A Set Based Probabilistic Approach to Threshold Design for Optimal Fault Detection

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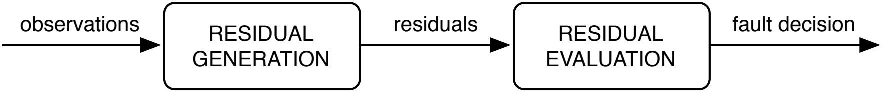
Model Based Fault Detection

General Concepts

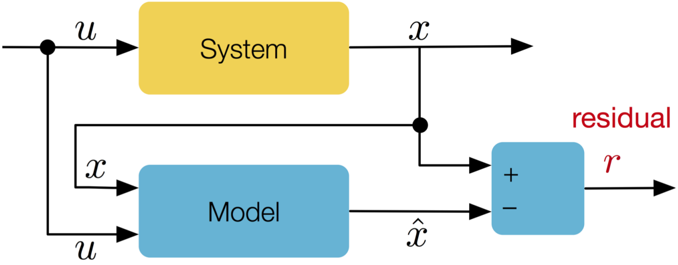
- The basis of model based fault detection concept is:



Model Based Fault Detection

General Concepts: Residual Generation

- A basic point is to have observer that generate residual:

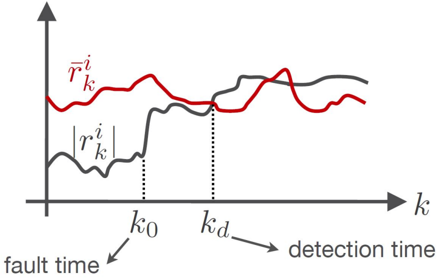


Model Based Fault Detection

General Concepts: Residual Evaluation

Define the residual signal to be as: $$r_k = y_k - \hat{x}_k$$

- $\bar{r}_k$: corresponding threshold
- faults can be detected by comparing $|r_k|$ with $\bar{r}_k$
- the threshold is needed to be robust w.r.t. uncertainties



Detectability Concept

A fault will be detected if the absolute value of at least one component of the residual crosses the threshold.

Why Go Set Based Probabilistic Residual Evaluation?

From Research Questions To Contributions

1. How one can develop a fault detection framework for **uncertain nonlinear systems**?
2. What sort of requirements are necessary to formalize the level of **robustness in a probabilistic sense**?
   - Existing approaches are based on the **robust notation of thresholds** w.r.t the uncertainties. How one can **relax** such a condition?
   - What are the requirements about the uncertainty sources?
   - Can we relax such a **requirements (assumptions)**?
3. How one can achieve to a **notation for the level of detectability** as a design variable?

Outline

1. Residual Generation Setup
2. Residual Evaluation Frameworks
3. Simulation Study
4. Conclusions

Uncertain Nonlinear Systems Fault Detection

- Uncertain nonlinear system under fault: additive terms of system **nominal behavior, uncertainties, and fault functions**

$$x_{k+1} = g(x_k, u_k) + \eta(x_k, u_k, w_k) + \phi(x_k, u_k, f_k)$$

$$y_k = Hx_k + v_k$$

nominal: $g(x_k, u_k)$, Hx_k

uncertainties: $\eta(x_k, u_k, w_k)$, v_k

fault function: $\phi(x_k, u_k, f_k)$

- w_k , v_k are two independent random variables

Problem Statement

1. In the presence of w_k , v_k with unknown distribution and no specific structure, can we detect a faulty parameter from uncertainties in a fairly enough general class of dynamical (nonlinear) system?
2. If so, can we develop a formalize flexible framework for such a problem?

Uncertain Nonlinear Systems Fault Detection

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Residual Generator System Dynamics

(1) State estimator can be built such as:

$$\hat{x}_{k+1} = g(y_k, u_k) + \Lambda\left(\hat{x}_k - y_k\right)$$

(2) The residual dynamics obey the following equation:

