{R}^2$ which satisfies the objective for the defender.

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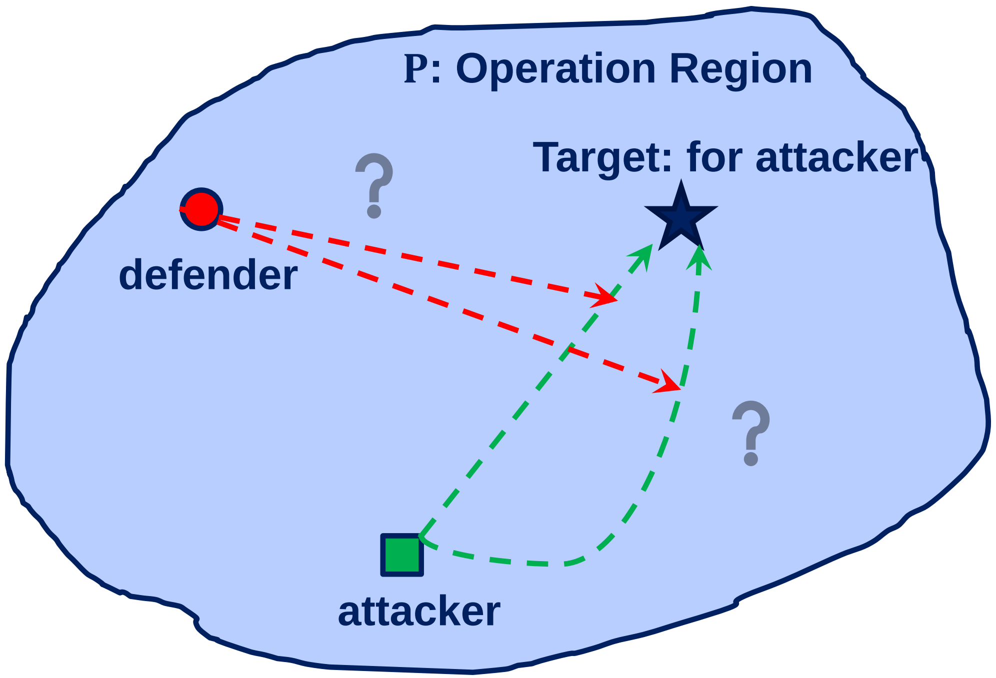

