Distributed Randomized Optimization for Large Scale Interconnected Systems

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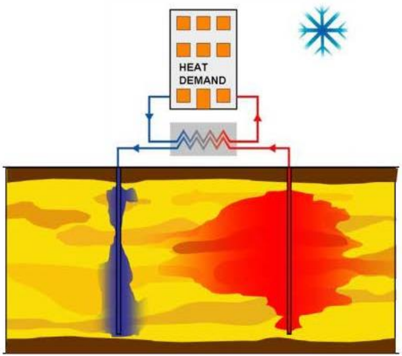
Aquifer Thermal Energy Storage (ATES)

- A large-scale natural subsurface storage for thermal energy
- An innovative method for thermal energy balance in smart grids

Cold season:

- The building requests thermal energy for the heating purpose
- Water is injected into cold well and is taken from warm well
- The stored water contains cold thermal energy for next season



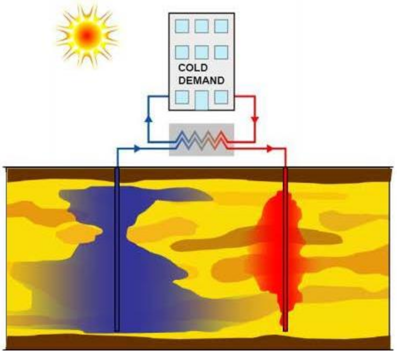
[Rostampour et al., JEP, 2016]

Aquifer Thermal Energy Storage (ATES)

- A large-scale natural subsurface storage for thermal energy
- An innovative method for thermal energy balance in smart grids

Warm season:

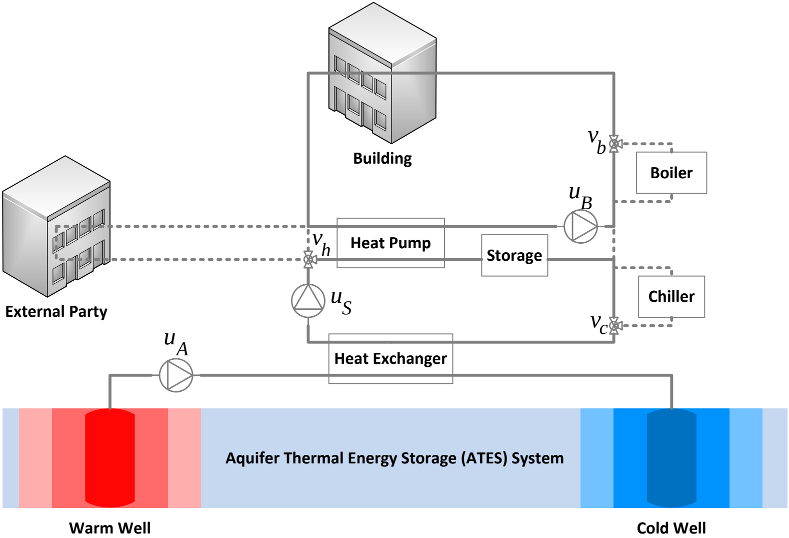
- The building requests thermal energy for the cooling purpose
- Water is injected into warm well and is taken from cold well
- The stored water contains warm thermal energy for next season



[Rostampour et al., JEP, 2016]

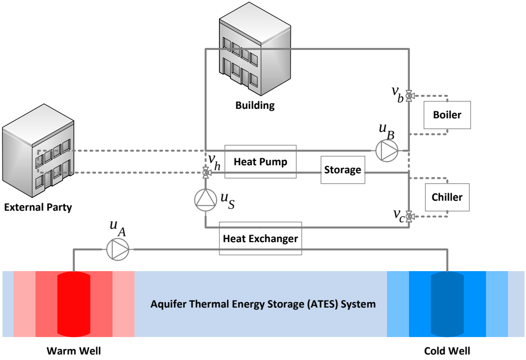
How to Deal with ATES Systems in Smart Thermal Grids?

Single Building with ATES System



[Rostampour et al., EGC, 2016], [Rostampour et al., HPC, 2017]

Single Building with ATES System

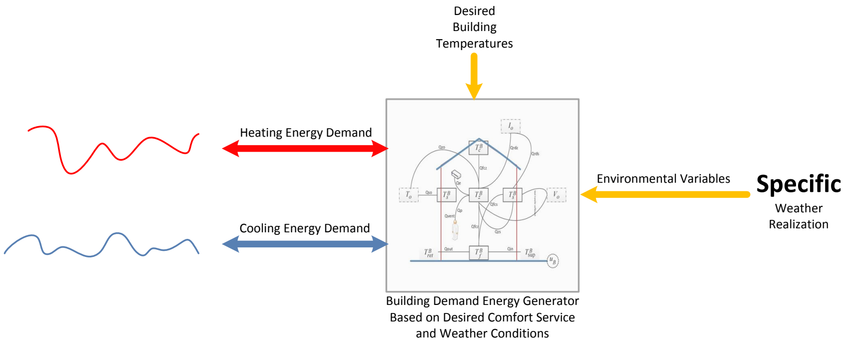


- A complete model of building thermal system integrated with ATES system
- Simulation results for more than a month

- Nonconvex problem formulation
- Computationally intractable optimization problem for long simulation study
- Infeasible setup for the network of ATES systems

[Rostampour et al., EGC, 2016], [Rostampour et al., HPC, 2017]