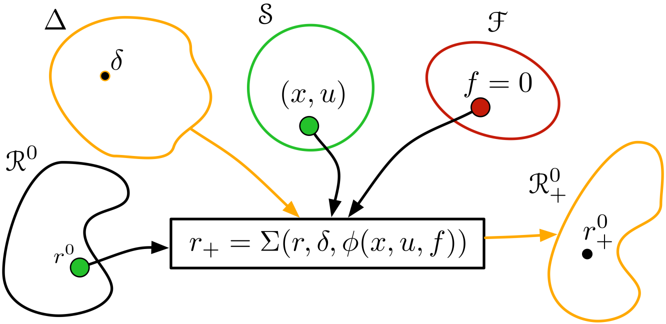
$$r_{k+1} = y_{k+1} - \hat{x}_{k+1} = \Lambda r_k + \underbrace{\delta_k}_{\text{total uncertainty}} + \underbrace{\phi(x_k, u_k, f_k)}_{\text{fault function}} := \Sigma(r_k, \delta_k, \phi(x_k, u_k, \underbrace{f_k}_{\text{fault parameter}}))$$

⇒ δ_k represents all the sources of uncertainty (model and output):

$$\delta_{k+1} = \underbrace{g(x_k, u_k) - g(y_k, u_k)}_{\text{effect of output uncertainty on nominal dynamics}} + \underbrace{\eta(x_k, u_k, w_k)}_{\text{model uncertainty}} + \underbrace{v_{k+1}}_{\text{output uncertainty}}$$

Set Based Threshold Design

- It is interesting to consider the **set of healthy residuals** that one can produce at next time



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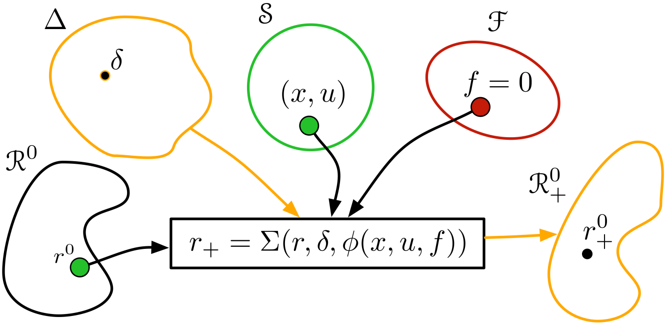

δ is a **random variable** and that leads to r_+ is a **random variable**

- We are therefore able to generate many samples of healthy residuals:

$$r_+^{0,(i)} = \Sigma(r, \delta^{(i)}, \phi(x, u, 0)) \ , \qquad \delta^{(i)} \in \Delta$$

A **deterministic threshold** should contain **all possible healthy residuals**

Outline

Existing Approaches for Residual Evaluation

- Deterministic threshold sets are **overly-conservative** and **limit performance**; Using sets, we can easily visualize why this is true!

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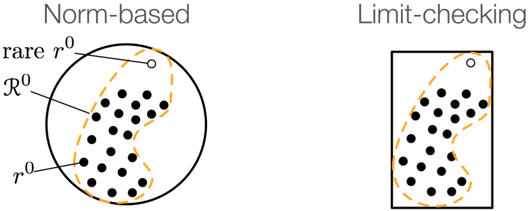
- Their shape may not be **tight** enough and they may be overly **influenced** by **large** and **rare** values of the uncertainty



(a) $\|r\| \leq \tau$ (b) $|r_{(i)}| \leq \tau_{(i)}$

Proposed Residual Evaluation

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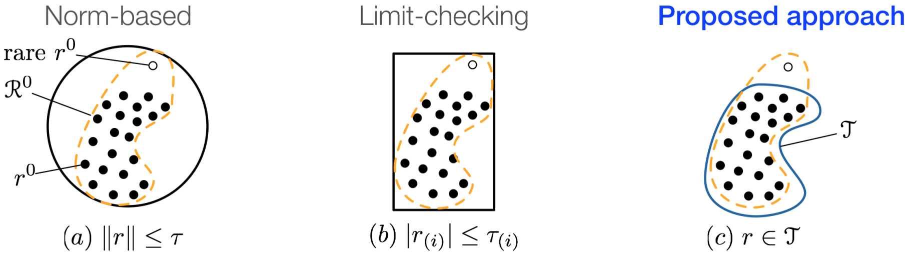


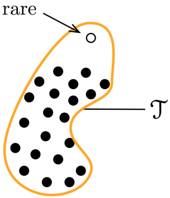
- We therefore proposed a **probabilistic approach** for determining **set-based thresholds** with shapes that can be as tight as desired

Proposed Probabilistic Threshold Set

Polynomial Level Sets

- We define $\mathcal{T}$ (threshold set) as the $c-$**superlevel set** using a suitable **parametrized indicator function** $\mathcal{I}_{\mathcal{T}}$:

$$\mathcal{T}_k := \{r \in \mathbb{R}^n \mid \mathcal{I}_{\mathcal{T}}(r, \theta_k) \geq c\}$$