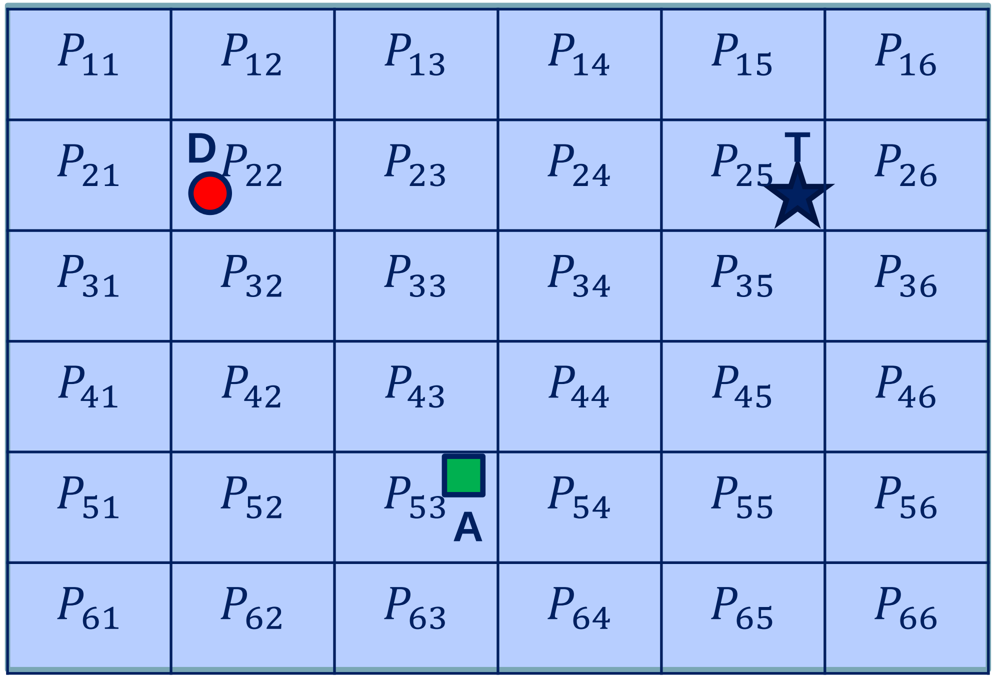

| | | | | | |
|---|---|---|---|---|---|
| $P_{11}$ | $P_{12}$ | $P_{13}$ | $P_{14}$ | $P_{15}$ | $P_{16}$ |
| $P_{21}$ | D $P_{22}$ | $P_{23}$ | $P_{24}$ | $P_{25}$ T | $P_{26}$ |
| $P_{31}$ | $P_{32}$ | $P_{33}$ | $P_{34}$ | $P_{35}$ | $P_{36}$ |
| $P_{41}$ | $P_{42}$ | $P_{43}$ | $P_{44}$ | $P_{45}$ | $P_{46}$ |
| $P_{51}$ | $P_{52}$ | $P_{53}$ A | $P_{54}$ | $P_{55}$ | $P_{56}$ |
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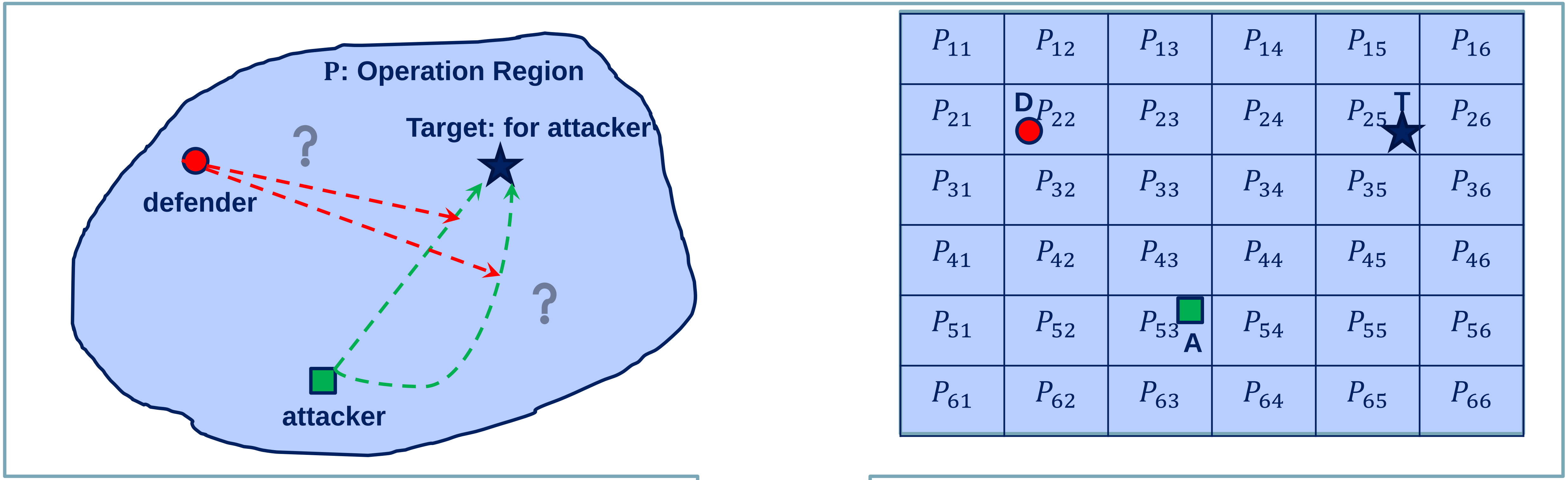

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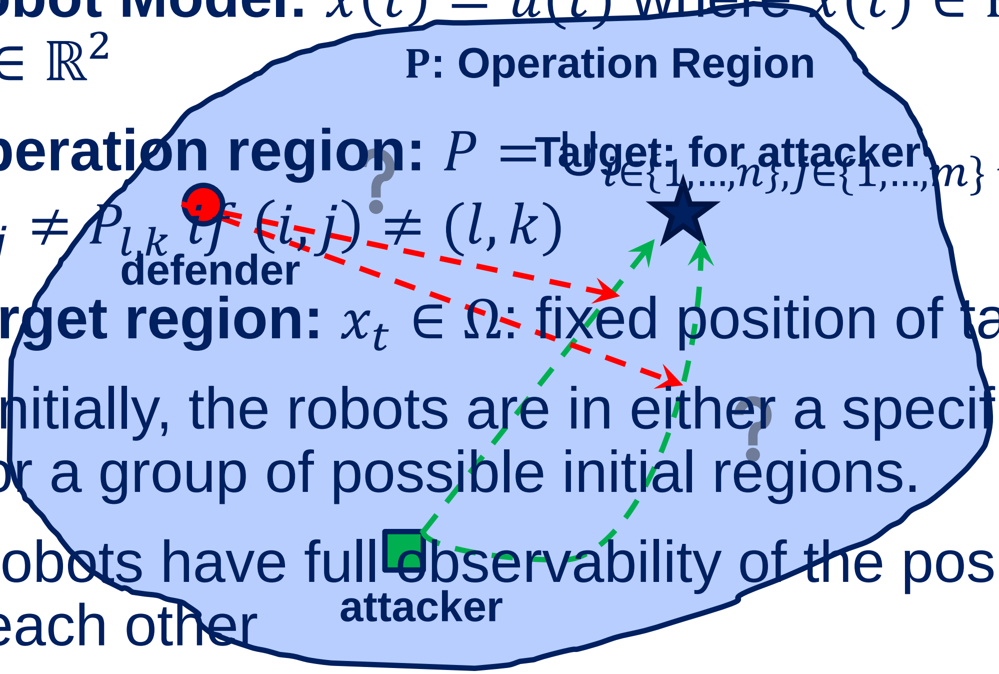
# Reach-avoid Problem Formulation

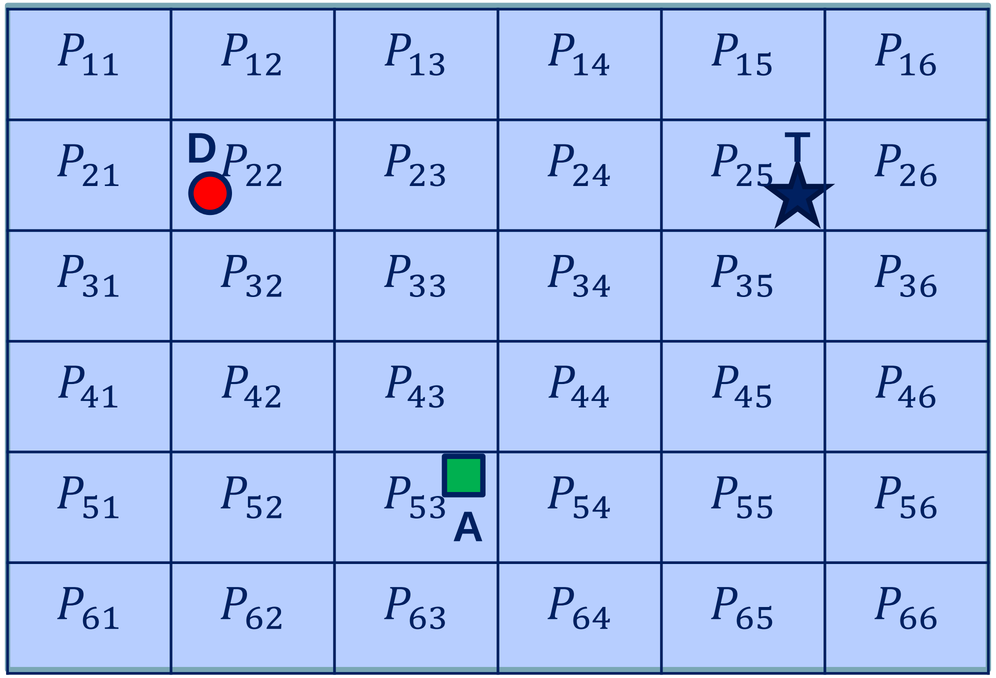
**Robot Model:** $\dot{x}(t) = u(t)$ where $x(t) \in \mathrm{P} \in \mathbb{R}^2, u \in U \in \mathbb{R}^2$