Single Building Thermal Energy Demand

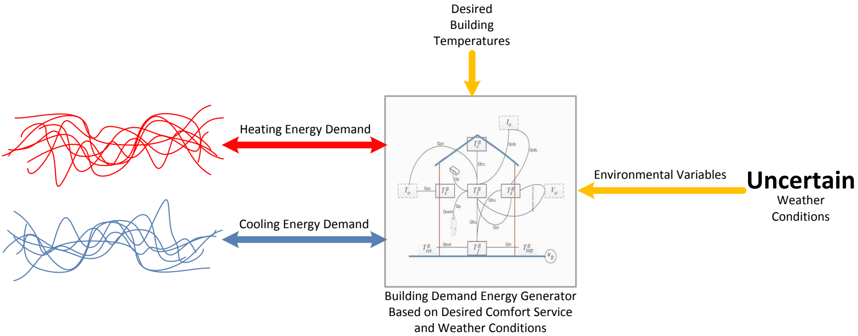


Thermal Energy Demand Profile:

- Complete and detailed building dynamical model
- Desired building temperatures (local controller unit)
- In specific weather realization, certain demand profiles are generated

[Rostampour & Keviczky, ECC, 2016]

Single Building Thermal Energy Demand

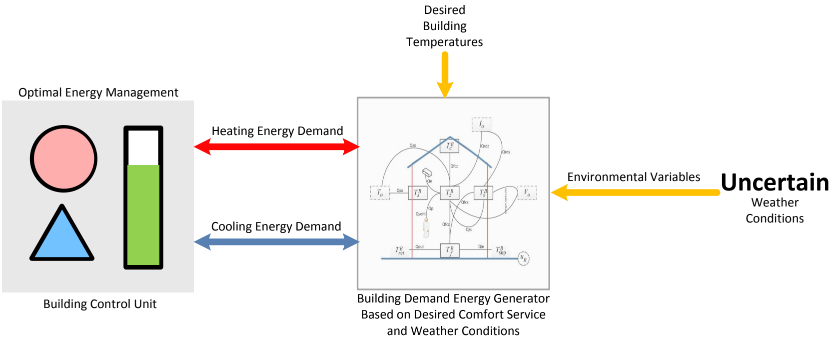


Thermal Energy Demand Profile:

- Complete and detailed building dynamical model
- Desired building temperatures (local controller unit)
- In uncertain conditions, uncertain demand profiles are generated

[Rostampour & Keviczky, ECC, 2016]

Single Building Climate Comfort System



Building Control Unit:

- Main components: Boiler, HP, HE, micro-CHP, Storage Tank
- **ON/OFF status** together with **production schedule** as decisions
- **Control Objective:** thermal **energy balance** for the overall systems

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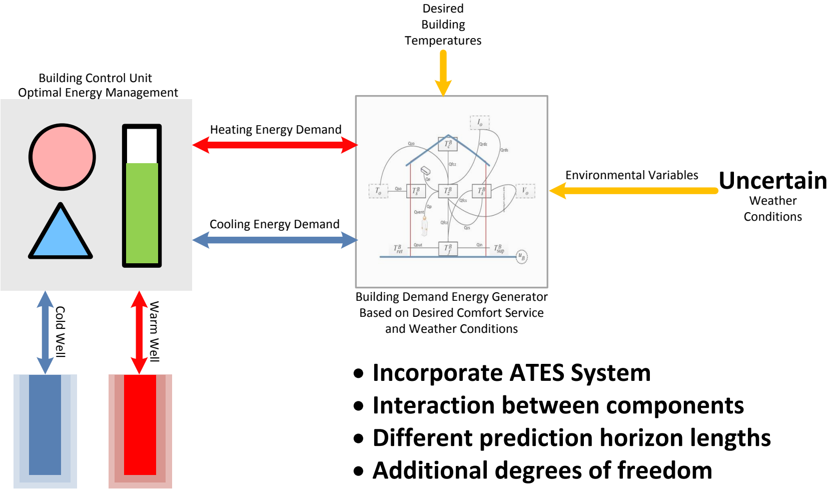
[Rostampour & Keviczky, ECC, 2016]

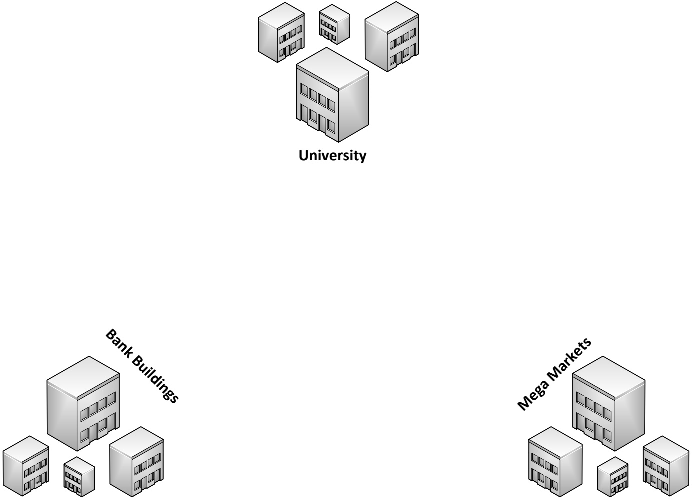
Building Climate Comfort with ATES Systems



- **Incorporate ATES System**
- **Interaction between components**
- **Different prediction horizon lengths**
- **Additional degrees of freedom**

[Rostampour & Keviczky, IFAC, 2017]

Smart Thermal Grids with ATES Systems



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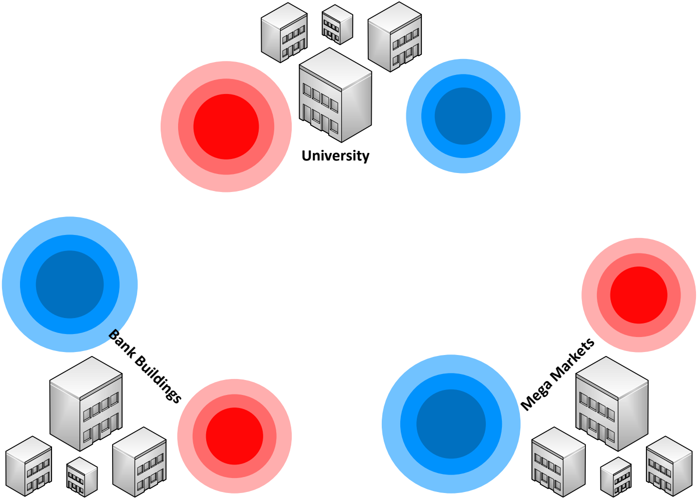
Smart Thermal Grids with ATES Systems