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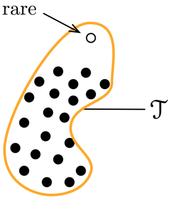
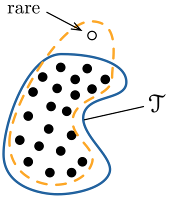


$(c)\ r \in \mathcal{T}$

- We next introduce the concept of **probabilistically** $\alpha-$**robust** threshold set:

$$\mathcal{V}(\mathcal{T}_+) := \mathbb{P}[r_+^0 \notin \mathcal{T}_+] \leq 1 - \alpha$$

$\Rightarrow$ Then the **optimal threshold** is the **smallest** with a given **robustness probability**



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Proposed Cascade Framework

(I): The optimal threshold problem can be formulated as:

$$\begin{cases} \min\limits_{\theta,\gamma} & \gamma \\ \text{s.t.} & \textbf{vol}\ \mathcal{T} \leq \gamma \quad \longrightarrow \text{ min-volume constraint} \\ & \mathcal{V}(\mathcal{T}) \leq 1 - \alpha \quad \longrightarrow \text{ chance constraint} \end{cases}$$

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(II): Next, the sensitivity w.r.t. faulty residuals can be maximized:

$$\begin{cases} \max\limits_{\theta} & \|\mathcal{T} - \mathcal{R}^{\mathcal{F}'}\|_{\infty} \quad \longrightarrow \ \text{max-distance from faulty samples} \\ \text{s.t.} & \textbf{vol}\ \mathcal{T} \leq \gamma^* \quad \longrightarrow \ \text{not worse than (I)} \\ & \mathcal{V}(\mathcal{T}) \leq 1 - \alpha \quad \longrightarrow \ \text{chance constraint} \end{cases}$$

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$\Rightarrow$ **chance-constrained** problems are **non-convex** and hard to solve

Proposed Cascade Framework

Sample Complexity in Cascade Setup

- We use **randomization technique** to obtain a tractable formulation by replacing the chance constraint with sample-based hard constraints
- **How many samples are enough to provide equivalent properties?**

Theorem: Probabilistic Guarantee for Cascade Setup

Fix α, β, and determine $N \geq N(\alpha, \beta, \ell)$, then, the obtained threshold set is $\alpha-$robust threshold set with high confidence level $1-\beta$, in the average.

$$N(\alpha, \beta, \ell) := \min \left\{ N \in \mathbb{N} \,\middle|\, d \sum_{i=0}^{\ell-1} \binom{N}{i} (1-\alpha)^i \alpha^{N-i} \leq \beta \right\}$$

number of samples (N)

required probability of robustness (α)

size of solution (ℓ)

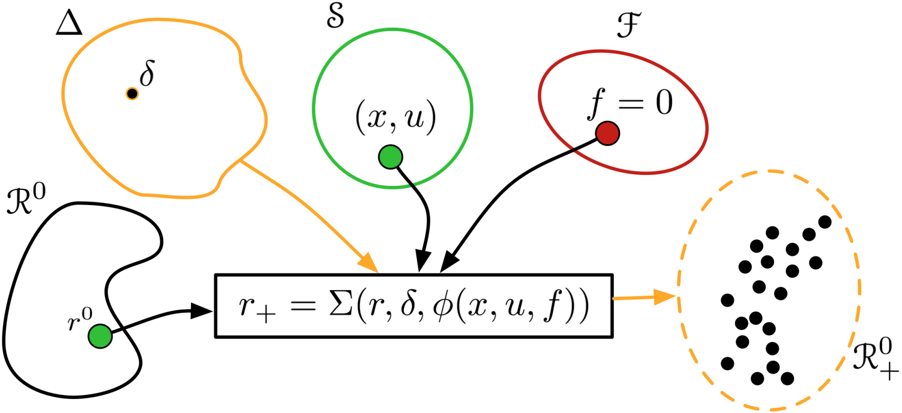
size of indicator function parameters (d)

desired level of robustness (α)

desired level of confidence (β)