**Operation region:** $P = \bigcup_{i\in\{1,\dots,n\},j\in\{1,\dots,m\}} P_{i,j}$ and $P_{i,j} \neq P_{l,k}$ if $(i,j) \neq (l,k)$

**Target region:** $x_t \in \Omega$: fixed position of target

- Initially, the robots are in either a specific region or a group of possible initial regions.
- robots have full observability of the position of each other





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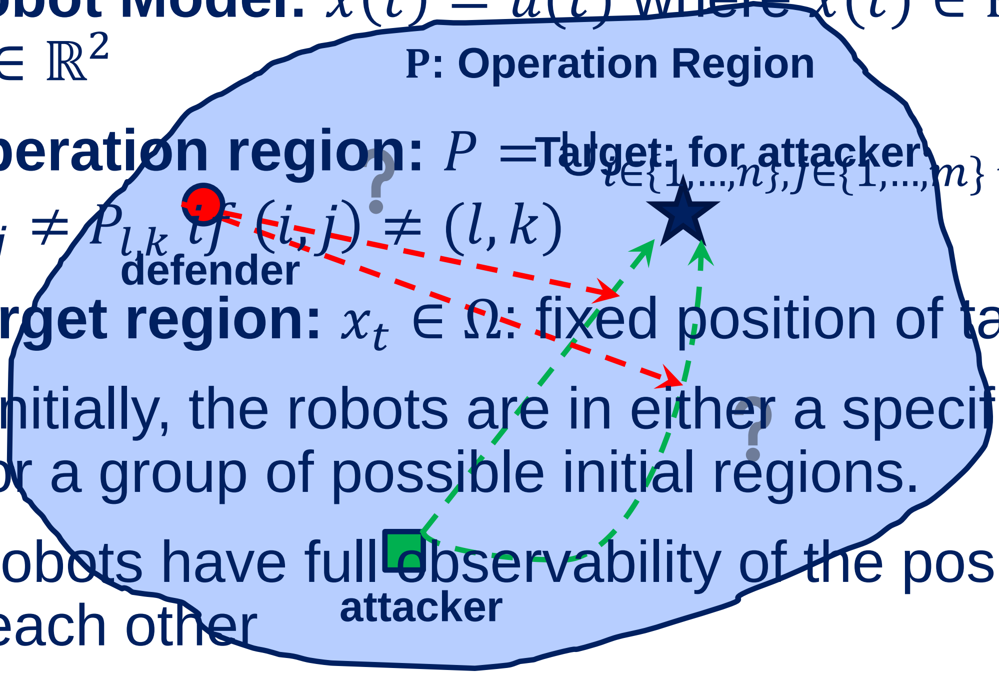

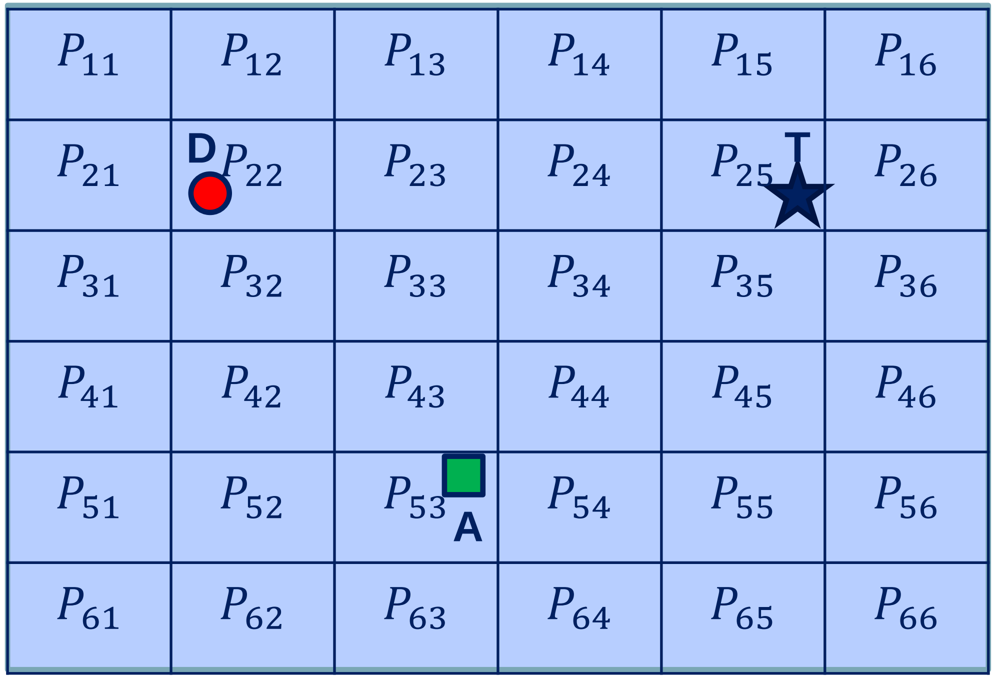

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# Symbolic planning: Specifying the tasks

The LTL formulas ($\varphi$) are constructed over ($\Sigma$) using Boolean operators and temporal operators.