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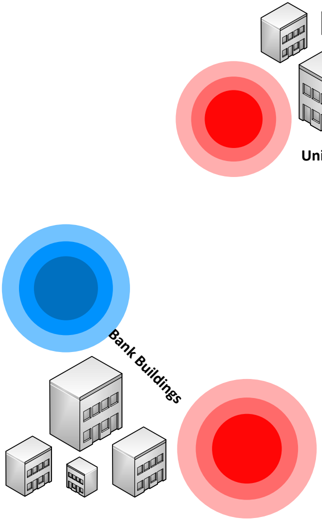

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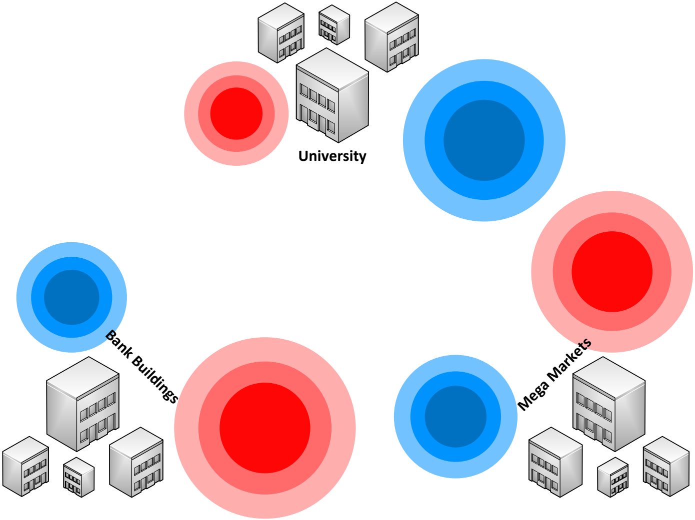

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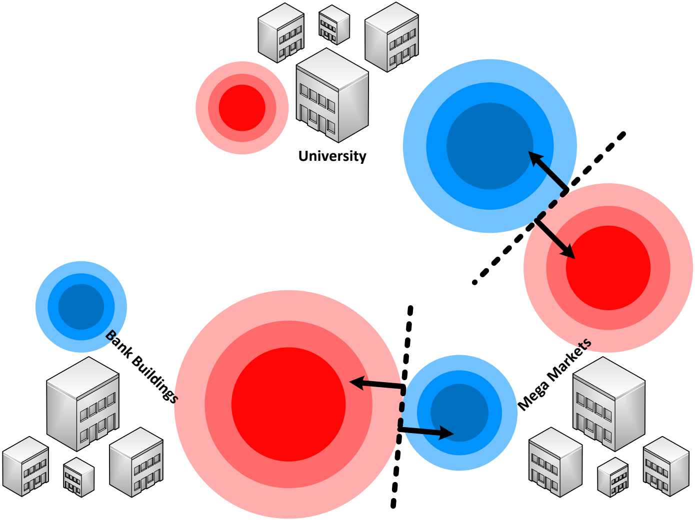
Smart Thermal Grids with ATES Systems



[Rostampour & Keviczky, IFAC, 2017]

Achievements & Developments

[Rostampour & Keviczky, IFAC, 2017]

Research Motivation

Challenge: Optimizing the performance of a network ...

1. Computation: Problem size is too large!
2. Communication: Communication bandwidth limitation
3. Information Privacy: Agents may not want to share information
4. Stochastic Nature:
    - Agents private uncertainty source (local); uncertain thermal energy demand of a single building climate comfort
    - Agents common uncertainty source (shared); uncertain common resource pool, e.g., ATES systems

Why Distributed?

1. Scalable Methodology
   - Communication: Only between neighbors
   - Computation: Only local; in parallel for all agents
2. Preserving Privacy
   - Agents do not reveal information about their preferences (encoded by objective and constraint functions) to each other
3. Numerous Applications
   - Wireless Networks
   - Electric Vehicle Charging Control
   - Optimal Power Flow with Reserve Scheduling *
   - Energy Management in STGs with ATES Systems †

* [Rostampour et. al., 2017]

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Outline

1. Centralized Framework
2. Distributed Framework
3. Soft Communication Scheme
4. Case Study: STGs with ATES Systems
5. Conclusions and Future work

Problem Setup

Centralized Deterministic Program

$$\min_{x} \quad \sum_{i} f_i(x) \qquad \longrightarrow \qquad f_i(\cdot) \ : \ \text{objective function of agent } i$$

$$\text{s.t.} \quad x \in \mathcal{X}_i \ , \ \text{for all } i \qquad \longrightarrow \qquad \mathcal{X}_i \ : \ \text{constraint set of agent } i$$

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- How one can deal with uncertain $\mathcal{X}_i(\delta)$?