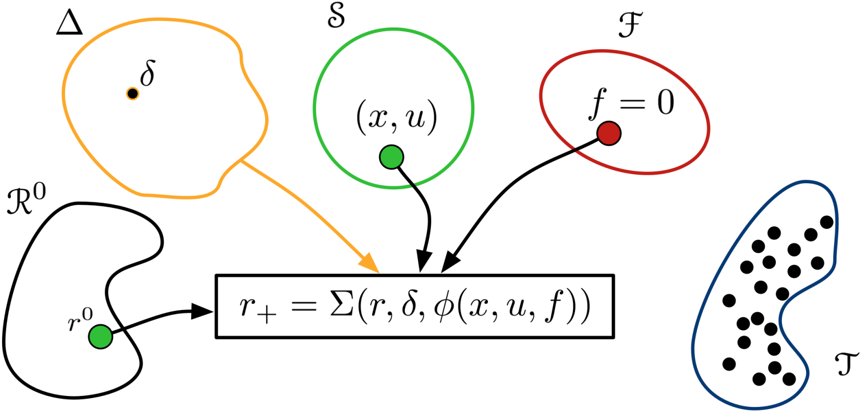
Proposed Cascade Framework — Visualization

Generating Samples of Healthy Residuals



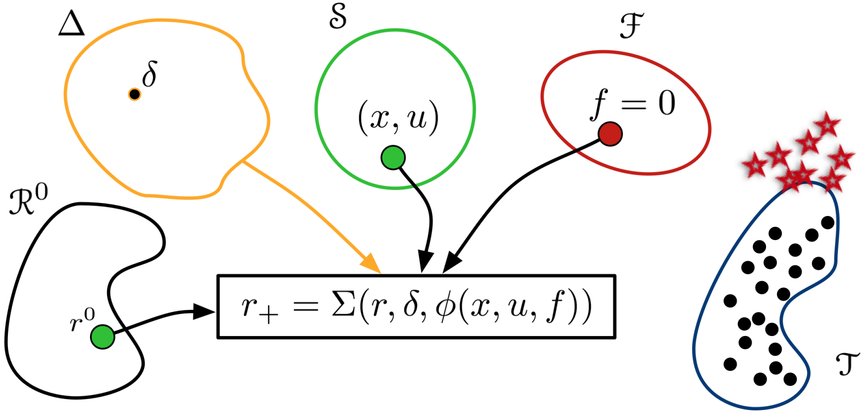
Proposed Cascade Framework — Visualization

Minimizing Volume of Polynomial Level Set: Solution of Problem (I)



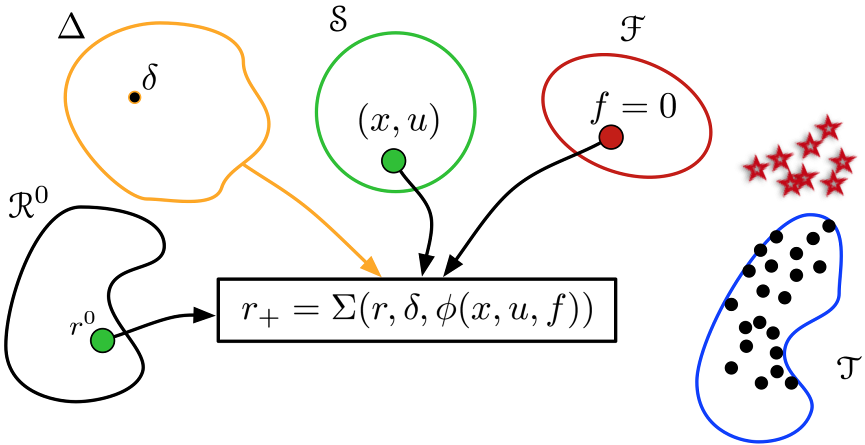
Proposed Cascade Framework — Visualization

Determining of Faulty Residual Set



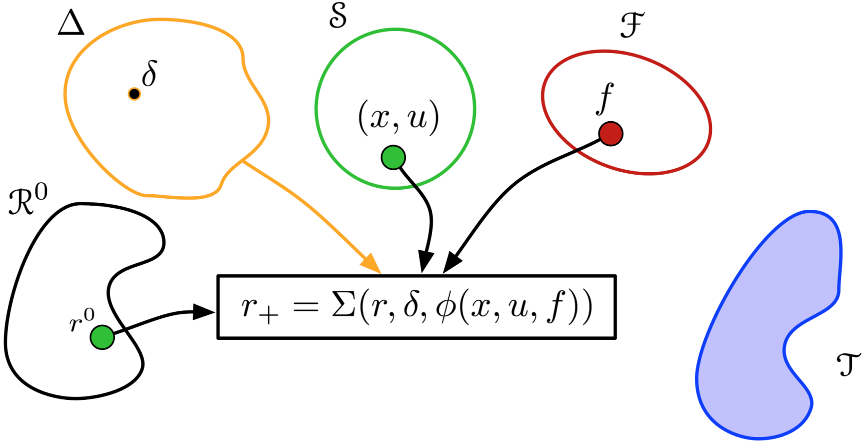
Proposed Cascade Framework — Visualization