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \bigcirc\varphi \mid \varphi\, \mathcal{U} \varphi$$

- $\boldsymbol{\Sigma}$ : A finite set of atomic proposition: $p \in \Sigma$ $(p\ can\ be\ either\ T\ or\ F)$
- **Boolean operators:** negation (¬), disjunction (∨), conjunction (∧), implication (→)
- **Modal temporal operators**: next ($O$), until ($\mathcal{U}$), eventually (◊) and always (□)

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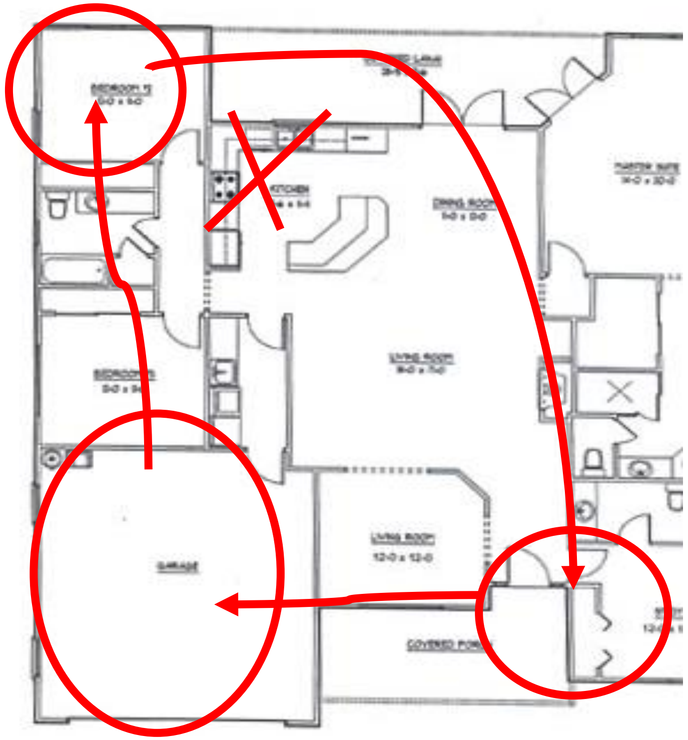
# Specifying the tasks: Why LTL?



- A formal high level language to describe a complex mission
- A wider class of properties than safety and stability
- Having well defined syntax and semantics, they can be easily used to specify complex behaviors

# Specifying the tasks: Specification: Reactive LTL formula

$$\boldsymbol{\varphi} = \boldsymbol{\varphi_a} \rightarrow \boldsymbol{\varphi_d}, \ \varphi_\alpha = \varphi_\alpha^i \wedge \varphi_\alpha^t \wedge \varphi_\alpha^r \wedge \varphi_\alpha^g, \ \alpha \in \{a, d\}$$

- $\boldsymbol{\varphi_a}$: all assumptions on the attacker
- $\boldsymbol{\varphi_d}$: all assumptions on the adversarial environment including defender and its desired behavior

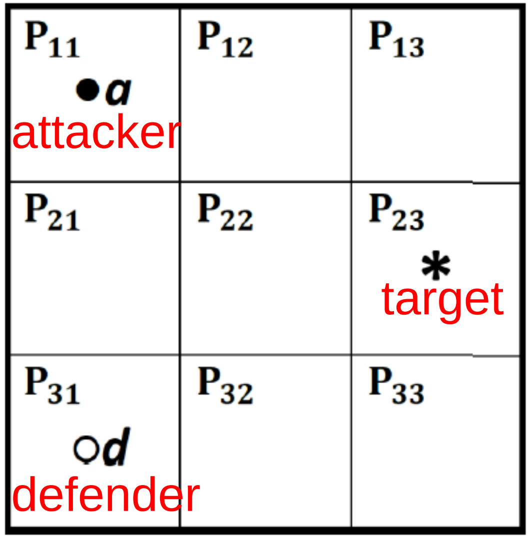
where:

- $\varphi_{\alpha\in\{a,d\}}^i$: initial positions of the attacker and the defender
- $\varphi_{\alpha\in\{a,d\}}^t$ : transition rules of robots over the partitioned space based on the information of the game at each step
- $\varphi_{\alpha\in\{a,d\}}^r$: the requirements that at each time only one robot proposition can be true as each robot cannot be in two regions at the same time.
- $\varphi_{\alpha\in\{a,d\}}^g$: the overall objective for the robots

# Players’ Decision-making

Objective function: $$L^k(u_a^{k+1}, u_d^{k+1}) = \begin{cases} \infty & \text{if } x_a^{k+1} = x_d^{k+1} \\ 0 & \text{if } x_a^{k+1} = x_t \\ \frac{\|x_a^{k+1} - x_t^{k+1}\|}{\|x_a^{k+1} - x_d^{k+1}\|} & \text{otherwise} \end{cases}$$

## Reach-avoid game configuration



## 2-player zero-sum game at first step

*attacker*

| | | Right | Down |
|---|---|---|---|
| *defender* | Right | 0.707 | 1.414 |
| | Up | 1 | $+\infty$ |

minmax{0.707; 1.414; 1;+∞} = 1
maxmin{0.707; 1.414; 1;+∞} = 1
The Nash equilibrium decision for this game is (Right,Up).

# Attacker’s Behavior

|  |  |  |
|---|---|---|
| $P_{11}$ | $P_{12}$ | $P_{13}$ |
| $P_{21}$ | $P_{22}$ | $P_{23}$ |
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$$\varphi_a = \varphi_i^a \wedge \varphi_t^a \wedge \varphi_r^a \wedge \varphi_g^a$$

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**Initial position of attacker**

$$\varphi_i^a = (a_{11} \wedge \neg a_{12} \wedge \cdots \wedge \neg a_{33})$$

**Requirements of position of attacker**

$$\begin{aligned}
&[(\bigcirc a_{11} \wedge \bigcirc\neg a_{12} \wedge \cdots \wedge \bigcirc\neg a_{33}) \\
&\quad (\bigcirc\neg a_{11} \wedge \bigcirc a_{12} \wedge \bigcirc\neg a_{13} \wedge \cdots \wedge \bigcirc\neg a_{33}) \\
&\quad \vdots \\
&\bigvee (\bigcirc\neg a_{11} \wedge \bigcirc\neg a_{12} \wedge \cdots \wedge \bigcirc a_{33})]
\end{aligned}$$

# Attacker’s Behavior

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**I**

$\varphi_i^a$

## Transition of attacker

$$\varphi_t^a = \varphi_{t,p}^a \wedge \varphi_{t,win}^a$$

possible transitions for the attacker

$$\varphi_{t,p}^a = \Box[(a_{11} \rightarrow (\bigcirc a_{11} \vee \bigcirc a_{12} \vee \bigcirc a_{21})) \bigwedge (a_{12} \rightarrow (\bigcirc a_{12} \vee \bigcirc a_{11} \vee \bigcirc a_{22} \vee \bigcirc a_{13}))$$