Problem Setup

Centralized Robust Program

$$\min_{x} \quad \sum_{i} f_i(x)$$

$$\text{s.t.} \quad x \in \bigcap_{i} \mathcal{X}_i(\delta) \; , \; \text{for all } \delta \in \Delta$$

Problem Setup

Centralized Robust Program

$$\min_{x} \quad \sum_i f_i(x)$$

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Stochastic Setting:

- δ : Uncertain parameter $\delta \sim \mathbb{P}$
- Δ : Possibly unknown distribution and unbounded set
- Semi-infinite optimization problem

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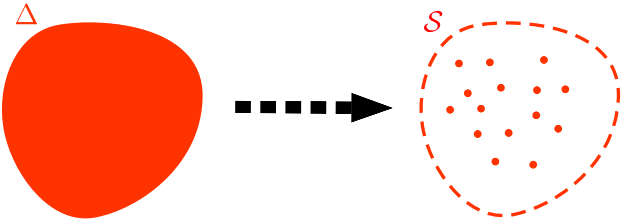
Scenario Based Approximation

Centralized Scenario Program

$$\min_x \quad \sum_i f_i(x)$$
$$\text{s.t.} \quad x \in \bigcap_i \bigcap_{\delta \in \Delta} \mathcal{X}_i(\delta)$$

Replace Δ with $\mathcal{S}$:



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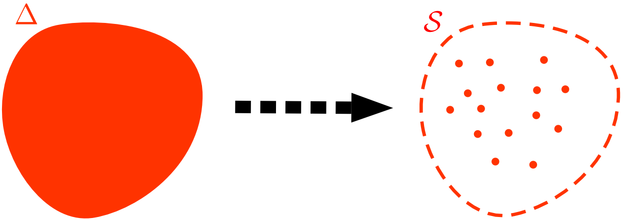

Scenario Based Approximation

Probabilistic Feasibility Certificate

Centralized Scenario Program $\mathcal{P}_{\mathcal{S}}$

$$\min_x \quad \sum_i f_i(x)$$
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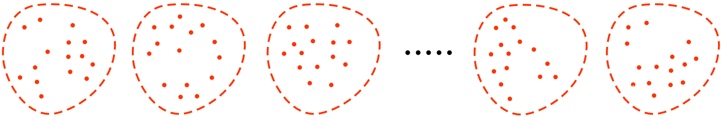
Centralized Stochastic Program $\mathcal{P}_{\Delta}$

$$\min_x \quad \sum_i f_i(x)$$
$$\text{s.t.} \quad \mathbb{P}\Big(\delta \in \Delta \, : \, x \notin \bigcap_i \mathcal{X}_i(\delta)\Big) \leq \varepsilon$$

- Is $x^*_{\mathcal{S}} \vDash \mathcal{P}_{\mathcal{S}}$ feasible for $\mathcal{P}_{\Delta}$?
- Is this true for any $\mathcal{S}$?



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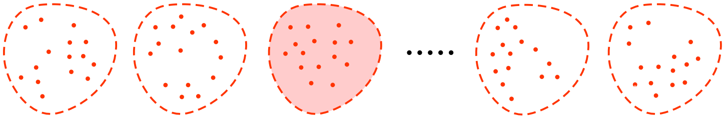

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Probabilistic Feasibility [Calafiore & Campi, TAC 2006]

Fix $\beta \in (0,1)$ and $\mathcal{S}$, then

$$\mathbb{P}^{|\mathcal{S}|}\left(\mathcal{S} \in \Delta^{|\mathcal{S}|} \,:\, \mathbb{P}\Big(\delta \in \Delta \,:\, x^*_{\mathcal{S}} \notin \bigcap_i \mathcal{X}_i(\delta)\Big) \leq \varepsilon(d, |\mathcal{S}|, \beta)\right) \geq 1 - \beta$$

Probabilistic Feasibility

Fix $\beta \in (0,1)$ and $\mathcal{S}$, then

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Complexity of $\varepsilon(d, |\mathcal{S}|, \beta)$:

- Logarithmic in β: β can be set close to "zero"
- Linear in $|\mathcal{S}|^{-1}$: the more data the better the result
- Linear in d: number of samples from $\mathcal{S}$ which "support" the solution, i.e. would leave it unchanged (# decision variables)

Outline

1 Centralized Framework

2 Distributed Framework

3 Soft Communication Scheme

4 Case Study: STGs with ATES Systems

5 Conclusions and Future work

Distributed Framework

There are two possible cases:

1. Private (local) uncertainty source: i.e. uncertain thermal energy demand of a single building climate comfort

$$x_i \in \mathcal{X}_i(\delta_i) \text{ , for all } \delta_i \in \Delta_i \text{ and for all agents } i$$

2. Common uncertainty source: i.e. uncertain common resource pool between neighboring agents

$$x \in \bigcap_k \mathcal{X}_{c_k}(\delta_{c_k}) \text{ , for all } \delta_{c_k} \in \Delta_{c_k}$$

Private Uncertainty Source

Multi Agent Problem

$$\begin{aligned} \min_{x} \quad & \sum_{i} f_i(x_i) \\ \text{s.t.} \quad & x \in \bigcap_{\delta \in \mathcal{S}} \prod_{i} \mathcal{X}_i(\delta_i) \end{aligned}$$

Private Uncertainty Source

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Decomposable Problem

$$\begin{aligned} \min_x \quad & \sum_i f_i(x_i) \\ \text{s.t.} \quad & x \in \prod_i \bigcap_{\delta_i \in \mathcal{S}_i} \mathcal{X}_i(\delta_i) \end{aligned}$$

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Requirements

- Decomposable uncertainty source:

$$\delta := [\delta_1, \cdots, \delta_i, \delta_j, \cdots] \qquad \text{and} \qquad \mathcal{S} := \prod_i \mathcal{S}_i$$

Private Uncertainty Source

Single Agent Problem

$$\min_{x_i} \quad f_i(x_i)$$
$$\text{s.t.} \quad x_i \in \bigcap_{\delta_i \in \mathcal{S}_i} \mathcal{X}_i(\delta_i)$$