Maximizing Sensitivity w.r.t. Faulty Residual Set: Solution of Problem (II)



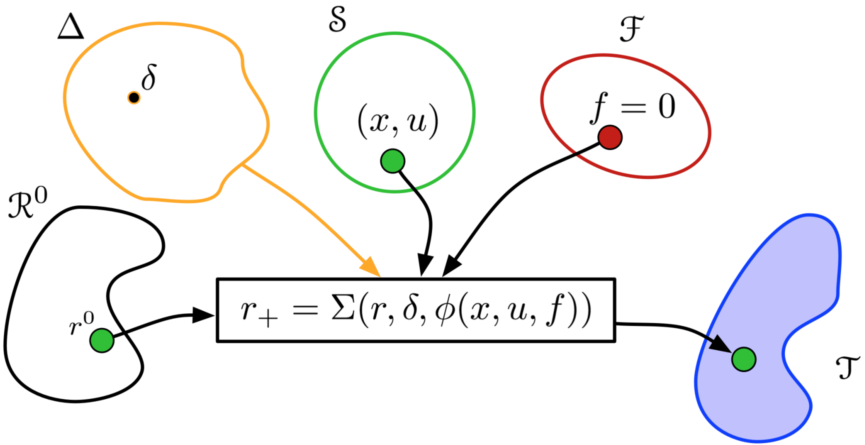
Proposed Cascade Framework — Visualization

Implementation Scheme



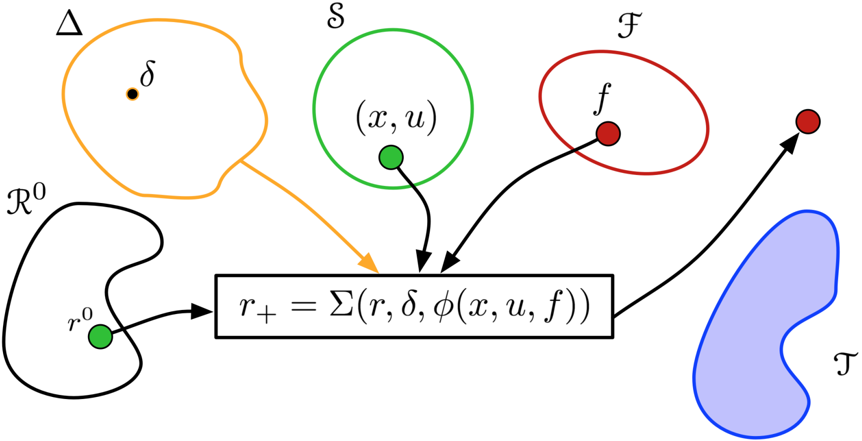
Proposed Cascade Framework — Visualization

Implementation Scheme: Healthy Residual



Proposed Cascade Framework — Visualization

Implementation Scheme: Faulty Residual

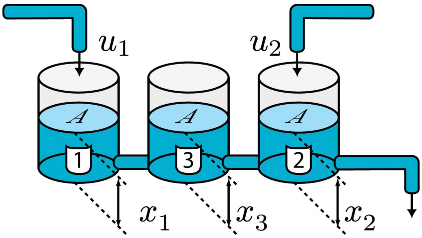


Outline

Benchmark Case Study: Three-Tank System[1]