Terminating transitions

$$\varphi_{t,win}^a = \Box(a_{23} \rightarrow \bigcirc a_{23})$$

$$\bigvee (\bigcirc \neg a_{11} \wedge \bigcirc \neg a_{12} \wedge \cdots \wedge \bigcirc a_{33})]$$

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Terminating transitions

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Terminating transitions

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# Defender’s Behavior

$$\varphi_d = \varphi_i^d \wedge \varphi_t^d \wedge \varphi_r^d \wedge \varphi_g^d$$

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**Initial position of defender**

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**Initial position of defender**

$$\varphi_i^d = (\neg d_{11} \wedge \neg d_{12} \cdots \wedge \neg d_{23} \wedge d_{31} \wedge \neg d_{32} \wedge \neg d_{33})$$

# Defender’s Behavior

$$\varphi_d = \varphi_i^d \wedge \varphi_t^d \wedge \varphi_r^d \wedge \varphi_g^d$$

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## Transition of defender

possible transitions

Terminating transitions

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$\varphi_i^d =$

## Transition of defender

$$\varphi_t^d = \varphi_{t,p}^d \wedge \varphi_{t,win}^d$$

possible transitions

Terminating transitions

$$\varphi_{t,p}^d = \Box[$$

$$\bigwedge((d_{11} \wedge a_{12}) \rightarrow \bigcirc d_{12})$$

$$\bigwedge((d_{11} \wedge a_{31}) \rightarrow \bigcirc d_{21})$$

$$\bigwedge((d_{11} \wedge (a_{13} \vee a_{21} \vee a_{22} \vee a_{32} \vee a_{33})) \rightarrow (\bigcirc d_{21} \vee \bigcirc d_{21}))$$

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# Defender’s Behavior

$$\varphi_d = \varphi_i^d \wedge \varphi_t^d \wedge \varphi_r^d \wedge \varphi_g^d$$

$\varphi_i^d =$

## Transition of defender

$$\varphi_t^d = \varphi_{t,p}^d \wedge \varphi_{t,win}^d$$

possible transitions

$$\varphi_{t,p}^d = \Box[$$
$$\bigwedge((d_{11} \wedge a_{12}) \rightarrow \bigcirc d_{12})$$
$$\bigwedge((d_{11} \wedge a_{31}) \rightarrow \bigcirc d_{21})$$
$$\bigwedge((d_{11} \wedge (a_{13} \vee a_{21} \vee a_{22} \vee a_{32} \vee a_{33})) \rightarrow (\bigcirc d_{21} \vee \bigcirc d_{21}))$$

Terminating transitions

$$\varphi_{t,win}^d = \Box[((d_{11} \wedge a_{11}) \rightarrow \bigcirc d_{11})$$
$$\bigwedge((d_{12} \wedge a_{12}) \rightarrow \bigcirc d_{12})$$
$$\bigwedge((d_{13} \wedge a_{13}) \rightarrow \bigcirc d_{13})$$
$$\bigwedge((d_{21} \wedge a_{21}) \rightarrow \bigcirc d_{21})$$
$$\bigwedge((d_{22} \wedge a_{22}) \rightarrow \bigcirc d_{22})$$
$$\bigwedge((d_{31} \wedge a_{31}) \rightarrow \bigcirc d_{31})$$
$$\bigwedge((d_{32} \wedge a_{32}) \rightarrow \bigcirc d_{32})$$
$$\bigwedge((d_{33} \wedge a_{33}) \rightarrow \bigcirc d_{12})]$$

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possible transitions

$$\begin{aligned}
&\varphi_{t,p}^d = \Box[\\
&\bigwedge((d_{11} \wedge a_{12}) \to \bigcirc d_{12})\\
&\bigwedge((d_{11} \wedge a_{31}) \to \bigcirc d_{21})\\
&\bigwedge((d_{11} \wedge (a_{13} \vee a_{21} \vee a_{22} \vee a_{32} \vee a_{33})) \to (\bigcirc d_{21} \vee \bigcirc d_{21}))
\end{aligned}$$

Terminating transitions

$$\begin{aligned}
\varphi_{t,win}^d = \Box[&((d_{11} \wedge a_{11}) \to \bigcirc d_{11})\\
&\bigwedge((d_{12} \wedge a_{12}) \to \bigcirc d_{12})\\
&\bigwedge((d_{13} \wedge a_{13}) \to \bigcirc d_{13})\\
&\bigwedge((d_{21} \wedge a_{21}) \to \bigcirc d_{21})\\
&\bigwedge((d_{22} \wedge a_{22}) \to \bigcirc d_{22})\\
&\bigwedge((d_{31} \wedge a_{31}) \to \bigcirc d_{31})\\
&\bigwedge((d_{32} \wedge a_{32}) \to \bigcirc d_{32})\\
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\end{aligned}$$

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$\bigwedge((d_{11} \wedge a_{12}) \rightarrow \bigcirc d_{12})$

$\bigwedge((d_{11} \wedge a_{31}) \rightarrow \bigcirc d_{21})$

$\bigwedge((d_{11} \wedge (a_{13} \vee a_{21} \vee a_{22} \vee$

## Requirements of position of defender

$$\varphi_{t,s}^d = \Box[(\bigcirc d_{11} \wedge \bigcirc\neg d_{12} \wedge \cdots \wedge \bigcirc\neg d_{33})$$
$$\bigvee(\bigcirc\neg d_{11} \wedge \bigcirc d_{12} \wedge \bigcirc\neg d_{13} \wedge \cdots \wedge \bigcirc\neg d_{33})$$
$$\bigvee \quad \vdots$$
$$\bigvee(\bigcirc\neg d_{11} \wedge \bigcirc\neg d_{12} \wedge \cdots \wedge \bigcirc d_{33})]$$

ons

$\rightarrow \bigcirc d_{11})$

$\rightarrow \bigcirc d_{12})$

$\rightarrow \bigcirc d_{13})$

$\rightarrow \bigcirc d_{21})$

$\bigwedge((d_{22} \wedge a_{22}) \rightarrow \bigcirc d_{22})$

$\bigwedge((d_{31} \wedge a_{31}) \rightarrow \bigcirc d_{31})$

$\bigwedge((d_{32} \wedge a_{32}) \rightarrow \bigcirc d_{32})$

$\bigwedge((d_{33} \wedge a_{33}) \rightarrow \bigcirc d_{12})]$

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possible transitions

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$\bigwedge((d_{11} \wedge a_{12}) \rightarrow \bigcirc d_{12})$