Probabilistic Feasibility for Single Agent Problem*

Fix $\varepsilon_i \in (0,1)$, $\beta_i \in (0,1)$ and $\mathcal{S}_i$, then

$$\mathbb{P}^{|\mathcal{S}_i|}\left(\mathcal{S}_i \in \Delta_i^{|\mathcal{S}_i|} \ : \ \mathbb{P}\Big(\delta_i \in \Delta_i \ : \ x^*_{\mathcal{S}_i} \notin \mathcal{X}_i(\delta_i)\Big) \leq \varepsilon_i\right) \geq 1 - \beta_i$$

*[Calafiore & Campi, TAC 2006]

Private Uncertainty Source

Multi Agent Problem

$$\begin{aligned} \min_{x} \quad & \sum_i f_i(x_i) \\ \text{s.t.} \quad & x \in \bigcap_{\delta \in \mathcal{S}} \prod_i \mathcal{X}_i(\delta_i) \end{aligned}$$

Probabilistic Feasibility for Multi Agent Problem*

If $\varepsilon = \sum_i \varepsilon_i \in (0,1)$, $\beta = \sum_i \beta_i \in (0,1)$ and given $\mathcal{S}$, then

$$\mathbb{P}^{|\mathcal{S}|}\left(\mathcal{S} \in \Delta^{|\mathcal{S}|} \ : \ \mathbb{P}\Big(\delta \in \Delta \ : \ x^*_{\mathcal{S}} \notin \prod_i \mathcal{X}_i(\delta_i)\Big) \le \varepsilon\right) \ge 1 - \beta$$

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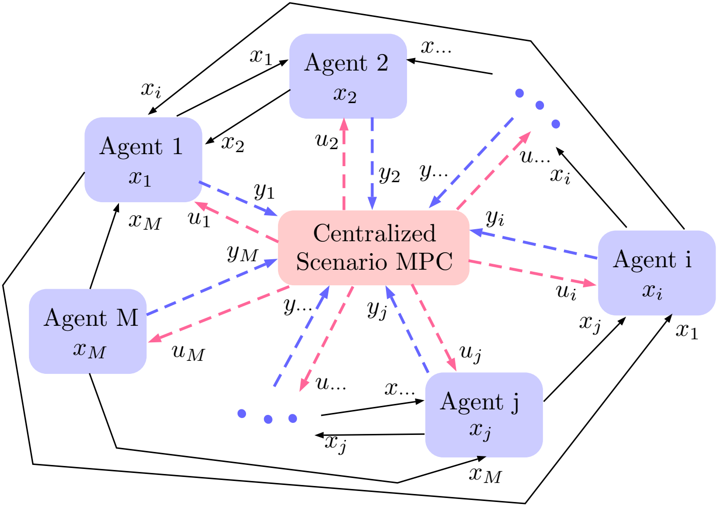
Proposed Distributed Implementation

Dynamically Coupled Systems



Proposed Distributed Implementation

Dynamically Coupled Systems

Centralized Scenario Program

$$
\begin{aligned}
\min_{\{\boldsymbol{u}_i\}_{\forall i \in \mathcal{N}}} \quad & \sum_{i \in \mathcal{N}} f_i(x_{i,k}, u_{i,k}) \\
\text{s.t.} \quad & x^{(i)}_{i,k+1} = A_{ii} x^{(i)}_{i,k} + B_i u_{i,k} + C_i \delta^{(i)}_{i,k} + \sum_{j \in N_i} A_{ij} x^{(i)}_{j,k} \ , \ x^{(i)}_{i,k} = x_{i,0} \\
& x^{(i)}_{i,k+\ell} \in \mathcal{X}_i \ , \quad \forall \ell \in \mathbb{N}_+ \ , \quad \forall \delta^{(i)}_{i,k} \in \mathcal{S}_{\delta_i} \\
& u_{i,k} \in \mathcal{U}_i \ , \quad \forall k \in \mathcal{T} \ , \forall i \in \mathcal{N}
\end{aligned}
$$

Proposed Distributed Implementation

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$$u_{i,k} \in \mathcal{U}_i \ , \quad \forall k \in \mathcal{T} \ , \forall i \in \mathcal{N}$$

Proposed Distributed Implementation

Dynamically Coupled Systems

Distributed Scenario Program

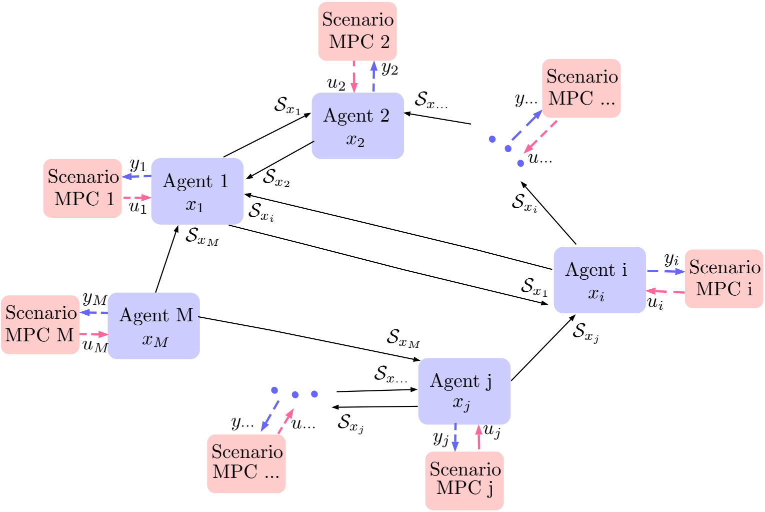
$$\begin{aligned} \min_{\{u_{i,k}\}_{\forall k \in \mathcal{T}}} \quad & f_i(x_{i,k}, u_{i,k}) \\ \text{s.t.} \quad & x^{(i)}_{i,k+1} = A_{ii} x^{(i)}_{i,k} + B_i u_{i,k} + q^{(i)}_{i,k} \ , \ x^{(i)}_{i,k} = x_{i,0} \\ & x^{(i)}_{i,k+\ell} \in \mathcal{X}_i \ , \quad \forall \ell \in \mathbb{N}_+ \ , \quad \forall q^{(i)}_{i,k} \in \mathcal{S}_{q_i} \\ & u_{i,k} \in \mathcal{U}_i \ , \quad \forall k \in \mathcal{T} \end{aligned}$$