- **Uncertainties source is model mismatch:** tanks and pipes' cross section and outflow coefficient
- **Fault classes:** the first or second pump shut down, leakage in the first tank
- A **fault corresponding to a reduction in the inflow** provided by the first pump is introduced

classical nonlinear system used as a **benchmark in FDI**

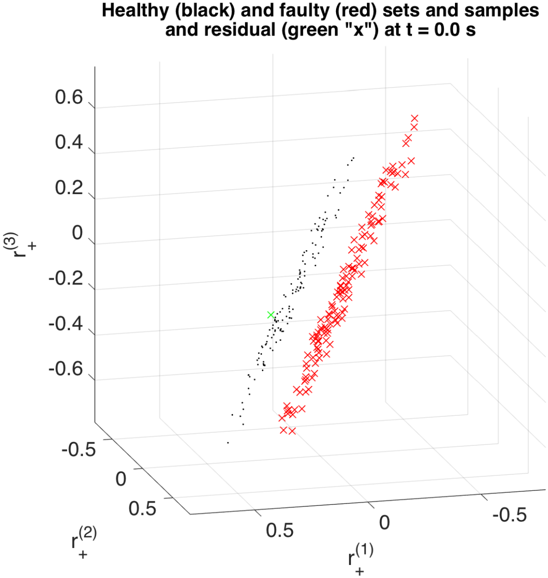


nominal dynamics can be easily written as a discrete-time nonlinear

[1] [R. Ferrari, et al. ACC 2008]