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## Requirements of position of defender

$$\varphi_{t,s}^d = \Box[(\bigcirc d_{11} \wedge \bigcirc\neg d_{12} \wedge \cdots \wedge \bigcirc\neg d_{33})$$
$$\bigvee(\bigcirc\neg d_{11} \wedge \bigcirc d_{12} \wedge \bigcirc\neg d_{13} \wedge \cdots \wedge \bigcirc\neg d_{33})$$
$$\bigvee \quad \vdots$$
$$\bigvee(\bigcirc\neg d_{11} \wedge \bigcirc\neg d_{12} \wedge \cdots \wedge \bigcirc d_{33})]$$

ons

$\rightarrow \bigcirc d_{11})$

$\rightarrow \bigcirc d_{12})$

$\rightarrow \bigcirc d_{13})$

$\rightarrow \bigcirc d_{21})$

$\bigwedge((d_{22} \wedge a_{22}) \rightarrow \bigcirc d_{22})$

$\bigwedge((d_{31} \wedge a_{31}) \rightarrow \bigcirc d_{31})$

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possible transitions

$\varphi_{t,p}^d = \square[$

$\bigwedge((d_{11} \wedge a_{12}) \rightarrow \bigcirc d_{12})$

$\bigwedge((d_{11} \wedge a_{31}) \rightarrow \bigcirc d_{21})$

$\bigwedge((d_{11} \wedge (a_{13} \vee a_{21} \vee a_{22} \vee$

R

$\varphi_{t,s}^d =$

## Overall goal of defender

$\rightarrow \bigcirc d_{21})$

$\bigwedge((d_{22} \wedge a_{22}) \rightarrow \bigcirc d_{22})$

$\bigwedge((d_{31} \wedge a_{31}) \rightarrow \bigcirc d_{31})$

$\bigwedge((d_{32} \wedge a_{32}) \rightarrow \bigcirc d_{32})$

$\bigwedge((d_{33} \wedge a_{33}) \rightarrow \bigcirc d_{12})]$

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$$\varphi_d = \varphi_i^d \wedge \varphi_t^d \wedge \varphi_r^d \wedge \varphi_g^d$$

$\varphi_i^d =$

possible transitions

$\varphi_{t,p}^d = \Box[$

$\bigwedge((d_{11} \wedge a_{12}) \rightarrow \bigcirc d_{12})$

$\bigwedge((d_{11} \wedge a_{31}) \rightarrow \bigcirc d_{21})$

$\bigwedge((d_{11} \wedge (a_{13} \vee a_{21} \vee a_{22} \vee$

**Re**

$\varphi_{t,s}^d =$

## Overall goal of defender

$$\varphi_g^d = \Box\Diamond[(a_{11} \wedge d_{11}) \vee (a_{12} \wedge d_{12}) \vee (a_{13} \wedge d_{13}) \vee (a_{21} \wedge d_{21}) \vee (a_{22} \wedge d_{22}) \vee (a_{31} \wedge d_{31}) \vee (a_{32} \wedge d_{32}) \vee (a_{33} \wedge d_{33})]$$