Requirements

$$\mathcal{S}_{q_i} = \left\{ q^{(i)}_{i,k} \ : \ q^{(i)}_{i,k} = C_i \delta^{(i)}_{i,k} + \sum_{j \in N_i} A_{ij} x^{(i)}_{j,k} \ , \ \forall \delta^{(i)}_{i,k} \in \mathcal{S}_{\delta_i} \ , \ \forall x^{(i)}_{j,k} \in \mathcal{S}_{x_j} \right\}$$

Proposed Distributed Implementation

Dynamically Coupled Systems



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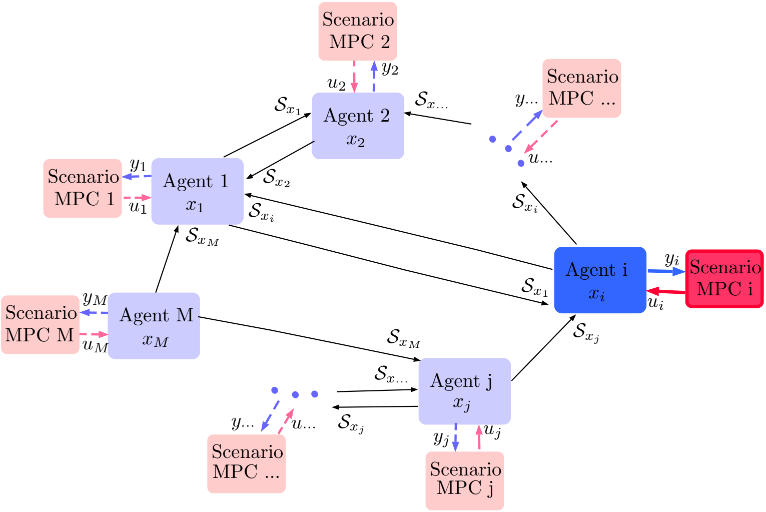

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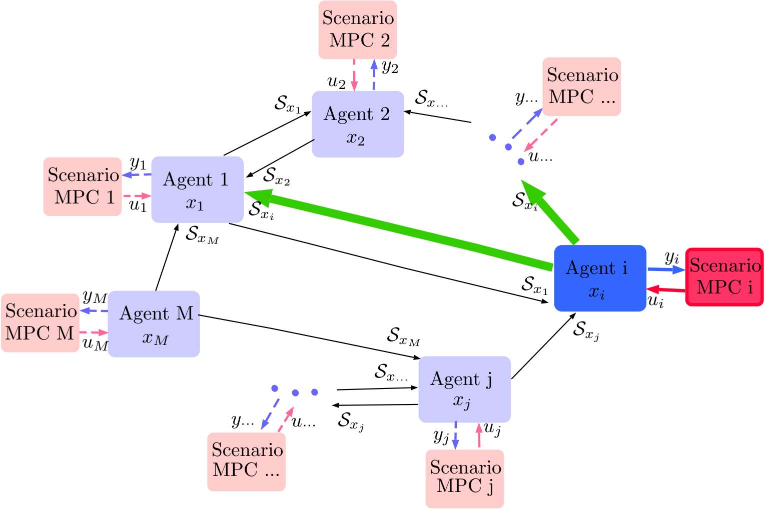

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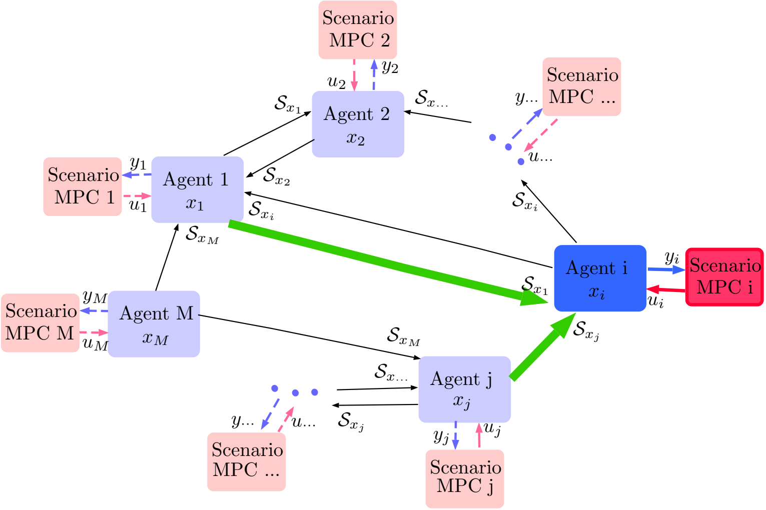

Outline

1. Centralized Framework
2. Distributed Framework
3. Soft Communication Scheme
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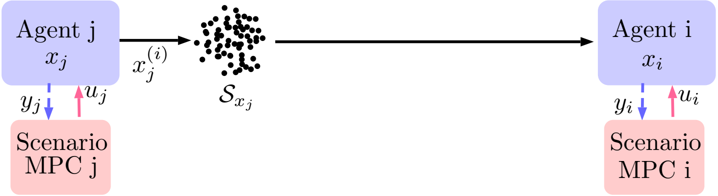
Communication Scheme

We consider two way to communicate between neighboring agents:

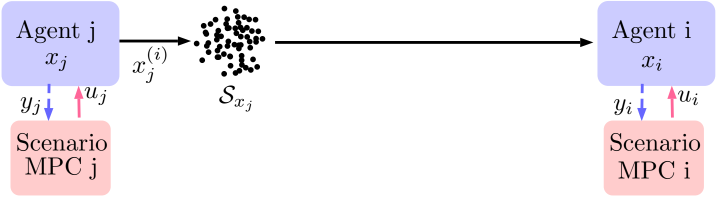
- Hard communication scheme: i.e. agent j has to send exactly the set $\mathcal{S}_{x_j}$ with cardinality N_{s_i} as it is requested by agent i



Communication Scheme

We consider two way to communicate between neighboring agents:

- Soft communication scheme: i.e. agent j sends a parametrized set $\tilde{\mathcal{B}}_j$ with its desired level of reliability $\tilde{\alpha}_j$ to agent i

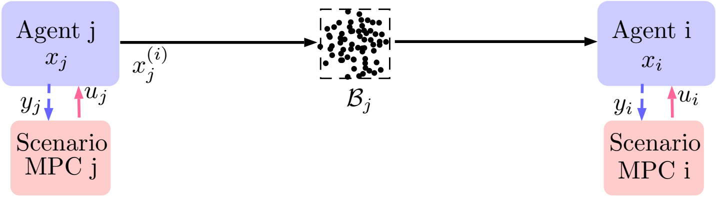


▶ It is an interest of agent j to decide about number of scenarios $\tilde{N}_j$ ◀

Communication Scheme

We consider two way to communicate between neighboring agents:

- Soft communication scheme: i.e. agent j sends a parametrized set $\tilde{\mathcal{B}}_j$ with its desired level of reliability $\tilde{\alpha}_j$ to agent i



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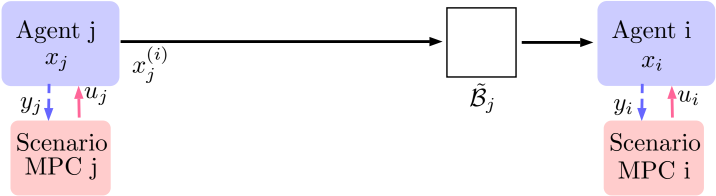

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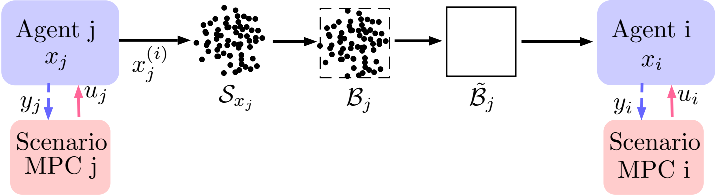
Soft Communication Scheme



Definition

A set $\tilde{\mathcal{B}}_j \subseteq \mathbb{R}^{m_j}$ is $\tilde{\alpha}_j-$reliable if

$$\mathbb{P}\left(\boldsymbol{x}_j \in \mathcal{X}_j \;:\; \boldsymbol{x}_j \notin \tilde{\mathcal{B}}_j\right) \leq 1 - \tilde{\alpha}_j \ ,$$

and we refer to $\tilde{\alpha}_j$ as the level of reliability of the set $\tilde{\mathcal{B}}_j$.

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Theorem*: how one can determine $\tilde{\alpha}_j$?

Fix $\tilde{\beta}_j \in (0,1)$, $\tilde{N}_{s_i}$ and let

$$\tilde{\alpha}_j = \sqrt[\tilde{N}_{s_i} - m_j]{\frac{\tilde{\beta}_j}{\binom{\tilde{N}_{s_i}}{m_j}}} \;.$$

We then have $\mathbb{P}\left(\boldsymbol{x}_j \in \mathcal{X}_j \;:\; \boldsymbol{x}_j \notin \tilde{\mathcal{B}}_j\right) \leq 1 - \tilde{\alpha}_j$, with prob. $1 - \tilde{\beta}_j$.

*[Rostampour et. al., 2017]

Soft Communication Scheme

Probabilistic Feasibility Certificate

The communicated information are reliable with certain level of probability. How one can accommodate such a probabilistically reliable information in the probabilistic feasibility certificate of the local agent?

Theorem*

Given $\tilde{\alpha}_j \in (0,1)$ and a fixed $\alpha_i \in (0,1)$, the state trajectory of a generic agent i is probabilistically $\bar{\alpha}_i$–feasible for all $\delta_i \in \Delta_i$, i.e.,

$$\mathbb{P}\left(x_{i,k+\ell} \in \mathcal{X}_i \, , \, \ell \in \mathbb{N}_+ \right) \geq \bar{\alpha}_i \, ,$$

where $\bar{\alpha}_i = 1 - \frac{1-\alpha_i}{\tilde{\alpha}_i}$ such that $\tilde{\alpha}_i = \prod_{j \in \mathcal{N}_i} (\tilde{\alpha}_j)$.

*[Rostampour et. al., 2017]

Outline

1. Centralized Framework
2. Distributed Framework
3. Soft Communication Scheme
4. Case Study: STGs with ATES Systems
5. Conclusions and Future work

Energy Management Problem

Optimization Problem

$$\begin{aligned} \min \quad & \text{thermal energy imbalance error} + \text{cost of equipment operation} \\ \text{s.t.} \quad & \text{1) equipment limits} \\ & \text{2) imbalance error dynamics} \\ & \text{3) ATES system dynamics} + \text{local thermal energy balance} \\ & \text{4) coupling constraint on the thermal radius between agents} \end{aligned}$$

[Rostampour & Keviczky, IFAC, 2017]

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Compact Form — x: Decision Variables*