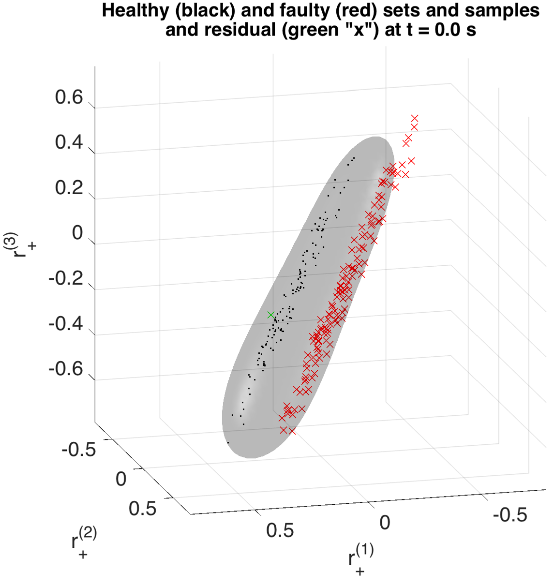
Results of Proposed Cascade Framework

(1) Healthy and Faulty Residual Samples



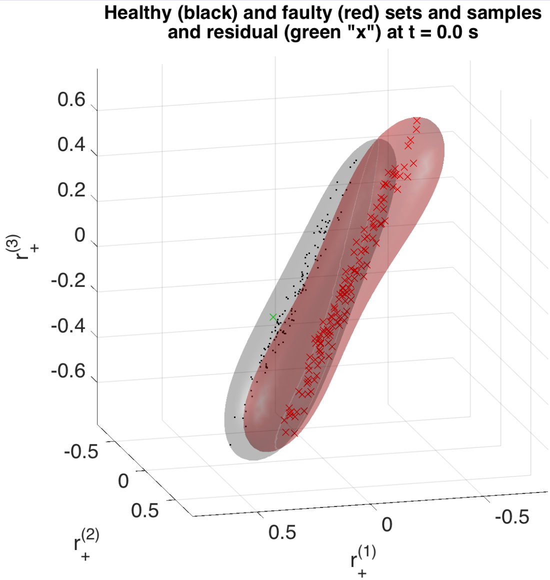
Results of Proposed Cascade Framework

(2) Polynomial Threshold Set Found in Problem I



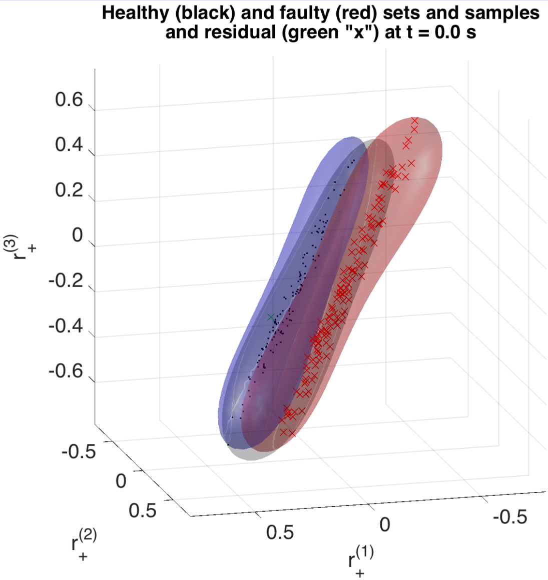
Results of Proposed Cascade Framework

(3) Polynomial Faulty Residual Set Used in Problem II



Results of Proposed Cascade Framework

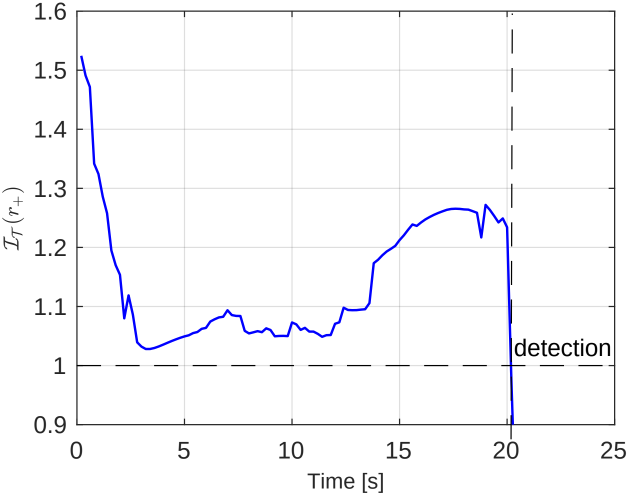
(4) Updated Polynomial Threshold Set Found in Problem II



Results of Proposed Residual Evaluation

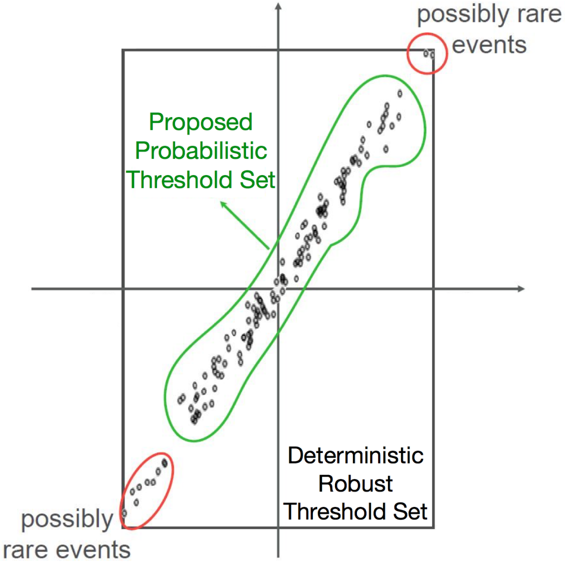
- Values of the proposed threshold set computed at the residual
- The fault is introduced at $T_f = 20s$ with sampling time $T_s = 0.1s$

- Values ≥ 1: **healthy**
- Values ≤ 1: **faulty**



Ratio of Volume of Threshold Sets

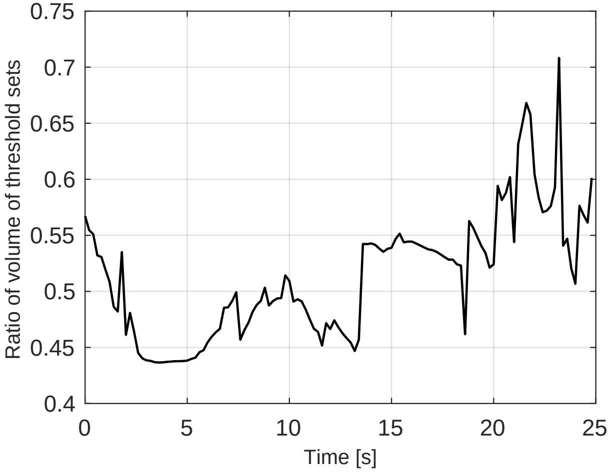
Polynomial vs. Rectangle



Ratio of Volume of Threshold Sets

Polynomial vs. Rectangle

Volume of proposed threshold is clearly smaller; **higher detectability**



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Concluding Remarks

Remarks:

1. Developed a novel approach to the design of fault detection thresholds for uncertain nonlinear systems
   - **Pros:** desired level of **false-alarm** and achievable level of **missed-detection**; **theoretically proven achievements**
   - **Cons:** in case of time-varying uncertainty sources, it requires to generate the required residual samples at each time step; **computationally demanding**
2. Provided a-priori probabilistic guarantees on the performance level of fault detection; this is **an extension of the existing results to the cascade setup**
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What comes next:

- Extending to fault isolation, identification, and tolerant control

Questions?

Thank you for your attention!

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