$\rightarrow \bigcirc d_{21})$

$\bigwedge((d_{22} \wedge a_{22}) \rightarrow \bigcirc d_{22})$

$\bigwedge((d_{31} \wedge a_{31}) \rightarrow \bigcirc d_{31})$

$\bigwedge((d_{32} \wedge a_{32}) \rightarrow \bigcirc d_{32})$

$\bigwedge((d_{33} \wedge a_{33}) \rightarrow \bigcirc d_{12})]$

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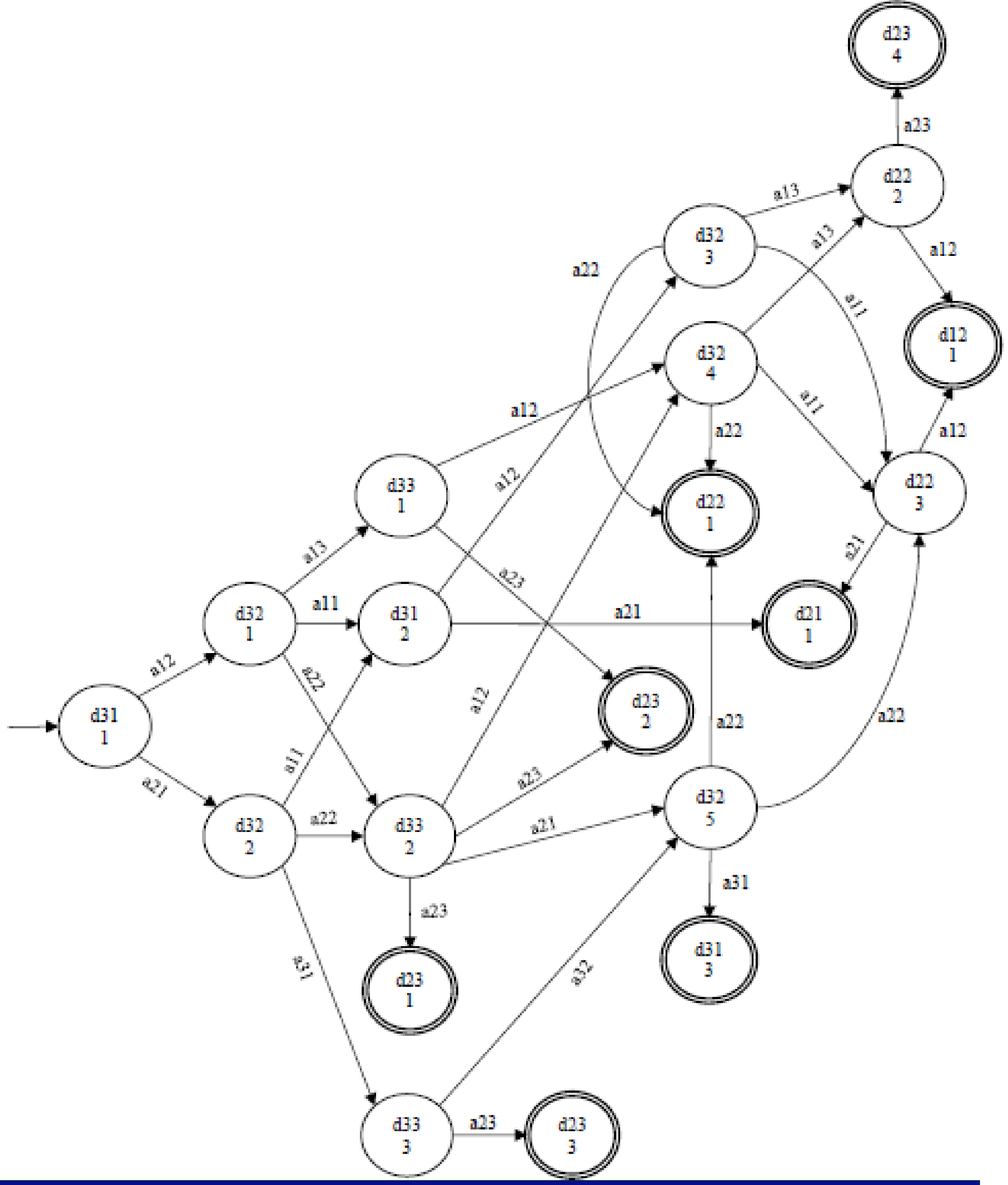
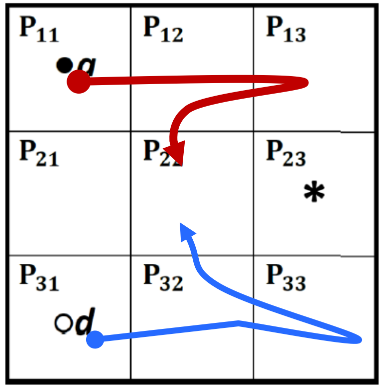
# Synthesizing the DES supervisor

$$G = (R, r_0, \mathcal{A}, \mathcal{D}, \delta, h)$$

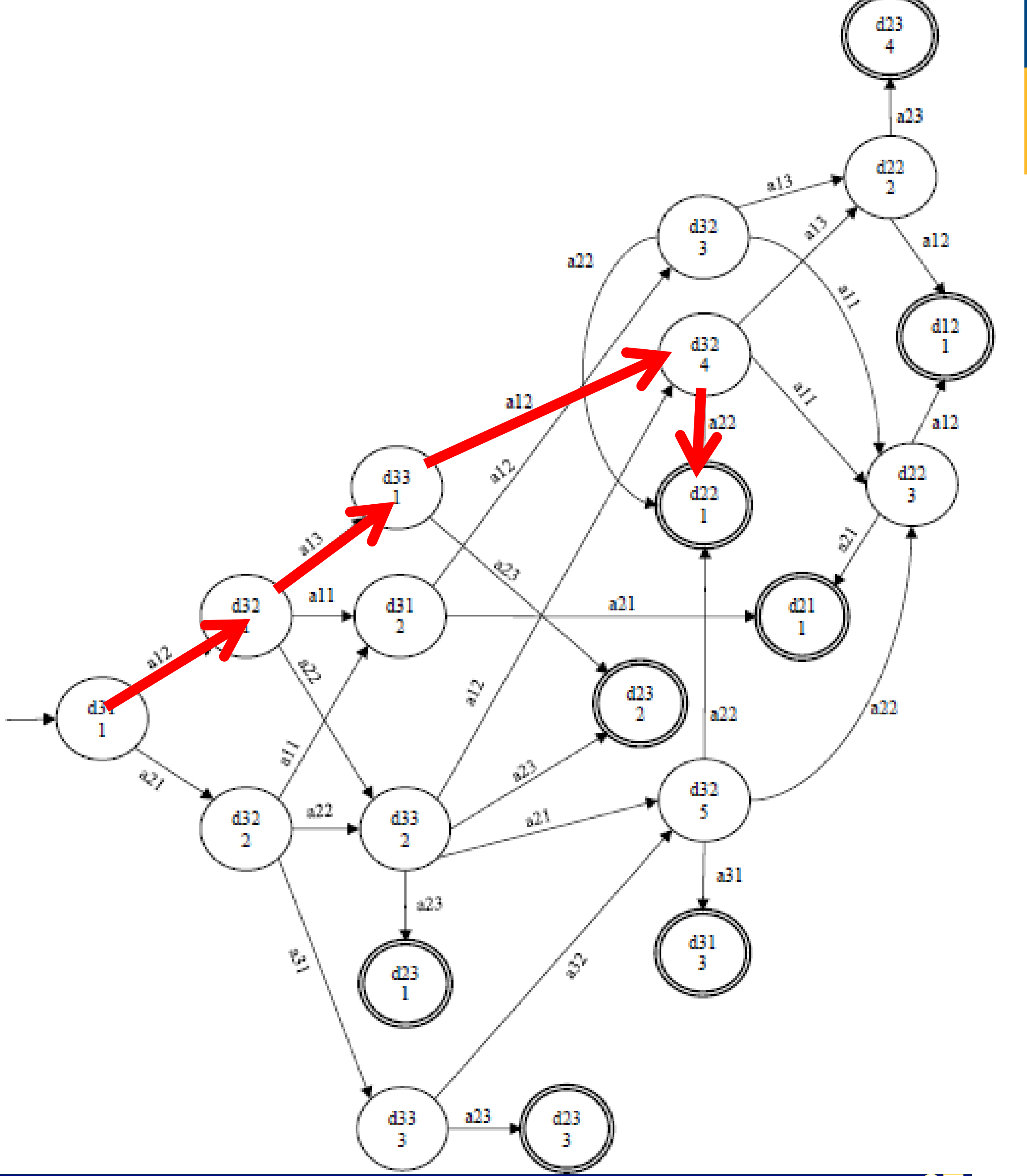
$R$ is the set of states,
$r_0$ is the initial state,
$\mathcal{A}$ is the set of input (attacker) propositions,
$\mathcal{D}$ is the set of output (defender) propositions,
$\delta : R \times 2^{\mathcal{A}} \rightarrow 2^{R}$ is the transition relation,
$h : Q \rightarrow 2^{\mathcal{D}}$ is the output function.