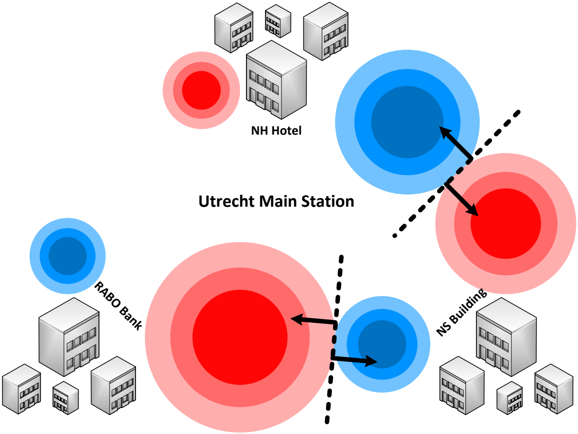
$$\min_{x_i} \quad \sum_i f_i(x_i) \qquad \longrightarrow \qquad f_i(\cdot) \; : \; \text{objective function of agent } i$$

$$\text{s.t.} \quad x_i \in \mathcal{X}_i(\delta_i) \, , \; \forall\, i \qquad \longrightarrow \qquad \mathcal{X}_i \; : \; \text{constraint set of agent } i$$

$$x \in \bigcap_k \mathcal{X}_{c_k}(\delta_{c_k}) \qquad \longrightarrow \qquad \mathcal{X}_{c_k} \; : \; \text{coupling constraint set}$$

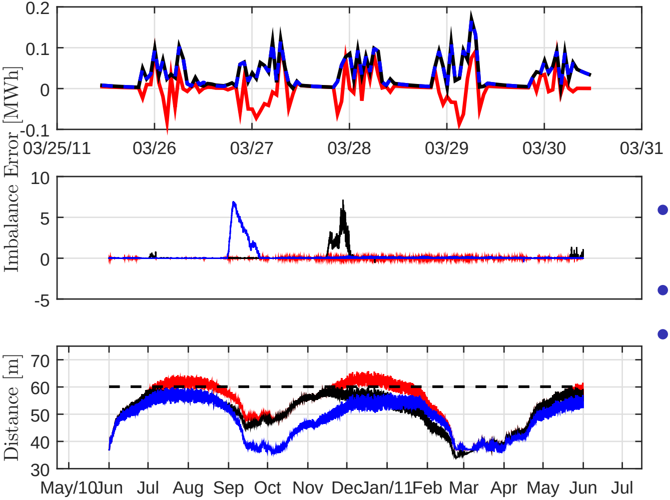
*set-points for control units of buildings and pump flow rate for ATES systems

STGs with ATES Systems



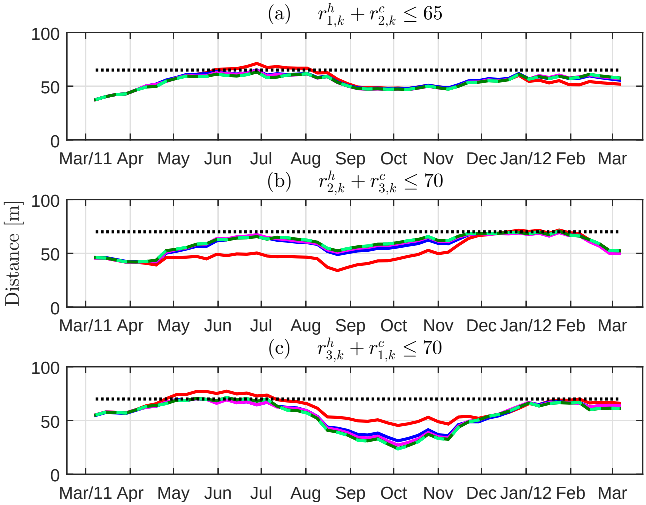
- Three buildings in Utrecht city with real parameters
- Real weather condition data from 2010 to 2012

STGs with ATES Systems



- Deterministic Decoupled MPC
- Centralized SMPC
- Move-Blocking Centralized SMPC

STGs with ATES Systems



- Decoupled SMPC
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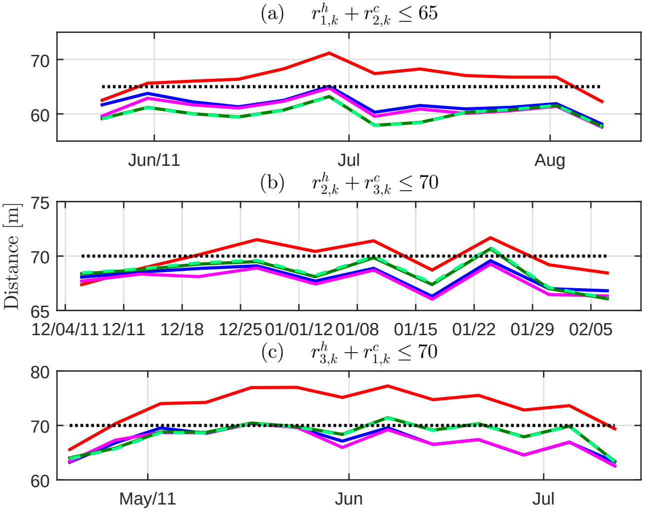
- DSMPC—0.85
- DSMPC—0.50

STGs with ATES Systems



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Concluding Remarks

- Distributed randomized optimization to deal with private (local) uncertainty source over a network of dynamically coupled systems
- Soft communication scheme with an extension of probabilistic feasibility guarantee
- Application to energy management of smart thermal grids (STGs) with aquifer thermal energy storage (ATES) system

Future Work

What comes next?

- Distributed randomized optimization for weakly coupled problems together with common uncertainty source
- Preserving privacy of individual agents in a network
- Distributed randomized optimization for hierarchical decision making in uncertain dynamical environment
- More applications — distributed (multi-area) reserve scheduling and optimal power flow over AC power networks

Acknowledgments

Thank you for your attention!
Questions?

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v.rostampour@tudelft.nl

Distributed Randomized Optimization for Large Scale Interconnected Systems

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April 12, 2017