# Example



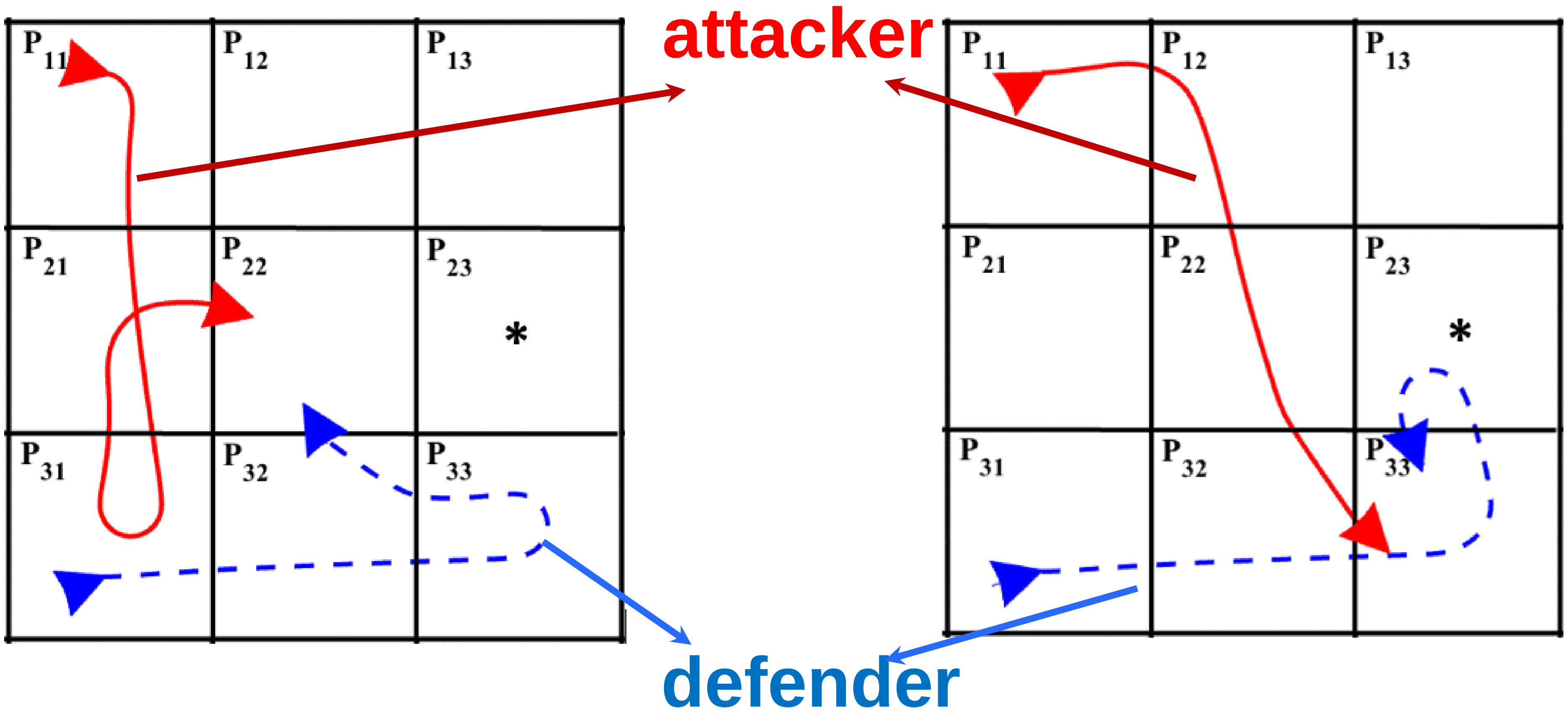
- If the attacker is in region $a_{11}$ and its decision is to transit to regions $a_{12}, a_{13}, a_{12}, a_{22}, a_{23}$, based on the automaton G, the defender will visit the regions $d_{31}, d_{32}, d_{33}, d_{32}, d_{22}$, and wins the game.



# Synthesizing the hybrid controller

- The generated discrete commands $d_{31}, d_{32}, d_{33}, d_{32}, d_{22}$ then need to be translated to continuous signals to be tracked by the lower levels of the control structure of the defender.
- For this purpose, for each of the transitions moving up, down, left, right or staying at the current region, we have designed a continuous controller that tune the vector fields over the partitioned region, so that starting at any point in any region, the planner can exit from the desired edge of the region.
- Due to the bisimulation relation between the original system and its abstracted partitioned model, we can be guarantee that the generated continues signals preserves the properties of the discrete paths.

# Simulation



attacker

defender

# Conclusion and Future Work

- We proposed a novel, formal, computationally effective symbolic framework to construct hybrid controllers for autonomous vehicles involved in reach-avoid scenarios with a dynamic and adversarial nature.
- In contrast to earlier approaches, this methodology considers less restrictions on the robot motion and assumes no knowledge about the model of the opponent is available.
- Moreover, due to the abstract nature of the proposed symbolic approach, it has less computational cost.
- The reactive nature of the proposed framework allows the defender to automatically react to the attacker's actions and make best decisions.
- Our future work include the extension of the proposed framework to more complex scenarios and environments, for example reach-avoid problem with more number of players.

# Acknowledgements

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http://accesslab.net/

http://techlav.ncat.edu/

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# Q&A

## A Symbolic Motion Planning Approach for the Reach-avoid Problem

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